Coupling of a windmill to a single acting piston pump by means of a crank mechanism

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Contains page

1 Introduction

During the period in between 1975 and 1990 I have worked as a designer of water pumping windmills at the former Wind Energy Group of the University of Technology Eindhoven. The Wind Energy Group was a member of the former CWD, Consultancy services Wind energy Developing countries financed by the Dutch Ministry of Developing Countries. A lot of research was done by CWD to all aspects of water pumping windmills and hundreds of reports were written. However, these reports are difficult to obtain now.

It appeared that development of a good water pumping windmill which can be manufactured in developing countries from local materials was very difficult and even after 15 years of development not all problems with our windmills were solved. CWD was closed down in 1990 because the ministry stopped financing it.

I started my one man engineering office Kragten Design in 1989 and I specialised in the development of small electricity generating windmills because water pumping was much too complex for a one man office. But I did some research to water pumping using an electricity generating windmill driving an electrical pump or using a windmill which drives a rotating pump by means of an accelerating transmission in the head and a vertical shaft in the tower. Personally I prefer these systems more than a windmill which drives a single acting piston pump. But there are still people who want to develop windmills with piston pumps and as they have no longer easy access to the reports written in the CWD time, I thought it is necessary that at least the very basic information about coupling of a windmill to a piston pump is available in a KD-report which can easily be sent by E-mail.

2 Description of the system

Traditional water pumping windmills have a multibladed rotor with a very low design tip speed ratio λ_d of about 1. The design tip ratio is the ratio in between the speed of the blade tip and the undisturbed wind speed for which maximum power is extracted from the wind. The rotor is driving a crank shaft by a reducing transmission using gear wheels turning in an oil bath. A crank mechanism with a connecting rod is used to transform the rotating movement of the crank into an oscillating movement of the pump rod. The speed of the pump rod is varying almost sinusoidal if the length of the connecting rod is long with respect to the length of the crank.

For the CWD windmills and many other recently designed water pumping windmills, the reducing transmission is cancelled so the crank is directly mounted to the rotor shaft. To reduce the weight of the rotor, a design tip speed ratio of about 2 is chosen. The rotational speed of the crank is therefore much higher than for traditional windmills. This has as main advantage that the pump dimensions are much smaller for a certain rotor diameter and water height. However, it has as main disadvantage that shock forces in the transmission are much higher. A rotor with λ_d is 2 has a much lower starting torque coefficient than a rotor with $\lambda_d = 1$ and this results in a higher starting wind speed except if some provision is made to reduce the peak torque of the pump at very low rotational speeds.

The pump is made of a cylinder in which the piston is moving. The upper part of the cylinder is connected to the raising main through which the water flows to the outlet pipe. The lower part of the cylinder contains the foot valve. The piston contains the piston valve and a leather cup to prevent water leaking along the piston. Most pumps are mounted under the water level of the well but some pumps have a suction pipe and are mounted some meters above the water level.

If the piston moves upwards, the foot valve is open and the piston valve is closed so the water is lifted upwards. If accelerating forces are neglected, the required force to lift the water is constant during this upwards stroke. This constant force requires a torque in the pump shaft which is varying sinusoidal and which has a maximum if the crank is in the horizontal position.

If the piston moves downwards, the foot valve is closed and the piston valve is open. If the friction in between the leather cup and the pressure drop over the piston valve is neglected, the force during the downwards stroke is zero. This means that the torque during the downwards stroke is zero too. It can be proven that the average torque during one revolution of the rotor shaft is $1/\pi$ times the peak torque which occurs if the crank is horizontal. So the peak torque is π times the average torque. Because of this high peak torque, a rotor with a high starting torque coefficient is required other wise the starting wind speed will be too high.

On the end of the CWD time we have developed a pump with a floating valve at the piston. This valve closes only at a certain speed of the water, flowing in between the valve and the valve seat. Therefore almost no torque is required for the upwards stroke at low rotational speeds and this results in a much lower starting wind speed and allows the use of rotors with a rather low starting torque coefficient.

3 Deriving of the formulas for windmill and pump

The general wind energy theory and the design procedure for a windmill rotor is given in report KD 35 (ref. 1). The rotor is characterised by the C_p-λ or the C_q-λ curve. C_p is the power coefficient, C_q is the torque coefficient and λ is the tip speed ratio. The rotor power P for a rotor perpendicular to the wind is given by formula 4.1 of KD 35 copied as formula 1.

$$P = C_p * ½\rho V^3 * \pi R^2 \qquad \text{(W)} \qquad (1)$$

The rotor torque Q for a rotor perpendicular to the wind is given by formula 4.3 of KD 35 copied as formula 2.

$$Q = C_q * ½\rho V^2 * \pi R^3 \qquad \text{(Nm)} \qquad (2)$$

In these formulas ρ is the density of air (about 1.2 kg/m^3 for a temperature of 20° C at sea level), V is the undisturbed wind speed (m/s) and R is the radius of the rotor at the blade tip (m). R is half the rotor diameter. The relation in between λ, C_p and C_q is given by formula 4.5 of KD 35 copied as formula 3.

