Calculations executed for the 4-bladed rotor of the VIRYA-2.8B4 windmill (λ_d = 2.5, galvanised steel blades)

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KD 319

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1 Introduction

The VIRYA-2.8B4 windmill is designed to drive a rope pump using an accelerating gearing with a gear ratio of 2.5 in the windmill head and a vertical shaft in the tower. For very low heads it might also be possible to couple the VIRYA-2.8B4 to an Archimedian screw pump. The gearing can be realised with a 12 mm string and one auxiliary wheel. The transmission is described in report KD 320 (ref. 1). The rope pump and the starting behaviour are described in report KD 321 (ref. 2). The VIRYA-2.8B4 is meant for use in developing countries. The use of stainless steel is therefore limited as much as possible. The VIRYA-2.8B4 windmill has a 4-bladed rotor with galvanised steel blades.

The windmill will be provided with the hinged side vane safety system which is used in all VIRYA windmills. This safety system will be equipped with a relatively light vane blade resulting in a rated wind speed of about 8 m/s.

The tower is probably the same as the tower of the VIRYA-3D windmill. It has a 6 m long lowest part made of angle iron and strip and a 2 m long upper part made of pipe. This tower is about similar to the tower which is used for the VIRYA-2.2S windmill except that the pipe diameter is larger.

2 Description of the rotor of the VIRYA-2.8B4 windmill

The 4-bladed rotor of the VIRYA-2.8B4 windmill has a diameter D = 2.8 m and a design tip speed ratio λ_d = 2.5. Two opposite blades are connected to each other by means of a twisted strip. Advantages of this construction are that no welded spoke assembly is required, that the rotor can be balanced easily and that the assembly of two blades can be transported completely mounted.

The rotor has blades with a constant chord and is provided with a 7.14 % cambered airfoil. A blade is made of a strip with dimensions of 333 * 1000 * 1.5 mm and 6 blades can be made from a standard sheet of 1 * 2 m. Because the blade is cambered, the chord c is a little less than the blade width, resulting in c = 328.5 mm = 0.3285 m.

Two opposite blades are connected to each other by a 1.5 m long twisted strip. The overlap in between blade and strip is 0.35 m which results in a free blade length of 0.65 m. This blade length in combination with a design tip speed ratio of 2.5 and a blade thickness of 1.5 mm, is expected to be enough to prevent flutter of the blade at high wind speeds. The blade is connected to the strip using three bolts. In between the strip and the blade is a thin strip to prevent too strong deformation of the blade. The bolts are also used for connection of the balancing weights.

The hub is made of square bar 60 * 60 mm with a tapered hole in the centre for connection to the rotor shaft. The two blade trips are clamped in between the hub and a square sheet by means of four bolts and that is why the strip is not loaded by a bending moment at the position of the holes. The hub is pulled on the tapered shaft end by one central bolt. A sketch of the rotor is given in figure 7.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 319) has its own formula numbering. Substitution of λ_d = 2.5 and R = 1.4 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 1.7857 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 4 and c = 0.3285 m in formula (5.4) of KD 35 gives:

$C_l = 19.127\ r\ (1 - \cos\phi)$ (-) (4)

Substitution of V = 4 m/s and c = 0.3285 m in formula (5.5) of KD 35 gives:

$R_{e\,r} = 0.876 * 10^5 * \sqrt{(\lambda_{r\,d}^2 + 4/9)}$ (-) (5)

The blade is calculated for six stations A till F which have a distance of 0.2 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 4). The Reynolds values for the stations are calculated for a wind speed of 4 m/s because this is a reasonable wind speed for a windmill with V_{rated} = 8 m/s. Those airfoil Reynolds numbers are used which are lying closest to the calculated values. The effect of the strip on the aerodynamic characteristics of the inner part of the blade is neglected.

sta-tion	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 4 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	1.4	2.5	14.5	0.3285	0.86	0.97	2.27	2.5	1.3	2.5	13.2	12	0.034
B	1.2	2.143	16.7	0.3285	0.97	0.96	1.97	1.7	3.0	2.9	13.7	13.8	0.041
C	1	1.786	19.5	0.3285	1.10	1.05	1.67	1.7	4.5	3.9	15.0	15.6	0.045
D	0.8	1.429	23.3	0.3285	1.25	1.23	1.38	1.2	6.1	5.9	17.2	17.4	0.045
E	0.6	1.071	28.7	0.3285	1.41	1.42	1.11	1.2	9.0	9.5	19.7	19.2	0.094
F	0.4	0.714	36.3	0.3285	1.49	1.29	0.86	1.2	-	15.3	-	21	0.235

table 1 Calculation of the blade geometry of the VIRYA-2.8B4 rotor

No value for α_{th} and therefore for β_{th} is found for station F because the required C_l value can not be generated. The theoretical blade angle β_{th} for stations A to E varies in between 13.2° and 19.7°. If the blade angle is linearised in between 12° at the blade tip and 21° at the blade root, the linearised angle of attack α_{lin} differs only a little from the theoretical value α_{th} for the most important outer side of the blade. The strip of the spoke assembly is twisted 21° in between the hub and the blade root.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.041. Figure 4.8 of KD 35 (for B = 4) en λ_{opt} = 2.5 and C_d/C_l = 0.041 gives $C_{p\ th}$ = 0.43. The blade is stalling in between station E and F and the airfoil is disturbed because of the blade connection to the strip. Therefore not the whole blade length k = 1 m, but only the part up to 0.08 m from station F is used for the calculation of the C_p. This gives an effective blade length k' = 0.92 m.

