Combination of the VIRYA-2.8B4 windmill with a rope pump

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KD 321

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Contains page

1 Introduction

Up to now Kragten Design has developed thirteen electricity generating windmills with rotor diameters in between 1.2 and 4.6 metre. Eight of these windmills are designed especially for manufacture in developing countries. The idea is to develop the VIRYA-2.8B4 water pumping windmill which has a vertical shaft in the tower which can be coupled to a rope pump or for low heads to an Archimedian screw pump. Both pumps have about the same characteristics because they loose water if they are not working for some time. This means that the starting torque is low from stand still position and therefore a windmill rotor can be used which has a not very high starting torque coefficient. It is expected that the starting torque coefficient of the VIRYA-2.8B4 rotor with λ_d = 2.5 is large enough. As an Archimedian screw pump is rather difficult to manufacture, coupling of the VIRYA-2.8B4 to a rope pump has first priority.

The VIRYA-2.8B4 windmill has a 4-bladed rotor which is provided with 7.14 % curved sheet blades. Two opposite blades are connected to each other by means of a long thin strip and that’s why the rotor is rather flexible. This eliminates vibrations due to wind turbulence and yawing. The advantage of a 4-bladed rotor is that balancing is easy and that a set of two blades and one strip can be transported even mounted. The rotor geometry and strength is calculated in report KD 319 (ref. 1).

The VIRYA-2.8B4 is equipped with the hinged side vane safety system which is used in all VIRYA windmill. This safety system has a rather large eccentricity in between the rotor shaft and the tower axis. A accelerating transmission with a gear ratio of 2.5 is described in report KD 320 (ref. 2) which is able to bridge the distance in between the rotor shaft and the vertical shaft. This transmission makes use of a 12 mm Polycord, a large wheel on the rotor shaft, a small wheel on the vertical shaft and one auxiliary wheel. In chapter 2 of KD 320 some simple theory is given which indicates that the starting coefficient of the VIRYA-2.8B4 rotor is high enough to realise a starting wind speed which is about the same as the design wind speed. This theory will be deepened in chapter 5 of this report KD 231.

An identical transmission but now with a reducing gear ratio will also be used to couple the vertical shaft of the windmill to the horizontal shaft of the rope pump. The starting behaviour of a rope pump has already been described in 2002 in report KD 103 (in Dutch, ref. 3). Parts of this report will be translated into English and used for this report KD 321.

2 Description of the rope pump

The rope pump described in this report KD 321, was developed by Bernard van Hemert and is used by Henk Holtslag in pump projects in for instance Nicaragua. The pump was originally developed as a hand pump but can also be used in combination with a windmill if this windmill is equipped with a transmission to bridge the distance in between the horizontal rotor shaft and the horizontal pump shaft. Only a transmission with a vertical shaft in the tower allows the yawing movement of the windmill head.

The pump has a horizontal axis which is running into two bearings which are placed above the well. In between the bearings is a spoke wheel which is provided with a rim which is made of two halve car tires. These halve tires are connected to each other at the outer side of the tire and this creates a V-shaped groove. A nylon or polypropylene rope is running in the groove. A large number of pistons are connected to the rope at a distance of about 1 m. There is enough friction in between the rope and the V-shaped groove to allow a certain pulling force in the rope.

The rising part of the rope is running in a PVC rising main which has a T-shaped part at the top side and a cone at the bottom side. The down going part of the rope is hanging free in the well but is guided at the bottom side by a small piece of pipe which has a cone on the upper part. Then the rope is making a loop and is going upwards again.

The lowest side of the rising main is situated at least halve a metre below the water level in the well. As soon as the pump is driven, a column of water is sucked by each piston. Because the pistons have a distance with respect to each other of only 1 m, the pressure drop over each piston is only about 0.1 bar. Therefore a the relatively large gab is allowed in between the piston and the inside diameter of the PVC pipe without resulting in a too large leaking flow. The raised water flows away through the horizontal part of the T-joint which is connected to the upper part of the rising main.

This pumping principle demands an almost constant pulling force in the rope and therefore an almost constant torque independent of the rotational speed. For a certain piston diameter and rubber wheel diameter, this torque is proportional to the height H in between the water level in the well and the outlet opening. The water can't be pumped higher than the outlet and the pump is therefore not able to supply a large pressure height above ground level.