$$\lambda = C_p / C_q \qquad \text{(-)} \qquad (3)$$

This means that the C_p-λ curve can be derived from the C_q-λ curve by multiplying every C_q value by λ. It also means that the C_q-λ curve can be derived from the C_p-λ curve by dividing every C_p value by λ (except for λ = 0). The rotational speed n for a rotor perpendicular to the wind is given by formula 4.8 of KD 35 copied as formula 4.

$$n = 30 * \lambda * V / \pi R \qquad \text{(rpm)} \qquad (4)$$

The rotational speed of the crank shaft or pump shaft n_p is given by:

$$n_p = n / i \qquad \text{(rpm)} \qquad (5)$$

In this formula i is the reducing gear ratio in between rotor shaft and pump shaft. The mechanical power supplied to the pump P_p is a little lesser than the rotor power because of the transmission efficiency η_{tr}. This gives:

$$P_p = \eta_{tr} * P \qquad \text{(W)} \qquad (6)$$

The transmission efficiency for a transmission without gear wheels is very high.

The hydraulic power P_{hyd} supplied to the water is lesser than the mechanical pump power because of the pump efficiency η_p. This gives:

$$P_{hyd} = \eta_p * P_p \qquad (W) \qquad (7)$$

The pump efficiency is caused by the friction of the leather cup, by the hydraulic losses at the valves and by the hydraulic losses in the suction and pressure pipes. For long and narrow pipes the pipe losses have to be taken separately from the pump losses but I take them as a part of the pump losses. This means that the static pressure height is taken as the total height.

(6) + (7) gives:

$$P_{hyd} = \eta_{tr} * \eta_p * P \qquad (W) \qquad (8)$$

(1) + (8) gives:

$$P_{hyd} = \eta_{tr} * \eta_p * C_p * ½\rho V^3 * \pi R^2 \qquad (W) \qquad (9)$$

The theoretical stroke volume of the pump ∇_{th} is given by:

$$\nabla_{th} = \pi/4\ D_p^2 * s \qquad (m^3) \qquad (10)$$

In this formula D_p is the piston diameter (m) and s is the stroke (m). The stroke is twice the length of the crank. The real stroke volume ∇_{real} is less than the theoretical stroke volume because of the volumetric efficiency η_{vol}. This gives:

$$\nabla_{real} = \eta_{vol} * \nabla_{th} \qquad (m^3) \qquad (11)$$

The volumetric efficiency is caused by closing of the valves not exactly at the upper or the lower dead centre and by leaking of water along the piston and the valves. Only leaking of water results in decrease of the pump efficiency. If the pump is equipped with an elastic element in the suction or pressure pipe, the volumetric efficiency can even be larger than 1 at rotational speeds were resonance occurs in the elastic element. But I assume that η_{vol} is smaller than 1. For a pump equipped with a floating valve, η_{vol} is zero for very low rotation speeds where the valve doesn't close. It is in between 0.5 and almost 1 for higher rotational speeds. The pump flow q (m^3/s) is given by:

$$q = \nabla_{real} * f \qquad (m^3/s) \qquad (12)$$

In this formula f is the frequency of the pump rod (osc/s). The relation in between the frequency f and the rotational speed n_p (rpm) of the pump shaft is given by:

$$f = n_p / 60 \qquad (osc/s) \qquad (13)$$

(5) + (13) gives:

$$f = n / (i * 60) \qquad (osc/s) \qquad (14)$$

(10) + (11) + (12) + (14) gives:

$$q = \eta_{vol} * \pi/4 * D_p^2 * s * n / (i * 60) \qquad (m^3/s) \qquad (15)$$

The hydraulic power P_{hyd} is given by:

$$P_{hyd} = \rho_w * g * H * q \qquad (W) \qquad (16)$$

In this formula ρ_w is the density of water (ρ_w = 1000 kg/m^3), g is the acceleration of gravity (g = 9.81 m/s^2) and H is the static height between the water level in the well and the outlet opening of the pressure pipe.

(15) + (16) gives:

$$P_{hyd} = \rho_w * g * H * \eta_{vol} * \pi/4 * D_p^2 * s * n / (i * 60) \qquad (W) \qquad (17)$$

(4) + (17) gives:

$$P_{hyd} = \rho_w * g * H * \eta_{vol} * D_p^2 * s * \lambda * V / (8 * i * R) \qquad (W) \qquad (18)$$

(9) + (18) gives:

$$4 * i * \eta_{tr} * \eta_p * C_p * \rho V^2 * \pi R^3 = \rho_w * g * H * \eta_{vol} * D_p^2 * s * \lambda \qquad (19)$$

Formula 19 is the formula in which the rotor parameters, the wind speed, the gear ratio, the pump parameters and the height are linked together. The pump geometry is normally determined for the design wind speed V_d. For this wind speed the rotor runs at the design tip speed ratio λ_d and has its maximum C_p value $C_{p\,max}$. Substitution of $C_p = C_{p\,max}$, $V = V_d$ and $\lambda = \lambda_d$ in formula 19 gives that:

$$4 * i * \eta_{tr} * \eta_p * C_{p\,max} * \rho V_d^2 * \pi R^3 = \rho_w * g * H * \eta_{vol} * D_p^2 * s * \lambda_d \qquad (20)$$

Formula 20 can be written in different ways depending on what parameter one wants to calculate. The use of formula 20 or a changed version of it, will now be explained for two examples

3.1 Example 1

Assume a rotor with R = 2.5 m, λ_d = 2, $C_{p\,max}$ = 0.38.
Assume a direct drive transmission so i = 1, assume η_{tr} = 0.99, and s = 0.24 m.
Assume D_p = 0.15 m, H = 6 m, η_p = 0.9 and η_{vol} = 0.98.
ρ = 1.2 kg/m^3, ρ_w = 1000 kg/m^3 and g = 9.81 m/s^2.
What is the design wind speed for this combination of windmill rotor and pump?

