Derivation of the formulas for torque and volumetric efficiency for a single acting piston pump with a floating valve

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Contains page

1 Introduction

Single acting piston pumps are widely used in combination with water pumping windmills. These pumps are rather simple and have a good energetic efficiency. However, a disadvantage is that the torque is very fluctuating and therefore a heavy multi-bladed windmill rotor with a high starting torque coefficient is required, other wise the starting wind speed will be too high. A high starting torque coefficient is only realised if the rotor has a very low design tip speed ratio of about 1. Rotors with a design tip ratio of about 2 are much lighter but have a much lower starting torque coefficient and this requires a starting help device in the pump. This problem is analysed in chapter 4 of report KD 294 (ref. 1).

Research to different options for a starting help device have been executed by the former CWD, Consultancy Service, Wind Energy, Developing Countries, which was active from 1975 up to 1990. I have worked as a designer of water pumping windmills during this period at the Wind Energy Group of the University of Technology Eindhoven, which was a member of CWD. The floating valve appeared to be the most promising starting help device and it was tested first in the CWD 67 S pump which belonged to the CWD 2000 windmill. The number 67 indicates the inside diameter of the cylinder in mm and the S indicates suction, which means that the pump is mounted just above ground level.

I have forgotten who came up with the idea of the floating valve but I have made the first pump drawings and I did some tests to find out how a strong floating valve can be manufactured. These tests are described in report R 1009 D (ref. 2). In this report some references are given to pump measurements but I don’t have these reports and as far as I know no derivation of the formula for the pump torque of a pump with a floating valve is given.

A pump with a floating valve can also be used in combination with a solar panel. This use is described in report R 1192 S (ref. 3). In chapter 3.5 of this report, the pump torque is given by formula 11. This formula originates from an internal note out of 1992 written by Jan Diepens and Paul Smulders. I have contacted Jan Diepens about this note but he said that he could not find it. So I decided to derive the formula myself.

The reason why I am enlivening the knowledge about the floating valve is that I was asked by the Dutch organisation Practica to make a composite drawing for a small, water pumping windmill, the Practica-2. This windmill will be equipped with a single acting piston pump with a floating valve. For correct matching of the pump with the windmill rotor and for the determination of the pump geometry I need the curves of the torque and the volumetric efficiency.

2 Description of the functioning of a normal piston pump

First the functioning of a piston pump with no floating valve will be described. The pump is made of a cylinder in which the piston is moving. The upper part of the cylinder is connected to the raising main through which the water flows to the outlet pipe. The lower part of the cylinder contains the foot valve. The piston is connected to the transmission by means of the pump rod. The piston contains the piston valve and a leather cup to prevent water leaking along the piston. Most pumps are mounted under the water level of the well but some pumps have a suction pipe and are mounted some meters above the water level.

If the piston moves upwards, the foot valve is open and the piston valve is closed so the water is lifted upwards. If accelerating forces are neglected, the required force to lift the water is constant during this upwards stroke. This constant force requires a torque in the pump shaft which is varying about sinusoidal if the crank radius is small with respect to the length of the crank rod. This torque has a maximum, half way the stroke.

If the piston moves downwards, the foot valve is closed and the piston valve is open. If the friction in between the leather cup and the pressure drop over the piston valve is neglected and if the weight of the pump rod is balanced, the force during the downwards stroke is zero. This means that the torque during the downwards stroke is zero too.

It can be proven that the average torque during one revolution of the rotor shaft is $1/\pi$ times the peak torque which occurs if the crank is half way the stroke, so for a crank angle $\alpha = 90°$. So the peak torque is π times the average torque. The torque depends on the pump geometry and the height. The torque is made dimensionless by dividing it by the average torque. So the dimensionless average torque is taken 1. The variation of the dimensionless torque and the dimensionless average torque as a function of the crank angle α is given in figure 1.