3 Determination of the efficiency of a rope pump

A rope pump has an energetic pump efficiency η_p which is determined by the following factors:

1 The friction losses in between the pistons and the rising main and the short piece of pipe in the well.
2 The friction losses in the bearings of the pump shaft.
3 The friction losses in between the rope and the rubber rim
4 The hydraulic losses of the rope and pistons in the down going part of the rope where it is in the water of the well.
5 The leaking flow along the pistons.

The pistons have a positive gap with the rising main and the friction losses mentioned at point 1 can therefore be neglected. The down going part is flowing for only a short length in the water of the well and the hydraulic losses mentioned at point 4 can therefore be neglected too.

The friction losses in the bearings and the friction losses in between the rope and the rubber rim will cause a friction torque which is almost independent of the rotational speed and will therefore result in an almost constant friction efficiency η_f. It is assumed that $\eta_f = 0.95$.

The leaking flow depends only on the pressure difference over the pistons and on the gap in between piston and rising main. It is almost independent of the piston velocity. Therefore an almost constant leaking flow per second q_l will leak away for a certain gap independent of the rotational speed. This leaking flow causes a certain volumetric efficiency η_{vol}. The volumetric efficiency for a rope pump has a direct influence on the pump efficiency η_p because the energy needed for the leaking flow has to be generated. This is in contradiction with piston pumps where the volumetric efficiency may be caused partly by valves not closing exactly at the upper and bottom dead centre. The leaking flow will have a larger influence on the volumetric efficiency as the rotational speed is lower. The volumetric efficiency η_{vol} (not in percent but as a factor of 1) is the real flow q divided by the theoretical flow q_{th} if there would be no leaking flow, or in formula:

$$\eta_{vol} = q / q_{th} \quad (-) \qquad (1)$$

The real flow q is the theoretical flow q_{th} minus the leaking flow q_l or in formula:

$$q = q_{th} - q_l \quad (m^3/s) \qquad (2)$$

(1) + (2) gives:

$$\eta_{vol} = 1 - q_l / q_{th} \quad (-) \qquad (3)$$

The theoretical flow is the piston area minus the rope area times the piston velocity V_p. The piston velocity is the same as the rope velocity V_r. It is assumed that the piston has a diameter D_p and that the rope has a diameter D_r. This gives for q_{th} that:

$$q_{th} = \pi/4 * (D_p^2 - D_r^2) * V_p \qquad (m^3/s) \qquad (4)$$

The total energetic pump efficiency η_p is given by:

$$\eta_p = \eta_f * \eta_{vol} \qquad (-) \qquad (5)$$

(3) + (5) gives:

$$\eta_p = \eta_f * (1 - q_l / q_{th}) \qquad (-) \qquad (6)$$

Now it is assumed that the pump has a energetic efficiency $\eta_p = 0.8$ for the design wind speed V_d. The design wind speed is the wind speed for which the rotor runs at its design tip speed ratio $\lambda_d = 2.5$. For this design wind speed, the design flow q_d is supplied and the piston is moving with the design piston speed V_{pd}.
Substitution of $\eta_p = 0.8$ and $\eta_f = 0.95$ and $\eta_{vol} = \eta_{vol\ d}$ in formula 5 gives that $\eta_{vol\ d} = 0.842$.

For the determination of the starting behaviour it is important to know how the volumetric efficiency varies with the piston velocity.

(3) + (4) gives:

$$\eta_{vol} = 1 - \frac{q_l}{\pi/4 * (D_p^2 - D_r^2) * V_p} \qquad (-) \qquad (7)$$

Substitution of $V_p = V_{pd}$ and $\eta_{vol} = \eta_{vol\ d} = 0.842$ in formula 7 gives that:

$$0.842 = 1 - \frac{q_l}{\pi/4 * (D_p^2 - D_r^2) * V_{pd}} \qquad (-) \qquad (8)$$

This can be written as:

$$q_l = 0.158 * \pi/4 * (D_p^2 - D_r^2) * V_{pd} \qquad (-) \qquad (9)$$

(7) + (9) gives that:

$$\eta_{vol} = 1 - 0.158 * V_{pd} / V_p \qquad (-) \qquad (10)$$

This function is given in figure 1 for $0.1 < V_p / V_{pd} < 3$. It appears to be easy to use V_p / V_{pd} in stead of V_{pd} / V_p on the x-axis. The function is intersecting with the x-axis for $\eta_{vol} = 0$. This gives that $V_p = 0.158 * V_{pd}$. The piston speed V_p is proportional to the rotational speed n of the rotor. So the ratio V_p / V_{pd} can be replaced by n / n_d.