Formula 20 can be written as:

$$V_d = \sqrt{\{\rho_w * g * H * \eta_{vol} * D_p^2 * s * \lambda_d / (4 * i * \eta_{tr} * \eta_p * C_{p\,max} * \rho * \pi R^3)\}} \qquad (m/s) \quad (21)$$

Substitution of the given values in formula 21 gives V_d = 2.79 m/s. This is a rather low wind speed except for regions with a very low wind regime. It indicates that the pump dimensions are rather small for the given rotor diameter and water height. If the calculation is done again for the same parameters except that H = 12 m, it is found that V_d = 3.95 m/s. This seems to be an acceptable value for a medium wind regime.

3.2 Example 2

Assume a rotor with R = 1.5 m, λ_d = 1, $C_{p\ max}$ = 0.34.
Assume a reducing transmission with i = 3.5, assume η_{tr} = 0.92 and s = 0.35 m.
Assume V_d = 4 m/s, H = 25 m, η_p = 0.9 and η_{vol} = 0.98.
ρ = 1.2 kg/m^3, ρ_w = 1000 kg/m^3 and g = 9.81 m/s^2.
What is the required piston diameter for this combination of windmill rotor and pump?

Formula 20 can be written as:

$$D_p = \sqrt{\{ 4 * i * \eta_{tr} * \eta_p * C_{p\ max} * \rho V_d^2 * \pi R^3 / (\rho_w * g * H * \eta_{vol} * s * \lambda_d)\}} \qquad \text{(m)} \qquad (22)$$

Substitution of the given values in formula 22 gives D_p = 0.0977 m = 97.7 mm. This is an acceptable piston diameter.

4 Matching of rotor and pump

In this chapter it will be explained for both examples of chapter 3 how rotor and pump will work together for other wind speed than the design wind speed. Therefore it is necessary to derive the Q-n curves of the rotor for different wind speeds. This requires the C_q-λ curve of the rotor, the δ-V curve of the safety system (δ is the yaw angle in between the rotor shaft and the wind direction) and the formulas for the torque and the rotational speed.

It is assumed that the windmill is provided with an ideal safety system with V_{rated} = 8 m/s. This means that the rotor is perpendicular to the wind for wind speeds lower than 8 m/s and that for higher wind speeds, it is turned out of the wind such that the component of the wind speed perpendicular to the rotor plane is kept constant. If this is realised, power, torque, thrust and rotational speed are kept constant above V_{rated}. So the Q-n curve for V = 8 m/s is also valid for higher wind speeds. The ideal safety system is described in chapter 5 of report R999D (ref. 2). It is impossible to design a real safety system which follows the ideal δ-V curve. In reality the rotor starts already turning out of the wind for lower wind speeds than V_{rated}. But if this is taken into account, this report would become too complicated.

4.1 Example 1

The CWD 2740 windmill has a design tip speed ratio λ_d = 2. A scale model of it has been measured in the wind tunnel. It is assumed that the C_p-λ and the C_q-λ curves of the rotor of example 1 are about the same as the measured curves of the CWD 2740 rotor. The C_p-λ and C_q-λ curves are given in figure 1 and figure 2. C_p is maximum for λ_d = 2 so the optimum C_q value $C_{q\ opt}$ is realised for λ_d = 2. $C_{q\ opt}$ is lower than the maximum C_q value which is realised for about λ = 1.4.