Substitution of $C_{p\ th}$ = 0.43, R = 1.4 m and effective blade length k' = 0.92 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.38. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.38 / 2.5 = 0.152.
Substitution of $\lambda_{opt} = \lambda_d$ = 2.5 in formula 6.4 of KD 35 gives λ_{unl} = 4.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

The average blade angle is 16.5°. For a non rotating rotor the average angle of attack α is therefore 90° - 16.5° = 73.5°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 73.5° it can be read that C_l = 0.54. The whole blade is stalling during starting and therefore now the whole blade length k = 1 m is taken.
Substitution of B = 4, R = 1.4 m, k = 1 m, C_l = 0.54 en c = 0.3285 m in formula 6 gives that $C_{q\ start}$ = 0.0556. The real starting torque coefficient is a little lower because we have used the average blade angle. Suppose $C_{q\ start}$ = 0.054. For the ratio in between the starting torque and the optimum torque we find that it is 0.054 / 0.152 = 0.355. This is good for a rotor with a design tip speed ratio of 2.5. The ratio is expected to be high enough for combination of the windmill with a rope pump because a rope pump looses its water in the pressure pipe after some time. This effect is explained in detail in report KD 321 (ref. 2).

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \qquad (m/s) \qquad (7)$$

At this moment the average starting torque Q_s of the rope pump is not yet known so the starting wind speed can't be calculated accurately. Assume Q_s = 3 Nm.
Substitution of Q_s = 3 Nm, $C_{q\ start}$ = 0.054, ρ = 1.2 kg/m^3 en R = 1.4 m in formula 7 gives that V_{start} = 3.3 m/s. This is acceptable low for a 4-bladed rotor with a design tip speed ratio of 2.5. The Q-n curve of the rotor for V = 3.3 m/s is rising rather fast and therefore it is allowed that the pump torque is also rising at increasing rotational speed. The pump torque is rising because the water level in the rising main is rising as the rotor has made more revolutions from stand still position. The maximum torque level is reached when the water has reached the top of the rising main. Only from this time, the pump will have a real output.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing a S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 5). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-2.8B4 rotor are given in figure 1 and 2.

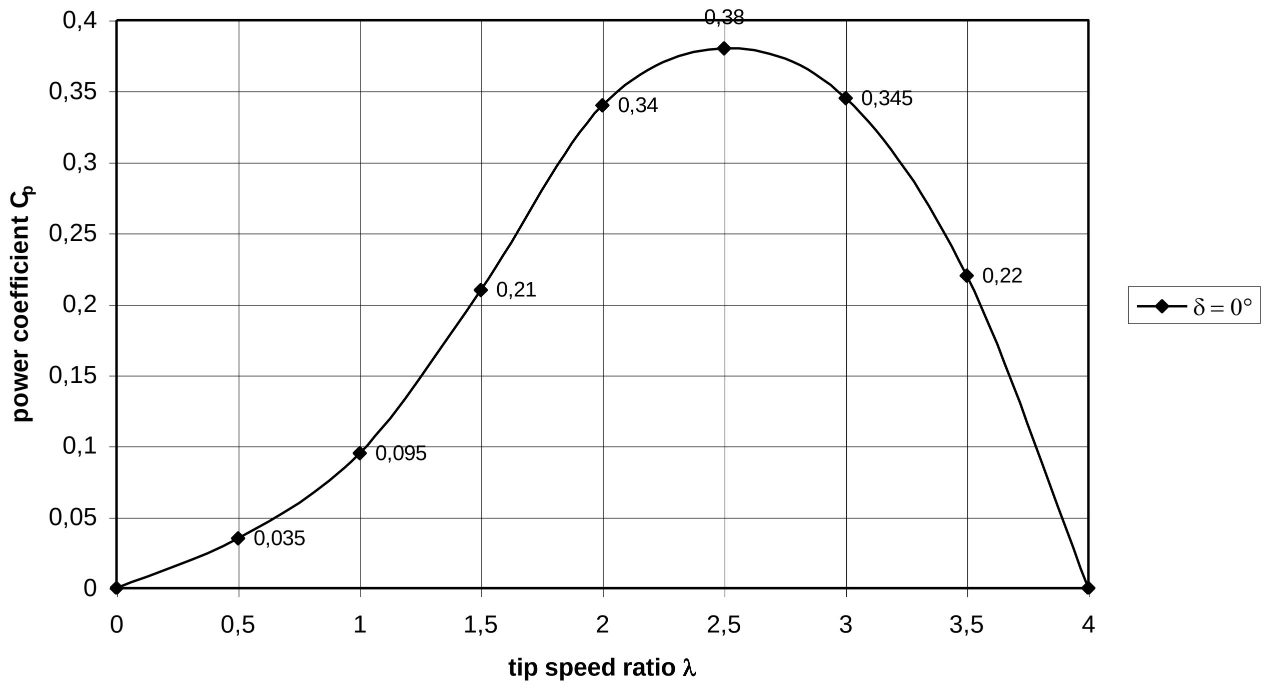

fig. 1 Estimated C_p-λ curve for the VIRYA-2.8B4 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