When the volumetric efficiency becomes smaller than zero it means that more water is leaking away than water that is pumped upwards. So below n / n_d = 0.158, no water will be pumped and the water level in the rising main will become smaller than the height H. Therefore the required torque will decrease too and it will become zero when the height of the water column has become zero.

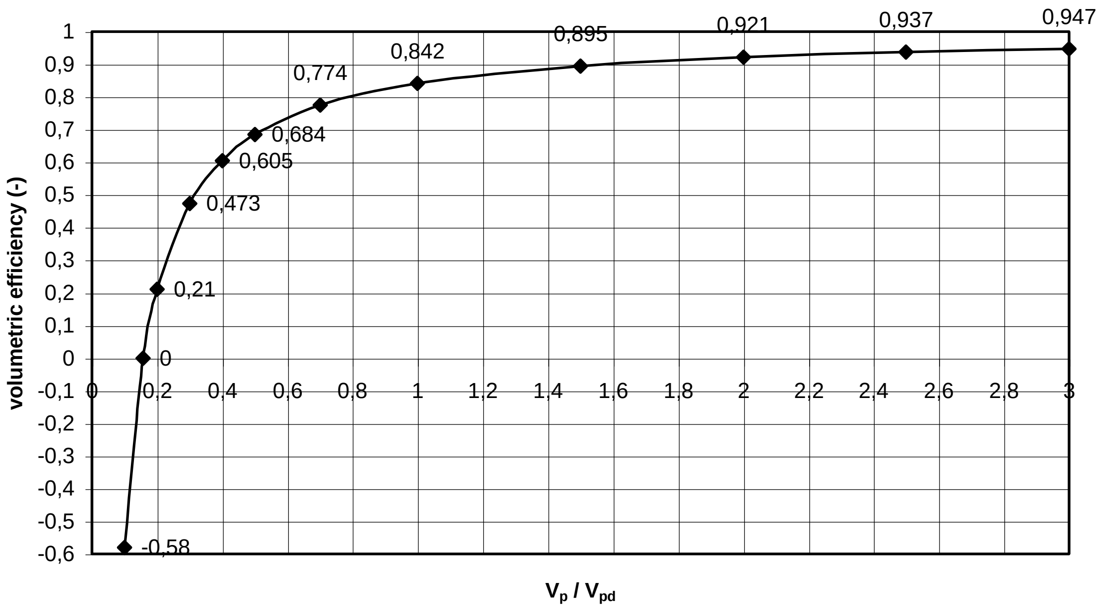

fig. 1 η_{vol} as a function of V_p / V_{pd}

4 Deriving of the formulas for windmill and rope pump

The general wind energy theory and the design procedure for a windmill rotor is given in report KD 35 (ref. 4). The rotor is characterised by the C_p-λ or the C_q-λ curve. C_p is the power coefficient, C_q is the torque coefficient and λ is the ratio in between the tip speed and the undisturbed wind speed V. The rotor power P for a rotor perpendicular to the wind is given by formula 4.1 of KD 35 copied as formula 11.

$$P = C_p * \tfrac{1}{2}\rho V^3 * \pi R^2 \qquad (W) \qquad (11)$$

The rotor torque Q for a rotor perpendicular to the wind is given by formula 4.3 of KD 35 copied as formula 12.

$$Q = C_q * \tfrac{1}{2}\rho V^2 * \pi R^3 \qquad (Nm) \qquad (12)$$

In these formulas, ρ is the density of air (about 1.2 kg/m^3 for a temperature of 20° C at sea level), V is the undisturbed wind speed (m/s) and R is the radius of the rotor at the blade tip (m). R is half the rotor diameter. The relation in between λ, C_p and C_q is given by formula 4.5 of KD 35 copied as formula 13.

$$\lambda = C_p / C_q \qquad (-) \qquad (13)$$

This means that the C_p-λ curve can be derived from the C_q-λ curve by multiplying every C_q value by λ. It also means that the C_q-λ curve can be derived from the C_p-λ curve by dividing every C_p value by λ (except for λ = 0). The rotational speed n for a rotor perpendicular to the wind is given by formula 4.8 of KD 35 copied as formula 14.