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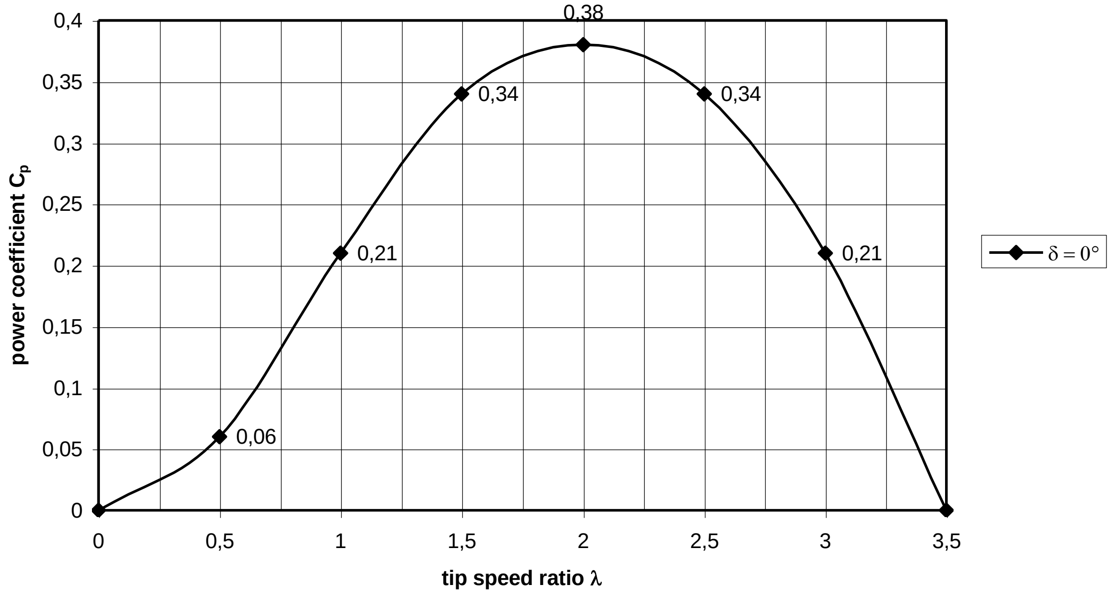

fig. 1 C_p-λ curve for a rotor with $\lambda_d = 2$ for the wind direction perpendicular to the rotor ($\delta = 0°$)

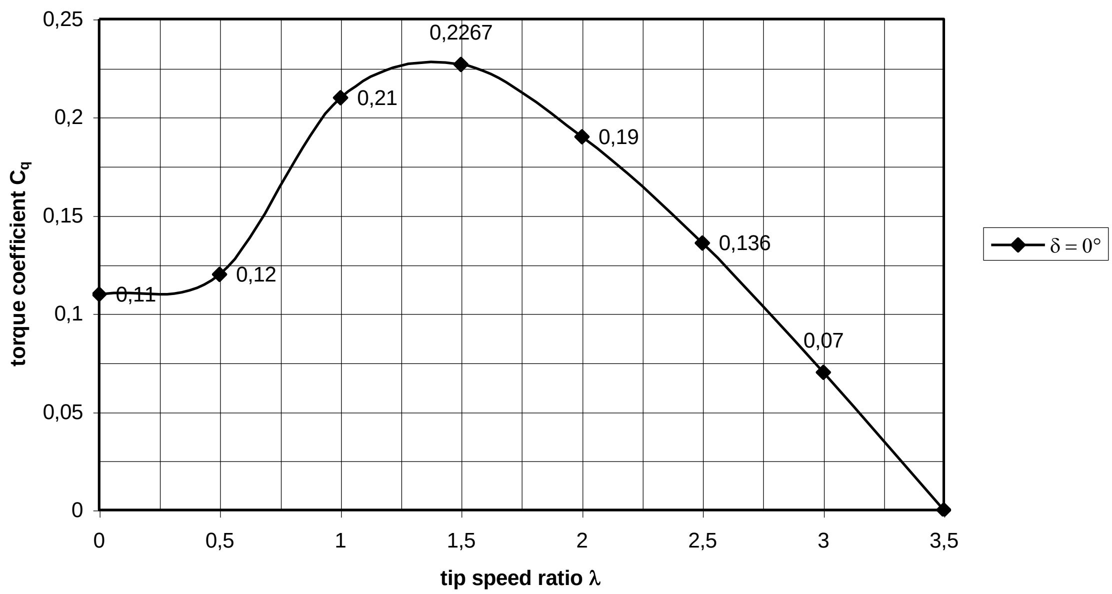

fig. 2 C_q-λ curve for a rotor with $\lambda_d = 2$ for the wind direction perpendicular to the rotor ($\delta = 0°$)

Substitution of $\rho = 1.2$ kg/m^3 and R = 2.5 in formula 2 gives:

$$Q = 29.452 * C_q * V^2 \quad \text{(Nm)} \qquad (23)$$

Substitution of R = 2.5 m in formula 4 gives:

$n = 3.820 * \lambda * V$ (rpm) (24)

The Q-n curve for a certain wind speed, for instance V = 2 m/s is determined for values of λ of 0, 0.5, 1, 1.5, 2, 2.5, 3 and 3.5 by substitution of corresponding values of λ and C_q in formulas 23 and 24. The same is done for wind speeds of 3, 4, 5, 6 ,7 and 8 m/s. The result of the calculations is mentioned in table 1 and figure 3. The optimum parabola which is going through the points for $\lambda_d = 2$ is also given in figure 3.