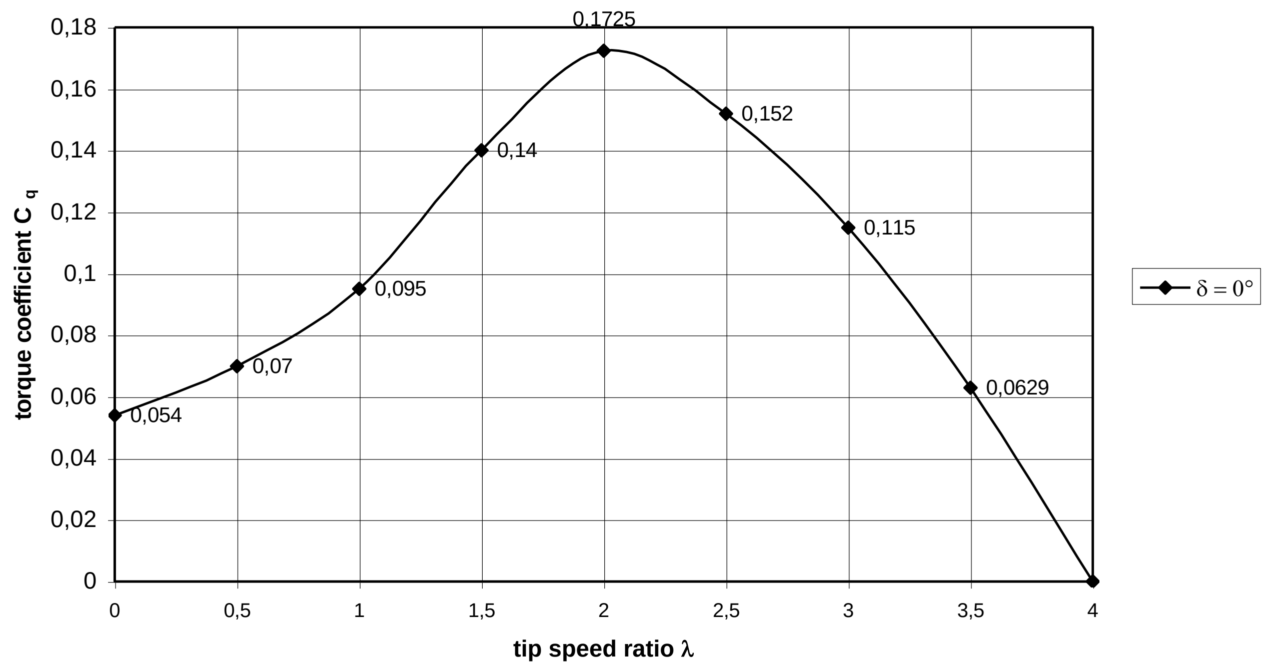

fig. 2 Estimated C_q-λ curve for the VIRYA-2.8B4 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

5 Determination of the Q-n curves

The determination of the Q-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_q-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the torque Q and the rotational speed n. The C_q-λ curve is given in figure 2. The δ-V curve of the safety system depends on the safety system. As this safety system is not yet designed, the δ-V curve of it is estimated. The estimated δ-V curve is given in figure 3.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 8 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The Q-n curve for 8 m/s will therefore also be valid for wind speeds higher than 8 m/s.

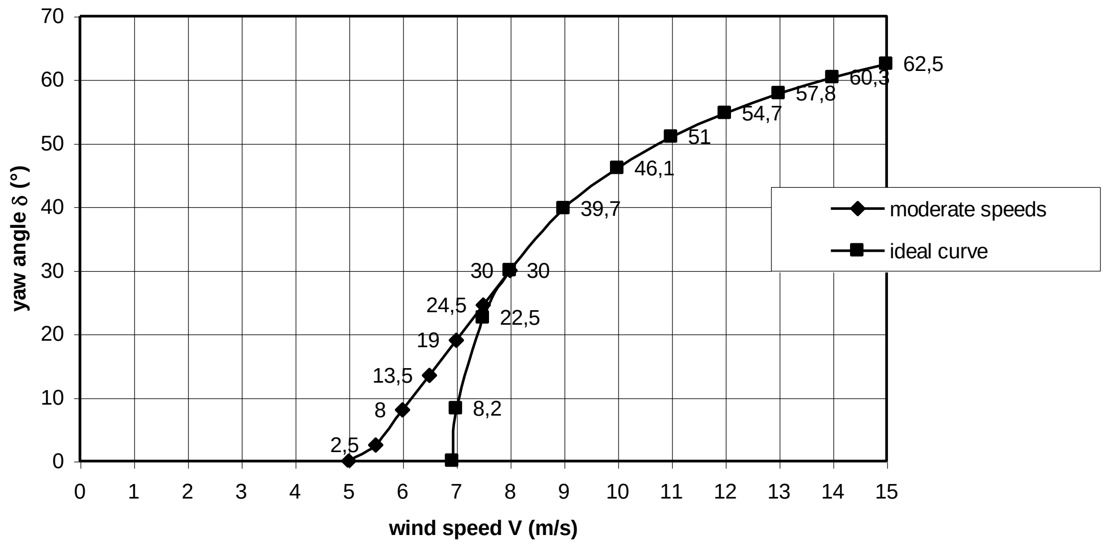

fig. 3 Estimated δ-V curve VIRYA-2.8B4 safety system with V_{rated} = 8 m/s

The Q-n curves are used to check the matching with the Q-n curve of the pump. The Q-n curves are determined for wind the speeds 2, 3, 4, 5, 6, 7 and 8 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for Q and n are used which are given in chapter 7 of KD 35.