$n = 30 * \lambda * V / \pi R$ (rpm) (14)

The rotational speed of the shaft of the rope pump n_p is given by:

$n_p = n * i_u * i_l$ (rpm) (15)

In this formula i_u is the accelerating gear ratio of the upper Polycord transmission in between the rotor shaft and the vertical shaft. In this formula i_l is the accelerating gear ratio of the lower Polycord transmission in between the vertical shaft and the shaft of the rope pump. In reality, this lower gearing must be decelerating for a rope pump, so the gear ratio will be smaller than 1. For instance, a decelerating gear ratio of 4 means an accelerating gear ratio of 1 / 4 = 0.25. So a gearing with for instance a 120 mm wheel on the vertical shaft and a 480 mm wheel on the pump shaft will have an accelerating lower gear ratio i_l = 120 / 480 = 0.25.

The mechanical power supplied to the pump P_p is lesser than the rotor power because of the transmission efficiency of the upper gearing $\eta_{tr\ u}$ and the transmission efficiency of the lower gearing $\eta_{tr\ l}$. This gives:

$P_p = \eta_{tr\ u} * \eta_{tr\ l} * P$ (W) (16)

The hydraulic power P_{hyd} supplied to the water is lesser than the mechanical pump power because of the pump efficiency η_p. This gives:

$P_{hyd} = \eta_p * P_p$ (W) (17)

The pump efficiency is explained in chapter 2.

(16) + (17) gives:

$P_{hyd} = \eta_{tr\ u} * \eta_{tr\ l} * \eta_p * P$ (W) (18)

(11) + (18) gives:

$P_{hyd} = \eta_{tr\ u} * \eta_{tr\ l} * \eta_p * C_p * ½\rho V^3 * \pi R^2$ (W) (19)

Now we go to the rope pump. The hydraulic power P_{hyd} is given by:

$P_{hyd} = \rho_w * g * H * q$ (W) (20)

In this formula ρ_w is the density of water (ρ_w = 1000 kg/m^3), g is the acceleration of gravity (g = 9.81 m/s^2), H is the static height between the water level in the well and the outlet opening of the T-joint and q is the real flow (m^3/s).

The theoretical flow q_{th} is the flow which would be gained if there is no leaking flow, so for η_{vol} = 1. The theoretical flow is given by:

$q_{th} = V_r * \pi/4 (D_p^2 - D_r^2)$ (m^3/s) (21)

In this formula V_r is the rope velocity which is the same as the piston velocity V_p. D_p is the piston diameter and D_r is the rope diameter. The real flow q is the theoretical flow times the volumetric efficiency or in formula:

$q = \eta_{vol} * V_r * \pi/4 (D_p^2 - D_r^2)$ (m^3/s (22)

The rope velocity depends on the pitch diameter of the rubber wheel of the rope pump D_w and on the rotational speed of the shaft of the rope pump n_p, in formula:

$$V_r = \pi * n_p * D_w / 60 \qquad (23)$$

(15) + (23) gives:

$$V_r = \pi * n * i_u * i_l * D_w / 60 \qquad (24)$$

(14) + (24) gives:

$$V_r = \lambda * V * i_u * i_l * D_w / 2 R \qquad (25)$$

(22) + (25) gives:

$$q = \eta_{vol} * \lambda * V * i_u * i_l * D_w * \pi (D_p^2 - D_r^2) / 8 R \quad (m^3/s) \qquad (26)$$

(20) + (26) gives:

$$P_{hyd} = \rho_w * g * H * \eta_{vol} * \lambda * V * i_u * i_l * D_w * \pi (D_p^2 - D_r^2) / 8 R \quad (W) \qquad (27)$$

(19) + (27) gives:

$$4 * \eta_{tr\,u} * \eta_{tr\,l} * \eta_p * C_p * \rho * V^2 * R^3 = \rho_w * g * H * \eta_{vol} * \lambda * i_u * i_l * D_w * (D_p^2 - D_r^2) \qquad (28)$$

In formula 28 all the parameters of the windmill, the transmissions, the rope pump, the wind speed and the height are linked together. The pump geometry is normally determined for the design wind speed V_d. For this wind speed, the rotor runs at the design tip speed ratio λ_d and has its maximum C_p value $C_{p\ max}$. The pump has the design volumetric efficiency $\eta_{vol\ d}$. Substitution of $C_p = C_{p\ max}$, $V = V_d$, $\eta_{vol} = \eta_{vol\ d}$ and $\lambda = \lambda_d$ in formula 28 gives that:

$$4 * \eta_{tr\,u} * \eta_{tr\,l} * \eta_p * C_{p\,max} * \rho * V_d^2 * R^3 = \rho_w * g * H * \eta_{vol\,d} * \lambda_d * i_u * i_l * D_w * (D_p^2 - D_r^2) \qquad (29)$$

Formula 29 can be written in different ways depending on what parameter one wants to calculate. The use of formula 29 or a changed version of it, will now be explained in two examples.