		V = 2 m/s δ = 0°		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 0°		V = 7 m/s δ = 0°		V = 8 m/s δ = 0°	
λ (-)	C_q (-)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)
0	0.11	0	12.96	0	29.16	0	51.84	0	80.99	0	116.63	0	158.75	0	207.34
0.5	0.12	3.82	14.14	5.73	31.81	7.64	56.55	9.55	88.36	11.46	127.23	13.37	173.18	15.28	226.19
1	0.21	7.64	24.74	11.46	55.66	15.28	98.96	19.10	154.62	22.92	222.66	26.74	303.06	30.56	395.83
1.5	0.2267	11.46	26.71	17.19	60.09	22.92	106.83	28.65	166.92	34.38	240.36	40.11	327.16	45.84	427.31
2	0.19	15.28	22.38	22.92	50.36	30.56	89.53	38.20	139.90	45.84	201.45	53.48	274.20	61.12	358.14
2.5	0.136	19.10	16.02	28.65	36.05	38.20	64.09	47.75	100.14	57.30	144.20	66.85	196.27	76.40	256.35
3	0.07	22.92	8.25	34.38	18.55	45.84	32.99	57.30	51.54	68.76	74.22	80.22	101.02	91.68	131.94
3.5	0	26.74	0	40.11	0	53.48	0	66.85	0	80.22	0	93.59	0	106.97	0

table 2 Calculated values of n and Q as a function of λ and V for a rotor with $\lambda_d = 2$

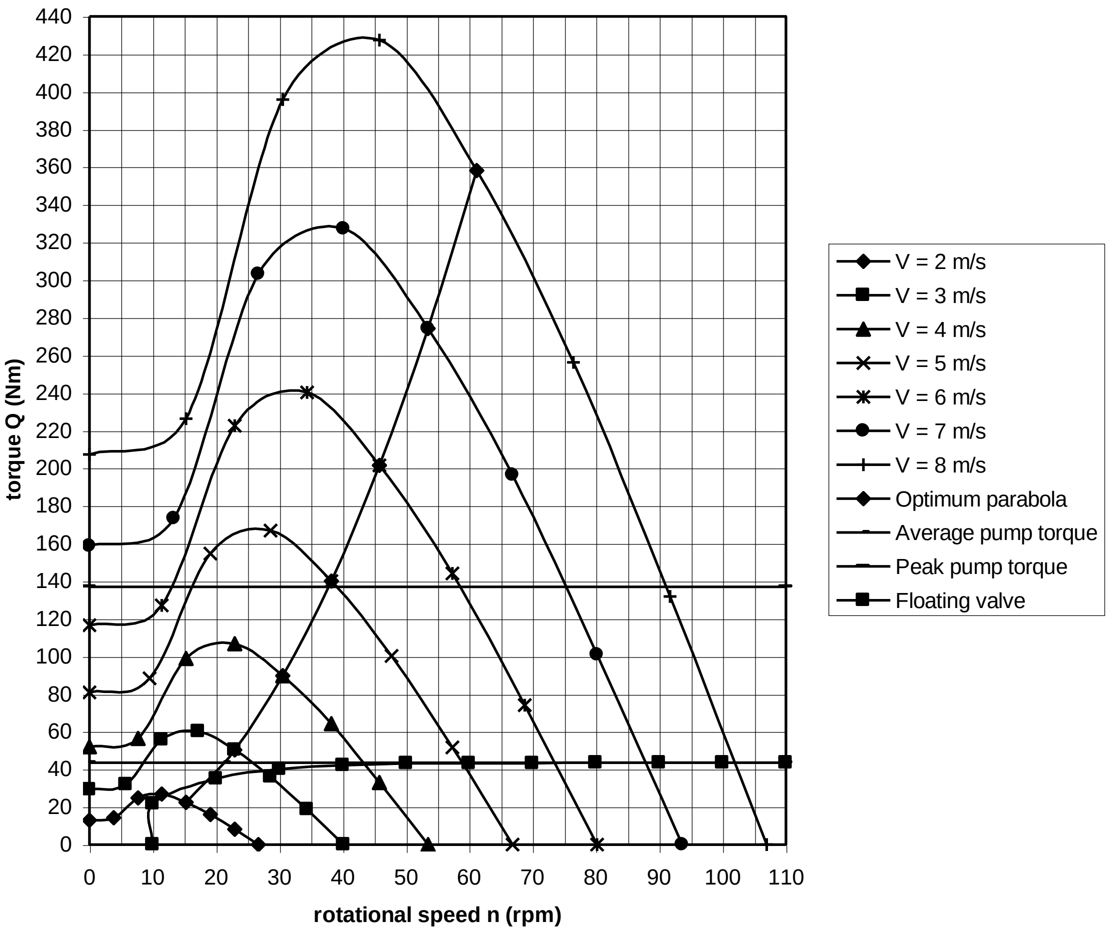

fig. 3 Q-n curves of a rotor with $\lambda_d = 2$ and Q-n curves of a single acting piston pump

The design torque Q_d for the design wind speed $V_d = 2.79$ m/s (for H = 6 m) can be calculated with formula 2. Substitution of $C_q = C_{q\ opt} = 0.19$, $\rho = 1.2$ kg/m^3, $V = V_d = 2.79$ m/s and R = 2.5 m in formula 2 gives $Q_d = 43.56$ Nm.
The design rotational speed n_d can be calculated with formula 4. Substitution of $\lambda = \lambda_d = 2$, $V = V_d = 2.79$ m/s and R = 2.5 m in formula 4 gives $n_d = 21.31$ rpm.

The design torque is equal to the average pump torque. If the dynamic water losses are neglected, the average pump torque is constant for every rotational speed. This means that the average pump torque is a horizontal line going through the design point n = 21.31 rpm and Q = 43.56 Nm. This average pump toque is also given in figure 3.