Substitution of R = 1.4 m in formula 7.1 of KD 35 gives:

$$n_\delta = 6.8209 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of ρ = 1.2 kg / m^3 en R = 1.4 m in formula 7.7 of KD 35 gives:

$$Q_\delta = 5.1723 * C_q * \cos^2\delta * V^2 \qquad \text{(W)} \qquad (9)$$

Formula 8 and 9 are only valid for values of λ in between about 2/3 λ_d and λ_{unl}. The thrust coefficient for very low values of λ is considerably lower than for values of λ in between 2/3 λ_d and λ_{unl}. Therefore the rotor will turn out of the wind lesser than according to the δ-V curve of figure 3 and this results in a higher rotational speed and torque. Another aspect is that the torque coefficient for low values of λ starts to decrease only for yaw angles δ larger than about 30°. These two effects make that the Q-n curves for wind speeds of 6, 7 and 8 m/s for λ in between 0 and about 1.5 are lying higher than the curves which are given in figure 4.

The Q-n curves are determined for C_q values belonging to λ is 0, 0,5, 1, 1.5, 2, 2.5, 3, 3.5 and 4 (see figure 2). For a certain wind speed, for instance V = 2 m/s, related values of C_q and λ are substituted in formula 8 and 9 and this gives the Q-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 5, is taken into account. The result of the calculations is given in table 2.

		V = 2 m/s δ = 0°		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 8°		V = 7 m/s δ = 19°		V = 8 m/s δ = 30°	
λ (-)	C_q (-)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n_δ (rpm)	Q_δ (Nm)	n_δ (rpm)	Q_δ (Nm)	n_δ (rpm)	Q_δ (Nm)
0	0.054	0	1.12	0	2.51	0	4.47	0	6.98	0	9.86	0	12.24	0	13.41
0.5	0.07	6.8	1.45	10.2	3.26	13.6	5.79	17.1	9.05	20.2	12.78	22.6	15.86	23.6	17.38
1	0.095	13.6	1.97	20.5	4.42	27.3	7.86	34.1	12.28	40.4	17.35	45.1	21.53	47.3	23.59
1.5	0.14	20.5	2.90	30.7	6.52	40.9	11.59	51.2	18.10	60.6	25.56	67.7	31.72	70.9	34.76
2	0.1725	27.3	3.57	40.9	8.03	54.6	14.28	68.2	22.31	80.8	31.50	90.3	39.08	94.5	42.83
2.5	0.152	34.1	3.14	51.2	7.08	68.2	12.58	85.3	19.65	101.1	27.75	112.9	34.44	118.1	37.74
3	0.115	40.9	2.38	61.4	5.35	81.9	9.52	102.3	14.87	121.3	21.00	135.4	26.06	141.8	28.55
3.5	0.0629	47.7	1.30	71.6	2.93	95.5	5.21	119.4	8.13	141.5	11.49	158.0	14.25	165.4	15.62
4	0	54.6	0	81.9	0	109.1	0	136.4	0	161.7	0	180.6	0	189.0	0

table 2 Calculated values of n and Q as a function of λ and V for the VIRYA-2.8B4 rotor

The calculated values for n and Q are plotted in figure 4.