4.1 Example 1

Assume the VIRYA-2.8B4 rotor is chosen. This gives R = 1.4 m, $\lambda_d = 2.5$ and $C_{p\ max} = 0.38$.
Assume the upper transmission has a gear ratio $i_u = 2.5$ and an efficiency $\eta_{tr\ u} = 0.95$.
Assume the lower transmission has a gear ratio $i_l = 1/4 = 0.25$ and an efficiency $\eta_{tr\ l} = 0.95$.
Assume the rubber wheel on the pump shaft has a pitch diameter $D_w = 0.5$ m.
Assume the rising main is made of 40 mm PVC pipe with a wall thickness of 3 mm. This means that the piston diameter $D_p = 0.034$ m. Assume that the rope diameter $D_r = 0.008$ m.
Assume the energetic pump efficiency $\eta_p = 0.8$ and the volumetric efficiency $\eta_{vol\ d} = 0.842$.
Assume H = 8.2 m, $\rho_w = 1000$ kg/m^3, $\rho = 1.2$ kg/m^3 and g = 9.81 m/s^2.
What is the design wind speed for this combination of windmill rotor and pump? What is the design rope speed and what is the design flow?

Formula 29 can be written as:

$$V_d = \sqrt{\{\rho_w * g * H * \eta_{vol\,d} * \lambda_d * i_u * i_l * D_w (D_p^2 - D_r^2) / (4\,\eta_{tr\,u} * \eta_{tr\,l} * \eta_p * C_{p\,max} * \rho * R^3)\}} \quad \text{(m/s)} \qquad (30)$$

Substitution of the assumed values in formula 30 gives V_d = 4.0 m/s. This is an acceptable wind speed except for regions with a very low wind regime.

The design rope speed V_{rd} can be calculated by formula 25. Substitution of $\lambda = \lambda_d = 2.5$, $V = V_d$ = 4 m/s, i_u = 2.5, i_l = 0.25, D_w = 0.5 m and R = 1.4 m gives V_{rd} = 1.116 m/s. This is rather fast but it seems acceptable. The maximum rope speed will occur at the rated wind speed and will be much higher.

The design flow q_d can be calculated by formula 26. Substitution of η_{vol} = 0.842, $\lambda = \lambda_d = 2.5$, $V = V_d$ = 4 m/s, i_u = 2.5, i_l = 0.25, D_w = 0.5 m, D_p = 0.034 m, D_r = 0.008 m and R = 1.4 m in formula 26 gives q_d = 0.000806 m^3/s or 2.90 m^3/hour. This is an acceptable flow which means that the VIRYA-2.8B4 with a rope pump can be used for irrigation.

4.2 Example 2

Assume the same windmill and the same pump are used in a region with rather low wind speeds. Assume the maximum design wind speed V_d = 3 m/s. Assume the height H = 20 m. What is the required gear ratio of the lower transmission?

Formula 29 can be written as:

$$i_l = 4\,\eta_{tr\,u} * \eta_{tr\,l} * \eta_p * C_{p\,max} * \rho * V_d^2 * R^3 / \{\rho_w * g * H * \eta_{vol\,d} * \lambda_d * i_u * D_w * (D_p^2 - D_r^2)\} \quad (-) \qquad (31)$$

Substitution of the assumed values in formula 31 gives that i_l = 0.1026. This is the increasing gear ratio. This means that the reducing gear ratio is 1 / 0.1026 = 9.75. If the pitch diameter of the Polycord wheel on the vertical shaft is taken 120 mm it means that the pitch diameter of the Polycord wheel on the pump shaft must be 120 * 9.75 = 1170 mm which is extremely large. So it seems to be not possible to realise the required gear ratio in one step if a Polycord transmission is used. Suppose the lower gearing is made in two steps. This results in a somewhat lower transmission efficiency but this effect is neglected at this moment.