In chapter 2 it was explained that the peak pump torque is a factor π higher than the average pump torque so the peak pump torque is also a horizontal line but one which is laying a factor π higher than the average pump torque. So $Q_{peak} = \pi * 43.56 = 136.85$ Nm. This line is also given in figure 3.

If the rotor is turning rather fast it is working as a big flywheel and the rotational speed will fluctuate only a little even for a strong fluctuating pump torque. So then it is allowed to use the average Q-n line of the pump. However, during starting, there is almost no kinetic energy in the rotor and for this situation one has to use the peak Q-n line of the pump.

In figure 3 it can be seen that for n = 0 rpm, the rotor can give a starting torque equal to the peak torque of the pump only at a wind speed of about 6.5 m/s. This is very high for a windmill with $V_d = 2.79$ m/s.

If the pump is equipped with a floating valve the rotor starts almost unloaded because the piston valve is open. At a certain critical rotational speed which depends on the buoyancy force of the floating valve, the piston valve closes a the maximum speed of the piston which is reached for the horizontal position of the crank so halfway the stroke. The volumetric efficiency at this point is 0.5 and the average pump torque is therefore halve the average torque of a pump with the same dimensions but with no floating valve. However, the peak torque of a pump with a floating valve will be the same or even larger than for a normal pump because the valve is closing when the piston has already reached a certain velocity and this means that the water column has to be accelerated suddenly. It depends on the elasticity of the pump rod and the rising main what maximum shock force may occur.

For rotational speeds higher than the critical rotational speed, the valve is closing at an earlier position of the upwards movement of the piston and this means that the volumetric efficiency becomes higher than 0.5 resulting in increase of the average pump torque. At high rotational speeds it is about the same as for a normal pump. The estimated average Q-n curve for the same pump but now equipped with a floating valve is also given in figure 3.

In figure 3 it can be seen that the pump with a floating valve will already start at a wind speed of about 2 m/s which is very much lower than for the same pump with no floating valve. The practical behaviour at very low wind speeds is that sometimes the pump makes an stroke followed by some strokes where no water is given. For wind speeds higher that a little below V_d, the pump is giving water for each stroke. Because the rotor doesn't stop, every little wind gust will be used and even at very low wind speeds there some water will be pumped. This is important, especially when the water is used for drinking water. This system has been tested on the CWD2000 windmill and has proven to work. But one has to be able to make a light floating valve which is strong enough to have the pressure pulses. Our experiments with manufacture of floating valves are described in report R1009D (ref. 3). Because the starting behaviour is very good, it is possible to use a much larger pump than a pump with no floating valve and this results in a much higher flow, especially at high wind speeds.

The flow for a normal pump is proportional to the rotational speed. At higher wind speeds the rotor is running almost unloaded because the optimum parabola of the rotor is not followed. The flow q is therefore increasing even stronger than the increase of the wind speed.

4.2 Example 2

No rotor with λ_d = 1 has been measured by CWD but the characteristics are known from literature. The estimated C_p-λ and C_q-λ curves of the rotor of example 3.2 are given in figure 4 and 5.

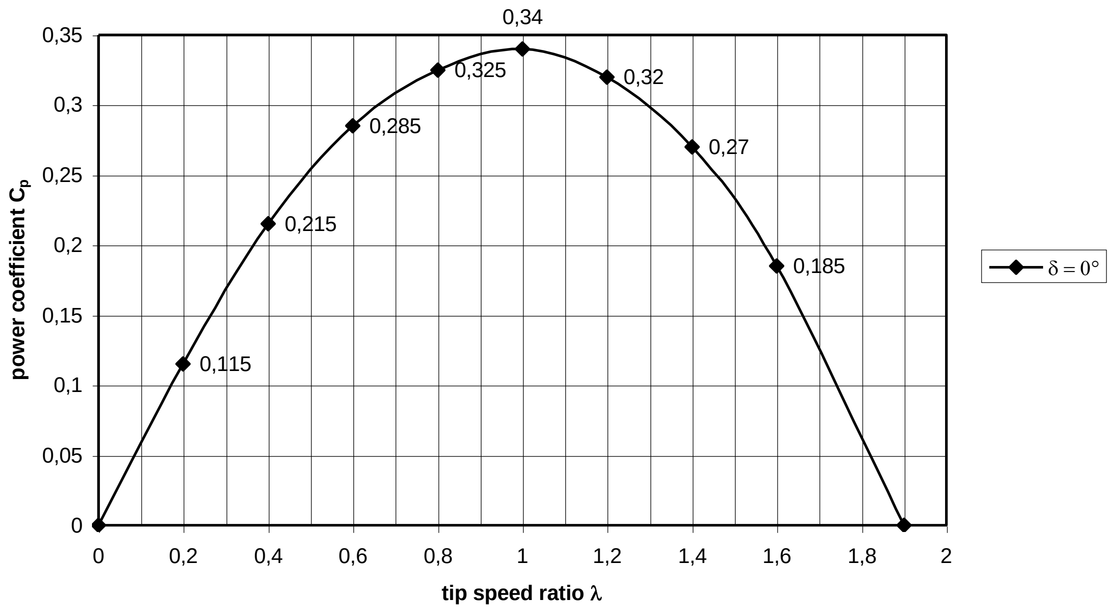

fig. 4 C_p-λ curve for a rotor with λ_d = 1 for the wind direction perpendicular to the rotor (δ = 0°)