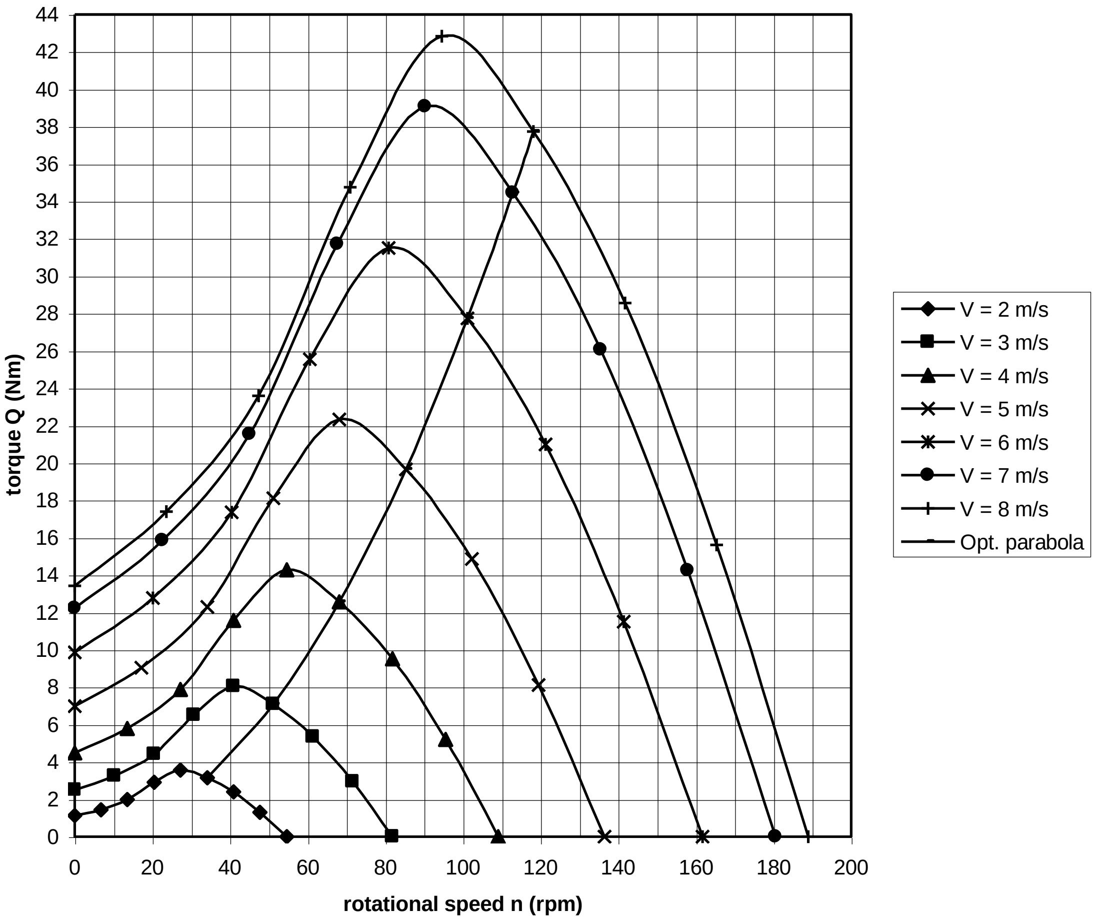

fig. 4 Q-n curves of the VIRYA-2.8B4 rotor

The optimum parabola which is going through the points with λ = 2.5, where C_p is maximum, is also drawn in figure 4.

6 Calculation of the strength of the strip which connects the blades

Two opposite blades are connected to each other by a strip with a length of 1500 mm, a width b = 60 mm and a height h = 6 mm. The strip is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The strip is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because the strip is thin and long it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:
1^e The load which appears for a rotating rotor at V_{rated} = 8 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 30° for V_{rated} = 8 m/s.
2^e The load which appears for a locked rotor. The rotor is locked by connecting a blade strip to the head (never connect a blade strip to the tower) or by locking the vertical shaft in the tower centre.

6.1 Bending stress in the strip for a rotating rotor and V = 8 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 / B \qquad \text{(N)} \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The effective blade length k' of the VIRYA-2.8B4 rotor is 0.92 m but the rotor radius R = 1.4 m. Therefore there is a disk in the centre with an area of about 0.118 of the rotor area on which almost no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.882 = 0.784. Because of the tip losses the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75.
Substitution of C_t = 0.75, δ = 30°, ρ = 1.2 kg/m^3, V = 8 m/s, R = 1.4 m and B = 4 in formula 10 gives $F_{t\,\delta\,bl}$ = 33.3 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 0.933 m. Because the effective blade length is only k', there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k' falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about r_1 = 0.98 m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at $r_2 = 0.03$ m. At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_1 - r_2) \qquad \text{(Nm)} \qquad (11)$$

Substitution of $F_{t\,\delta\,bl} = 33.3$ N, $r_1 = 0.98$ m and $r_2 = 0.03$ m gives $M_{b\,t} = 31.6$ Nm = 31600 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$$\sigma_b = M / W \qquad (\text{N/mm}^2) \qquad (12)$$

The moment of resistance W of a strip is given by:

$$W = 1/6\ bh^2 \qquad (\text{mm}^3) \qquad (13)$$

(12) + (13) gives:

$$\sigma_b = 6\ M / bh^2 \qquad (\text{N/mm}^2) \qquad (\text{M in Nmm}) \qquad (14)$$

Substitution of M = 31600 Nmm, b = 60 mm and h = 6 mm in formula 14 gives $\sigma_b = 88$ N/mm^2. For this stress the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the strip is bending only in between the hub and the inner connection bolt of blade and strip. So it is assumed that the blade itself is not bending. The inner connection bolt is lying at $r_3 = 0.425$ m = 425 mm. So the length of the strip l which is loaded by bending is given by:

$$l = r_3 - r_2 \qquad \text{(mm)} \qquad (15)$$

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$$M = F * (r_1 - r_3) \qquad \text{(Nmm)} \qquad (16)$$

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$$\phi = l * (M + ½\ Fl) / EI \qquad \text{(rad)} \qquad (17)$$

The bending moment of inertia I of a strip is given by:

$$I = 1/12\ bh^3 \qquad (\text{mm}^4) \qquad (18)$$

(15) + (16) + (17) + (18) gives:

$$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½\ (r_3 - r_2)\} / (E * bh^3) \qquad \text{(rad)} \qquad (19)$$