Suppose for the first step a Polycord transmission with i = 0.25 is used. This first step transforms the vertical shaft into an horizontal auxiliary shaft. For the second step it is necessary to use a belt with can have a larger pulling force than a 12 mm Polycord because the wheel diameter of the small wheel of the second step will be smaller than the big wheel of the first step. Assume a V-belt is used for the second step. This belt must have an acceleration gear ratio of 0.1026 / 0.25 = 0.4104 or a decelerating gear ratio of 1 / 0.4104 = 2.44. This is possible with a small size V-belt.

Another possibility to realise a reducing gear ratio of about 9.75 in one step is to use a worm wheel transmission which is coupled directly to the vertical shaft. This transmission automatically transforms the vertical shaft into a horizontal shaft. But it is uncertain if worm wheel transmissions are available in developing countries for a reasonable price. An extra disadvantage of a worm wheel transmission is that the efficiency is much lower than for a Polycord transmission. But mechanically, the construction is rather simple, especially if the worm wheel transmission has a hollow shaft so that the gear box can be hung directly on the pump shaft.

5 Determination of the starting behaviour of the VIRYA-2.8B4 rotor

The first part of this chapter is derived from chapter 2 of report KD 320 (ref. 2). The determination of the Q-n curves of the rotor for different wind speeds is given in figure 4 of chapter 5 of report KD 319 (ref. 1). The Q-n curves are given for a 6 mm plywood vane blade which is so light that the rated wind speed is 8 m/s. Figure 4 of report KD 319 is copied as figure 2. In this figure, n is the rotational speed of the rotor shaft and Q is the rotor torque.

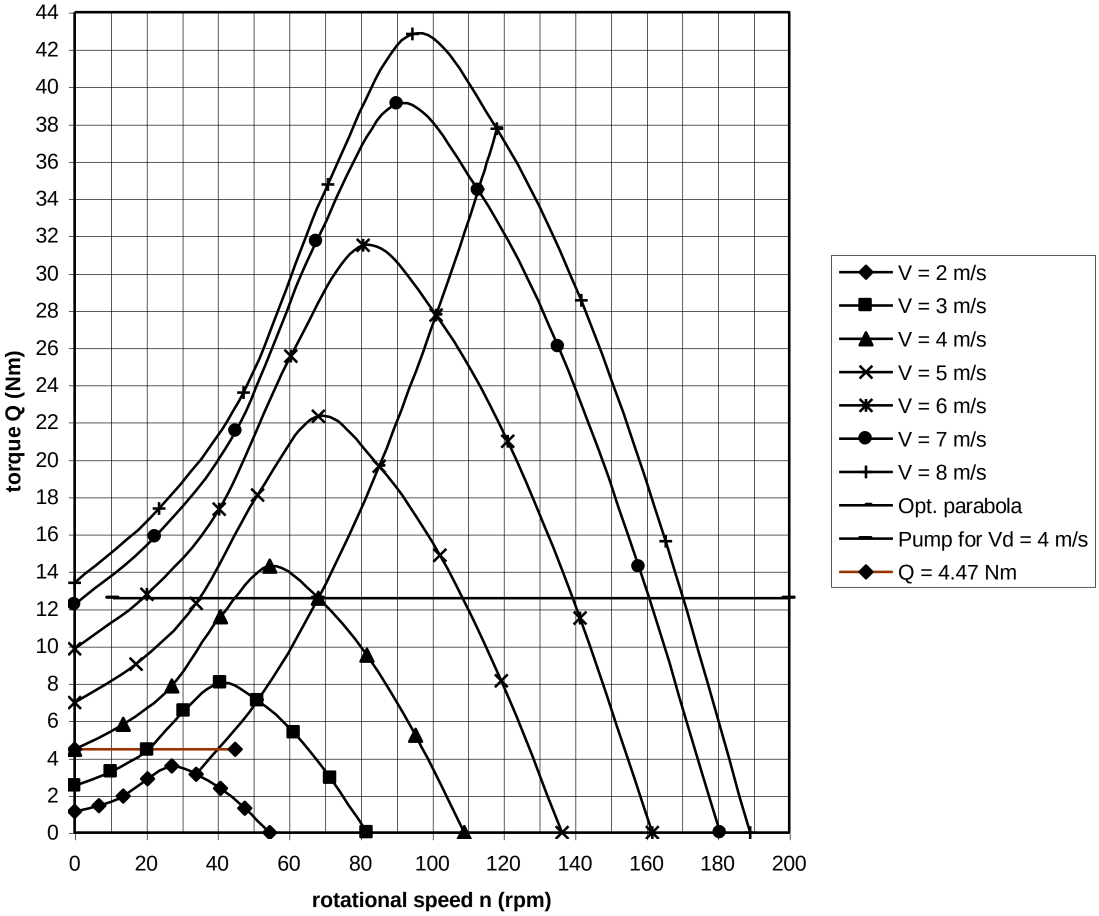

fig. 2 Q-n curves for VIRYA-2.8B4 rotor for a 6 mm plywood vane blade

In figure 2 the optimum parabola is also given. For this curve, the rotor is running at the design tip speed ratio λ_d = 2.5 and it has its maximum power coefficient $C_{p\,max}$ = 0.38.