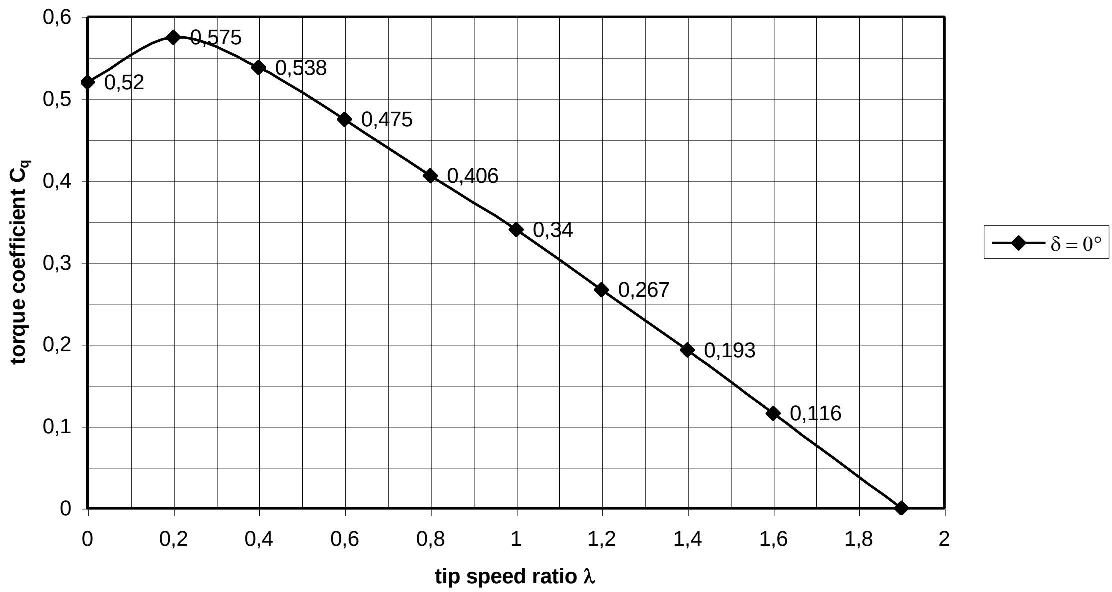

fig. 5 C_q-λ curve for a rotor with λ_d = 1 for the wind direction perpendicular to the rotor (δ = 0°)

Substitution of ρ = 1.2 kg/m^3 and R = 1.5 in formula 2 gives:

$$Q = 6.362 * C_q * V^2 \quad \text{(Nm)} \tag{25}$$

Substitution of R = 1.5 m in formula 4 gives:

$$n = 6.366 * \lambda * V \quad \text{(rpm)} \tag{26}$$

The Q-n curves are made in the same way as for example 4.1 using formulas 25 and 26. The result of the calculations is mentioned in table 2 and figure 6. The optimum parabola which is going through the points for λ_d = 1 is also given in figure 6.

		V = 2 m/s δ = 0°		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 0°		V = 7 m/s δ = 0°		V = 8 m/s δ = 0°	
λ (-)	C_q (-)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)
0	0.52	0	13.23	0	29.77	0	52.93	0	82.71	0	119.10	0	162.10	0	211.73
0.2	0.575	2.55	14.63	3.82	32.92	5.09	58.53	6.37	91.45	7.64	131.69	8.91	179.25	10.19	234.12
0.4	0.538	5.09	13.69	7.64	30.80	10.19	54.76	12.73	85.57	15.28	123.22	17.82	167.72	20.37	219.06
0.6	0.475	7.64	12.09	11.46	27.20	15.28	48.35	19.10	75.55	22.92	108.79	26.74	148.08	30.56	193.40
0.8	0.406	10.19	10.33	15.28	23.25	20.37	41.33	25.46	64.57	30.56	92.99	35.65	126.57	40.74	165.31
1	0.34	12.73	8.65	19.10	19.47	25.46	34.61	31.83	54.08	38.20	77.87	44.56	105.99	50.93	138.44
1.2	0.267	15.28	6.79	22.92	15.29	30.56	27.18	38.20	42.47	45.84	61.15	53.47	83.23	61.11	108.71
1.4	0.193	17.82	4.91	26.74	11.05	35.65	19.65	44.56	30.70	53.47	44.20	62.39	60.17	71.30	78.58
1.6	0.116	20.37	2.95	30.56	6.64	40.74	11.81	50.93	18.45	61.11	26.57	71.30	36.16	81.48	47.23
1.9	0	24.19	0	36.29	0	48.38	0	60.48	0	72.57	0	84.67	0	96.76	0

table 2 Calculated values of n and Q as a function of λ and V for a rotor with λ_d = 1