Substitution of $F = 33.3$ N, $r_3 = 425$ mm, $r_2 = 30$ mm, $r_1 = 980$ mm, $E = 2.1 * 10^5$ N/mm^2, b = 60 mm and h = 6 mm in formula 19 gives: $\phi = 0.04364$ rad = 2.50°. This is an angle which can not be neglected. In report R409D (ref. 6) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-2.8B4 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin\left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2}\right) \quad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 333 mm and a plate thickness of 1.5 mm it is found that $A_{pr} = 0.0005$ m^2. The blade is made of steel sheet with a density ρ_{pr} of about $\rho_{pr} = 7.8 * 10^3$ kg/m^3. If the rotor is coupled to a rope pump it will run with a rather high tip speed ratio at high wind speeds. It is supposed that the tip speed ratio is 3.6 for V = 8 m/s. Substitution of $C_t = 0.75$, $\rho = 1.2$ kg/m^3, R = 1.4 m, B = 4, $A_{pr} = 0.0005$ m^2, $\rho_{pr} = 7.8 * 10^3$ kg/m^3 and $\lambda = 3.6$ in formula 20 gives: $\varepsilon = 1.57°$. This angle is smaller than the calculated angle of 2.50° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.57°.

The real bending angle ε is determined a follows. A thrust moment $M_t = 31.6$ Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. $M_b = 31.6$ Nm for $\varepsilon = 2.50°$. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. $M_c = 31.6$ Nm for $\varepsilon = 1.57°$. The path of these three moments is given in figure 5. The sum total of $M_b + M_c$ is determined and the line $M_b + M_c$ is also given in figure 5.

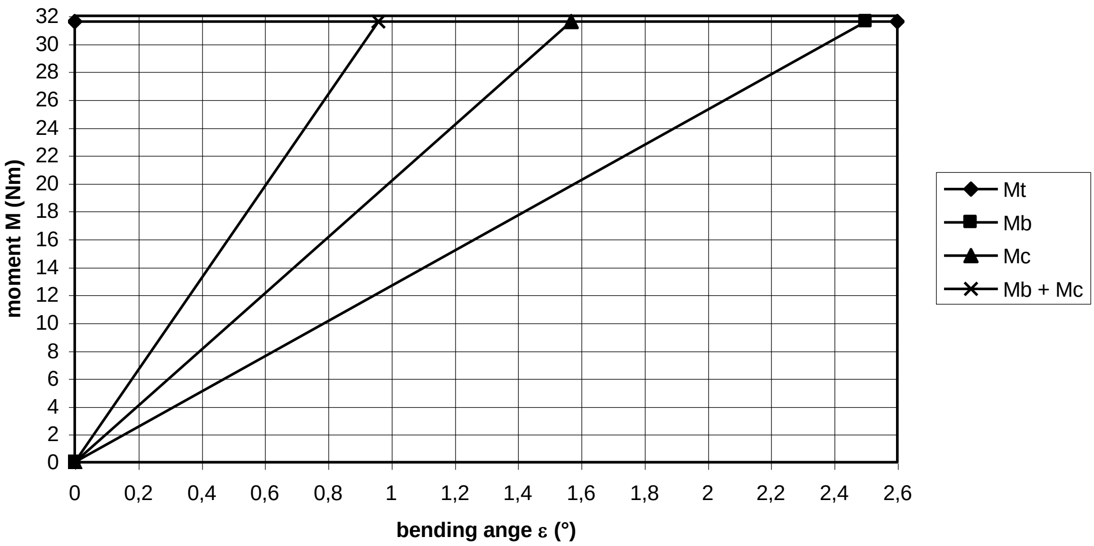

fig. 5 Path of M_t, M_b, M_c, and $M_b + M_c$ as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 5 it can be seen that $\varepsilon = 0.96°$. This is a factor 0.384 of the calculated angle of 2.50°. Because the bending stress is proportional to the bending angle it will also be a factor 0.384 of the calculated stress of 88 N/mm^2 resulting in a stress of about 34 N/mm^2. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with $\sin\alpha$ with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment $dF * r$ which is exerted by this force with respect to the blade is therefore varying sinusoidal too.

However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \tag{21}$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a four bladed rotor because adding four $\sin^2\alpha$ functions which make an angle of 90° which each other, appear to result in a constant value. The four functions are given in figure 6. It can be proven that the sum moment for all four blades is two times the peak value of one blade.