Now suppose that the gear ratios and the pump dimensions and the water height are chosen such that the design wind speed is 4 m/s like it was found for example 4.1. This means that the torque curve of the pump seen on the rotor shaft is a horizontal line which goes through the point of intersection of the optimum parabola and the P-n curve of the rotor for V = 4 m/s. This point is lying at a rotational speed of 68.2 rpm and a torque of 12.58 Nm (see figure 2). The rotational speed where the volumetric efficiency becomes zero will lie at a rotational speed of 0.158 * 68.2 = 10.8 rpm (see fig. 1). So above this rotational speed, water is pumped and the torque is constant. The horizontal torque line of the pump seen on the rotor shaft for V_d = 4 m/s is also drawn in figure 1. This line is intersecting with the P-n curve of the rotor for V = 8 m/s (and higher) at a rotational speed of about 170 rpm which is a little lower than the maximum unloaded rotational speed of 189 rpm. The tip speed ratio at V = 8 m/s is about 3.6 which is also only a little lower than the unloaded tip speed ratio of 4. The maximum loaded rotational speed of 170 rpm is a factor 170 / 68.2 = 2.49 higher than the design rotational speed.

This means that the maximum rope speed $V_{r\,max}$ is also a factor 2.49 higher. So the maximum rope speed becomes 2.49 * 1.116 = 2.78 m/s. It has to be checked if this maximum rope speed is allowed. The maximum flow is even more than a factor 2.49 higher than the design flow of 0.000806 m^3/s because the volumetric efficiency at the maximum rotational speed is higher than at the design rotational speed (about 0.937 in stead of 0.842, see figure 1). So the maximum flow q_{max} is about a factor 2.49 * 0.937 / 0.842 = 2.77 higher than at the design rotational speed. So the maximum flow becomes 2.77 * 0.000806 = 0.002233 m^3/s or 8.04 m^3/ hour.

In figure 2 it can be seen that the beginning point n = 10.8 rpm of the horizontal Q-n line of the pump is intersecting with the estimated the P-n curve of the rotor for V = 6.5 m/s. The first impression therefore is that a wind speed of 6.5 m/s is required to start the rotor. However, this is not true because, if the rotor is standing still for a certain time, all the water in the rising main has leaked back into the well. So for this condition, the pump torque is zero. The pump torque increases proportional to the water level in the rising main.

Now suppose that the wind speed increases suddenly from 0 to 4 m/s. In figure 2 it can be seen that the starting torque of the rotor for V = 4 m/s is 4.47 Nm and that the torque is rising directly if the rotational speed increases. So the rotor will start and almost the whole torque will be used for acceleration of the rotor. But during this acceleration, the water level in the rising main will rise and the required pump torque will rise too.

In figure 2 is can be seen that the Q-n line of the pump has a left point of intersection with the Q-n line of the rotor for V = 4 m/s at a rotational speed of 45 rpm. For higher rotational speeds than 45 rpm, the rotor is certainly able to supply the required pump torque. A rotational speed of 45 rpm is a factor 0.66 of the design rotational speed of 68.2 rpm. In figure 1 it can be seen that η_{vol} = 0.78 for this factor. It is expected that the rotor accelerates so fast from stand still position that it reaches this rotational speed of 45 rpm before the water in the rising main has reached the maximum level of 8.2 m. The real starting wind speed will therefore be much lower than 6.5 m/s and it is expected that the rotor will start at least at a wind speed equal to the design wind speed which was chosen 4 m/s. It will now be investigated if this assumption is true.

It is assumed that the water level in the rising main is zero and that the wind speed rises suddenly from 0 m/s to 4 m/s. To describe exactly what will happen is rather difficult for the following reasons:

1 The rotor torque is not constant but is increasing as the rotational speed increases. How it increases is given by the shape of the Q-n curve of the rotor for V = 4 m/s for rotational speeds in between 0 and 45 rpm (see figure 2).