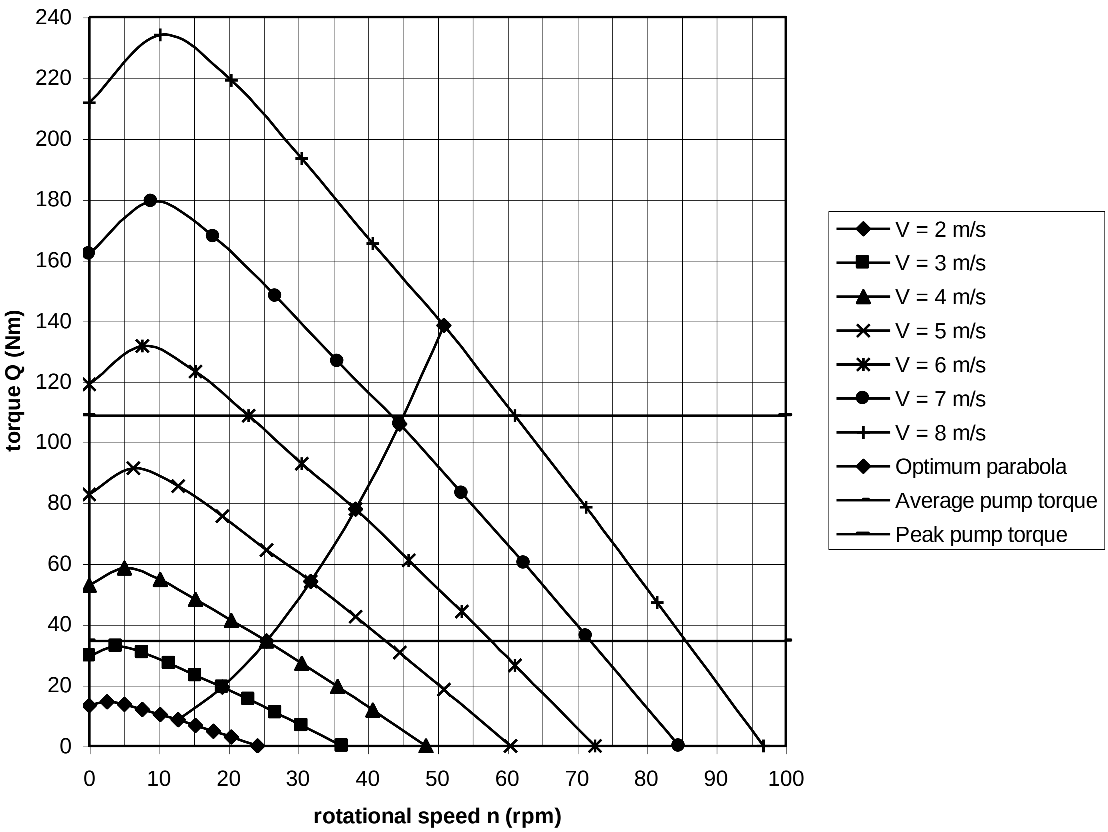

fig. 6 Q-n curves of a rotor with λ_d = 1 and Q-n curves of a single acting piston pump

The design torque Q_d for the design wind speed V_d = 4 m/s can be calculated with formula 2. Substitution of $C_q = C_{q\ opt} = 0.34$, $\rho = 1.2$ kg/m^3, $V = V_d = 4$ m/s and R = 1.5 m in formula 2 gives Q_d = 34.61 Nm.
The design rotational speed n_d can be calculated with formula 4. Substitution of $\lambda = \lambda_d = 1$, $V = V_d = 4$ m/s and R = 1.5 m in formula 4 gives n_d = 25.46 rpm.

The design torque is equal to the average pump torque. If the dynamic water losses are neglected, the average pump torque is constant for every rotational speed. This means that the average pump torque is a horizontal line going through the design point n = 25.46 rpm and Q = 34.61 Nm. This average pump toque is also given in figure 6.

In chapter 2 it was explained that the peak pump torque is a factor π higher than the average pump torque so the peak pump torque is also a horizontal line but one which is laying a factor π higher than the average pump torque. So $Q_{peak} = \pi * 34.61 = 108.73$ Nm. This line is also given in figure 6.

If the rotor is turning rather fast it is working as a big flywheel and the rotational speed will fluctuate only a little even for a strong fluctuating pump torque. So then it is allowed to use the average Q-n line of the pump. However, during starting, there is almost no kinetic energy in the rotor and for this situation one has to use the peak Q-n line of the pump.

In figure 6 it can be seen that for n = 0 rpm, the rotor can give a starting torque equal to the peak torque of the pump only at a wind speed of about 5.7 m/s. Although the rotor has a very high starting torque coefficient of 0.52, the starting wind speed is higher than the design wind speed V_d = 4 m/s. This is acceptable for a good wind regime because if the one minute average wind speed is 4 m/s, there might be short wind gusts of 5.7 m/s which makes the rotor starting.

The flow for a normal pump is proportional to the rotational speed. At higher wind speeds the rotor is running almost unloaded because the optimum parabola of the rotor is not followed. The flow q is therefore increasing even stronger than the increase of the wind speed.

5 References

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