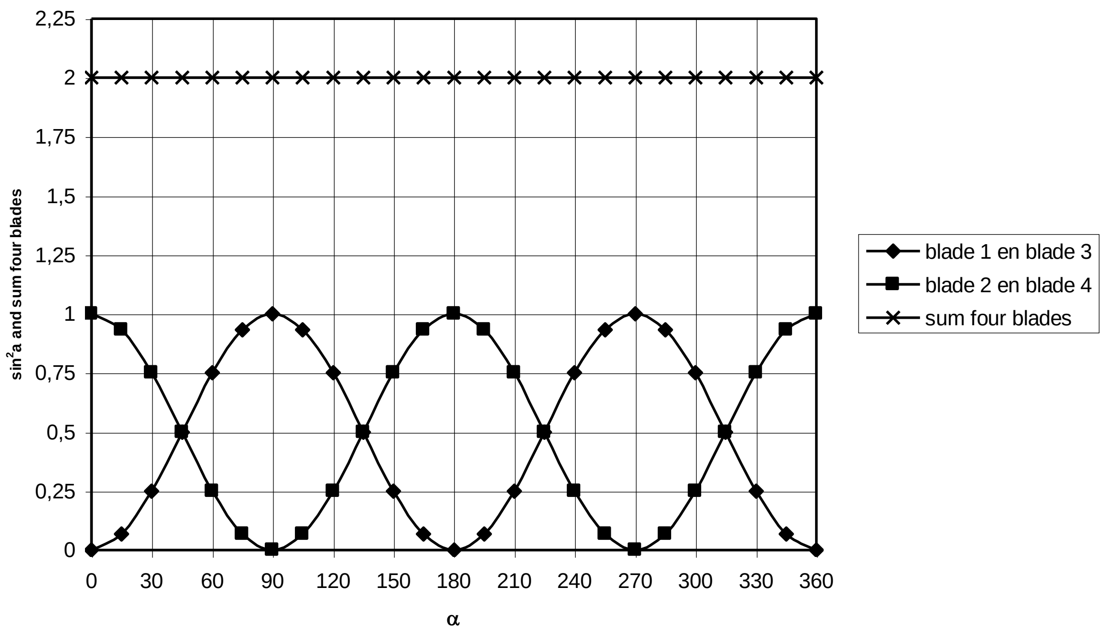

fig. 6 Variation of $\sin^2\alpha$ and the sum of four moments

We are not interested in the variation of the gyroscopic moment in a blade with respect to the x-axis but in the variation of the moment with respect to an axis frame with rotates with the blade. If the blade is vertical both axis coincide and the moment is the same for both axis frames. The peak moment in one blade is therefore half the value of the moment given by formula 21, so:

$M_{gyr\ bl\ max} = 0.5 * I_{rot} * \Omega_{rot} * \Omega_{head}$ (Nm) (22)

For a four bladed rotor, the moment of inertia of the whole rotor I_{rot} is four times the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$M_{gyr\ bl\ max} = 2\ I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (23)

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 21 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $\sin^2\alpha$ function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened peak value of $M_{gyr\ bl\ max}$ is given by:

$M_{gyr\ bl\ max\ flattened} = 1.2\ I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (24)

For the chosen blade geometry it is calculated that I_{bl} = 3.88 kgm^2. The maximum loaded rotational speed of the rotor can be read in figure 4 for λ = 3.6 and it is found that n_{max} = 170 rpm. This gives $\Omega_{rot\ max}$ = 17.8 rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-2.B2 will be with a safety system which has a large moment of inertia of the head around the tower axis and therefore sudden variations in wind speed and wind direction will be followed only slowly. It is assumed that the maximum angular velocity of the head can be 0.3 rad/s at very high wind speeds.

Substitution of I_{bl} = 2.64 kgm^2, $\Omega_{rot\ max}$ = 16.8 rad/s en $\Omega_{head\ max}$ = 0.3 rad/s in formula 24 gives: $M_{gyr\ bl\ max}$ = 24.9 Nm = 24900 Nmm.

Substitution of M = 24900 Nmm, b = 60 mm and h = 6 mm in formula 14 gives $\sigma_{b\ max}$ = 69 N/mm^2. This value has to be added to the bending stress of 34 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max}$ = 103 N/mm^2. The minimum stress is 34 – 69 = - 35 N/mm^2. So the stress is becoming negative and therefore it is necessary to take the load as a fatigue load.

For the strip material hot rolled strip Fe360 is chosen. For hot rolled strip the allowable stress for a load in between zero and maximum is about 190 N/mm^2 and for a fatigue load it is about 140 N/mm^2. The calculated stress is lower than the allowable fatigue stress so the strip is strong enough.

In reality the blade is not extremely stiff and will also bend somewhat. This reduces the bending of the strip and therefore the stress is the strip will be somewhat lower.

6.2 Bending stress in the strip for a locked rotor

The rotational speed for a rotor which is stopped by locking of the rotor is zero. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 33.3 N for $V = V_{rated}$ = 8 m/s and δ = 30°. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 30° if the force on one blade is 33.3 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at r_4 = 0.9 m because the relative wind speed is constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 11 changes into:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_4 - r_2) \quad \text{(Nm)} \qquad (25)$$

Substitution of $F_{t\,\delta\,bl}$ = 33.3 N, r_4 = 0.9 m en r_2 = 0.03 m in formula 25 gives $M_{b\,t}$ = 29.0 Nm = 29000 Nmm. Substitution of M = 29000 Nmm, b = 60 mm and h = 6 mm in formula 14 gives σ_b = 81 N/mm^2. This is lower than the calculated stress for a rotating rotor. The load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable stress is 190 N/mm^2 for hot rolled strip, so the strip is strong enough.