2 The volumetric efficiency of the pump is zero below n = 10.8 rpm but it is increasing very strongly at increasing rotational speed above n = 10.8 rpm (see figure 1). Therefore it is difficult to determine what water height will be realised after a certain time.

3 The available rotor torque is not only used to supply the required pump torque but also to accelerate the rotor. In the beginning when the pump torque is zero the whole rotor torque is used for acceleration of the rotor.

Description of the starting behaviour is now simplified. A horizontal line is drawn through the starting torque Q = 4.47 Nm for V = 4 m/s and n = 0 rpm. It is assumed that this constant torque is available for acceleration of the rotor and that the part of the Q-n curve of the rotor above this horizontal line is used to supply the pump torque. This horizontal line for $0 < n < 45$ rpm is also given in figure 2.

Now it will first be calculated how long it takes for the rotor to reach a rotational speed of 45 rpm and how high the water is risen in the rising main for this time. The relation in between the torque Q (Nm), the angular acceleration dω/dt (rad/s^2) and the mass moment of inertia of the rotor I (kgm^2) is given by:

$$Q = I * d\omega/dt \qquad (Nm) \qquad (32)$$

Formula 32 can be written as:

$d\omega/dt = Q / I$ (rad/s^2) (33)

The mass moment of inertia I of the VIRYA-2.8B4 rotor is determined and is about 15.5 kgm^2. Assume Q = 4.47 Nm. Substitution of these values in formula 33 gives that $d\omega/dt = 0.288$ rad/s^2.

The relation in between the rotational speed n and the angular velocity ω is given by:

$\omega = \pi * n /30$ (rad/s) (34)

Substitution of n = 45 rpm in formula 35 gives that $\omega = 4.71$ rad/s. The relation in between the rotational speed ω which is realised after acceleration with an angular acceleration $d\omega/dt$ during a certain time t from stand still position is given by:

$\omega = d\omega/dt * t$ (rad/s) (35)

formula 35 can be written as:

$t = \omega / d\omega/dt$ (s) (36)

Substitution of $\omega = 4.71$ rad/s and $d\omega/dt = 0.288$ rad/s^2 in formula 36 gives t = 16.35 s.

The total angle α (rad) for which the rotor has rotated from stand still position is given by:

$\alpha = ½ \, d\omega/dt * t^2$ (rad) (37)

Substitution of $d\omega/dt = 0.288$ rad/s^2 and t = 16.35 s in formula 37 gives $\alpha = 38.49$ rad = 6.13 revolutions (because 1 rad = 1 / 2π revolutions). If it is chosen that i_u is 2.5 and that $i_l = 0.25$ like it was done in example 4.1, it means that the rotational speed of the pump shaft is a factor 0.625 lower than the rotational speed of the rotor. So 6.13 revolutions of the rotor corresponds to 6.16 * 0.625 = 3.85 revolutions of the pump shaft.

For each revolution of the pump shaft a length of $\pi * D_w = \pi * 0.5 = 1.571$ m rope will be lifted. So for 3.85 revolutions of the pump shaft a total length of 3.85 * 1.571 = 6.048 m rope will be lifted. If the volumetric efficiency would be 1, this means that the water would have been risen up to a height of 6.048 m which is lower than the static height H = 8.2 m. But the average volumetric efficiency will be much lower during starting. It is zero below n = 10.8 rpm and it is 0.78 for n = 45 rpm. If it is supposed that the average volumetric efficiency is only 0.33, the water level would have been risen only till a level of 0.33 * 6.048 = 2 m. This is a factor 0.242 of the static height H = 8.2 m. This means that the required torque is only a factor 0,242 of the design torque Q_d = 12.58 Nm. So the required torque to lift the water over a height of 2 m is 3.07 Nm.

For the acceleration of the rotor a torque of 4.47 Nm was used. So the total required torque at n = 45 rpm is 4.47 + 3.07 = 7.54 Nm. This is lower than the torque of 12.58 Nm which can be supplied at n = 45 rpm. This indicates that it is acceptable to assume that almost all torque is used to accelerate the rotor. So the real situation is more favourable than the situation as used for the calculation. It can be concluded that the rotor will certainly start from stand still position if the wind speed becomes the same as the design wind speed. In reality the starting wind speed might be somewhat lower and will be about 3.5 m/s for a design wind speed of 4 m/s. This has to be verified in the field for a real prototype.

5 References

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