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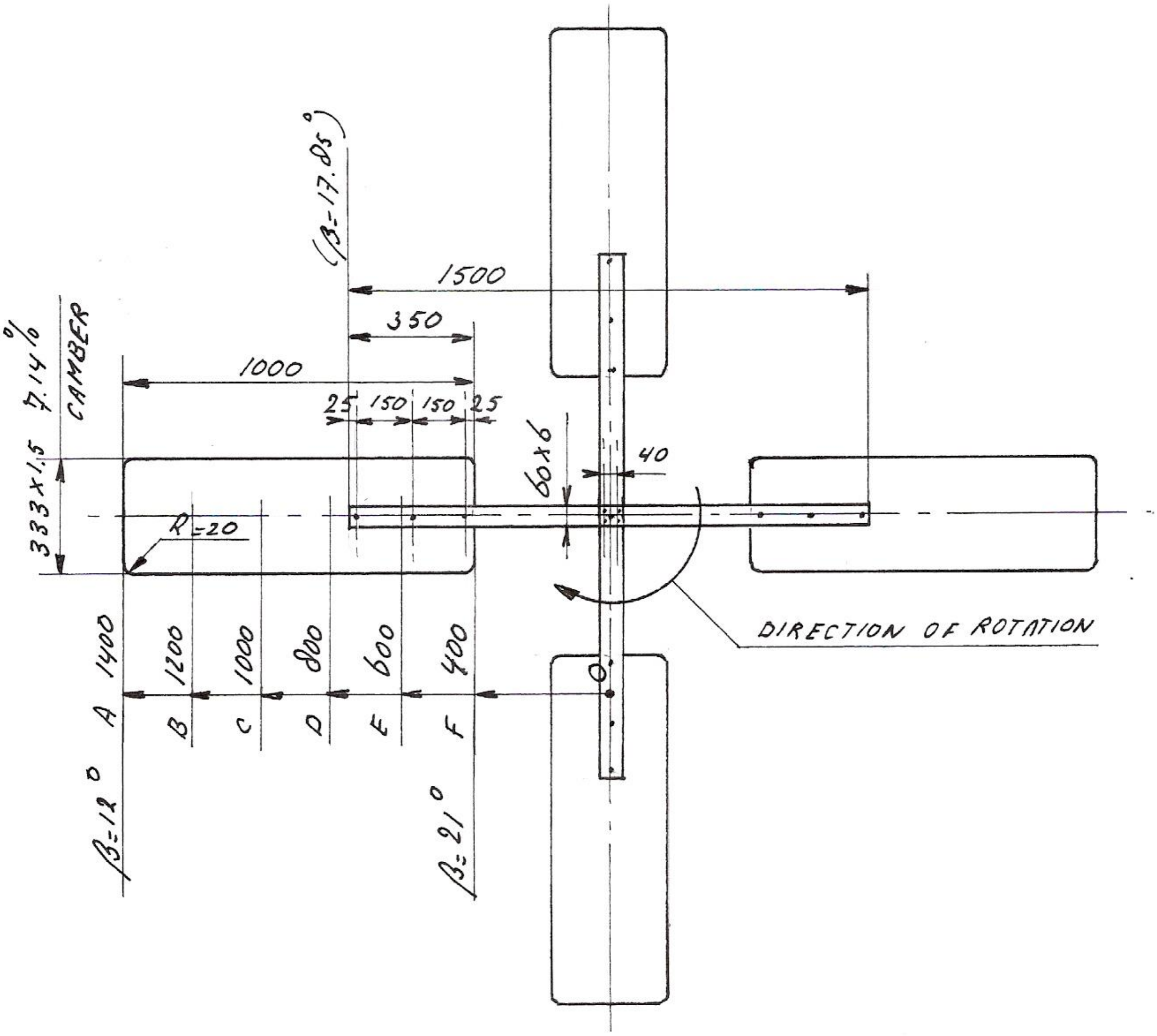

fig. 7 Sketch VIRYA-2.8B4 rotor

If the rotor is used to drive a rope pump or a generator, a shaft diameter of 25 mm will be enough. The shaft must have a halve cone angle of 5° over 45 mm. The hub length must be 47 mm. The hub is pulled to the shaft by one central M10 bolt quality at least 8.8, which has to be turned very tightly. The other four bolts at pitch 40 mm can also be M10 but can be of lower quality. One should not forget to place a clamping sheet of 60 * 60 * 6 mm on front side of the front strip to prevent too high bending stresses at the holes. The three bolts at which a blade is connected to the strip can be M10. These bolts must have a cylindrical part where the blade touches the bolt to guarantee the fitting and to prevent high surface stresses. The holes in the blade and the strip have to be made as small as possible (try 10 mm holes and make the hole pattern very accurately) to realise a rigid geometry for which the rotor balance is not changing during mounting because of clearance in between bolts and holes. A steel ring of large outer diameter has to be placed in between the bolt head of the outer bolt and the sheet to spread the thrust load at that point. A galvanised strip of 20 * 1 * 350 mm has to be placed in between blade and strip to prevent large deformation of the blade at the bolt heads. The bolt heads should be at the back side of the blade.

If the rotor is used to drive a piston pump there will be large shock forces in the pump rod and therefore also shock variation of the torque which may result in rotation of the hub over the shaft. In this case it is advised to use a 30 mm shaft with a cone angle of 5° over 50 mm and a hub length of 52 mm. Now one can use a M12 mm central bolt quality 8.8 and the clamping torque will be much larger. The other bolts can be M10.

The blade is twisted 9° in between station A and F. It might be possible to use a standard roll bender and to feed the blade into the roll bender with a certain angle in between the long side of the blade and the roll axis and then automatically realise the correct twist. One should try this to find the correct angle. Another advantage of this method is that one will have almost no straight parts at the beginning and the end of the camber.