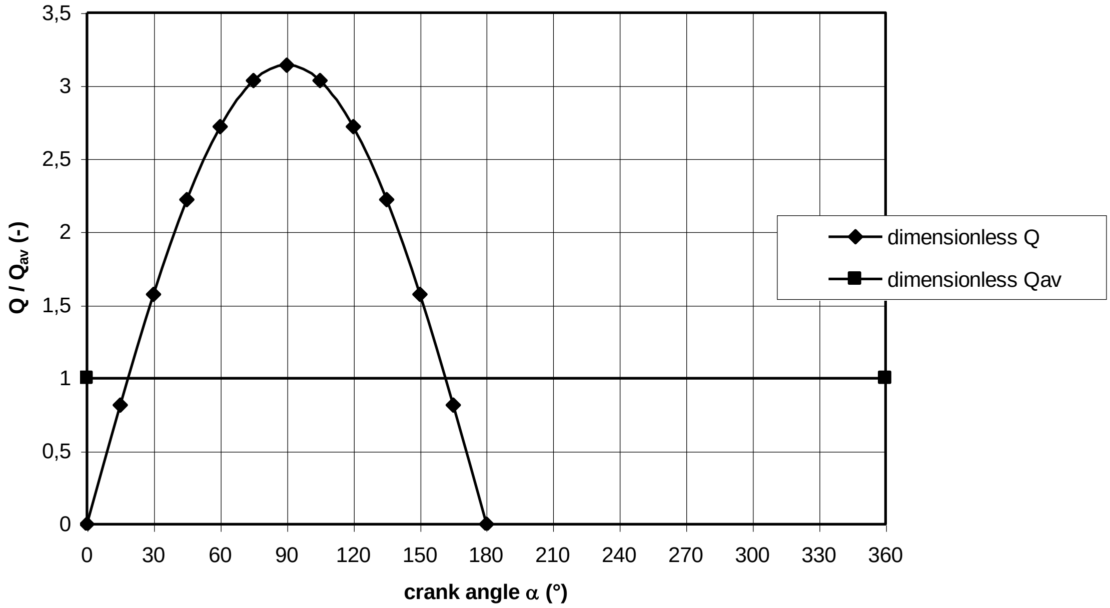

fig. 1 Dimensionless torque Q / Q_{av} and dimensionless average torque Q_{av} / Q_{av} as a function of the crank angle α

The Q-α curve is intersecting with the Q_{av}-α line at $\alpha = 18.56°$ and $\alpha = 161.44°$. Now suppose that the rotor is supplying a torque equal to the average pump torque. This means that the real pump torque is larger than the rotor torque for $18.56° < \alpha < 161.44°$. So the shortage of torque and so of energy, is taken from the kinetic energy available in the rotor which results in a decreasing rotor speed. How much the rotor speed decreases depends on the flywheel effect of the rotor and this depends on the moment of inertia of the rotor and on the rotational speed. For the interval $161.44° < \alpha < 378.56°$, the rotor torque is larger than the pump torque and for this interval the rotor is accelerating. Because of this variation of the rotor speed, the tip speed ratio of the rotor will vary too. This results in variation of the torque coefficient and so in variation of the torque. For the time being it is assumed that the fluctuation of the rotational speed and the rotor torque because of the flywheel effect can be neglected.

3 Description of the functioning of a piston pump with a floating valve

For a pump with a floating valve, the piston valve is replaced by a floating valve. This valve must have an average density of $0.5 * 10^3$ kg/m^3 or less to create a buoyancy force which is large enough without resulting in a very large valve height. Several options of valve material and valve manufacture have been tried. No material was found which was light enough to make massive valves which are strong enough and which are not absorbing too much water after a certain time. The best result was to make a hollow valve out of two polypropylene halves which are melted together (see R 1009 D chapter 3.8).

The piston is made of two brass disks and in between both a leather cup is clamped (see figure 2). The lowest disk has an outside diameter which is about 2 mm smaller than the inside diameter of the cylinder. The leather cup has a thickness of about 4 mm and the diameter of the upper disk is about 10 mm smaller than the inside diameter D of the cylinder. Both disks have a central hole and the diameter of this hole depends on the required strength of the pump rod. This diameter was about 10 mm for the CWD 67 S pump. I have no drawings of this pump available so dimensions for this pump given in between brackets are estimated! Assume the lowest part of the pump rod is made of massive stainless steel rod of 12 mm which is reduced to 10 mm at the upper side of the upper disk. So the central holes in the disks and the leather cup have a diameter of 10 mm. The lowest part of the pump rod is provided with thread M10 and the discs and leather cup are clamped together with one self locking nut M10.

Six holes are drilled in both disks and in the leather cup. The diameter of a hole D_h (12 mm) must be chosen such that enough material is available in between the holes to keep the piston strong enough. The floating valve is centred around the 12 mm pump rod, so the inside diameter of the floating valve must be about 12.5 mm. The floating valve has a certain height H (60 mm), and there must be a stop on the pump rod at a distance H + s_v. This stop must have a rubber sheet below it to prevent damage of the floating valve when it hits the stop. The valve stroke s_v must have a certain value. Assume that the total area of the six holes in the piston is called A_{6h}. A_{6h} is given by:

$$A_{6h} = 6 * \pi/4 * D_h^2 \quad (mm^2) \qquad (1)$$

Assume that the pitch diameter at the holes is called D_p (32 mm). The gap area A_{gp} at the pitch diameter D_p is given by:

$$A_{gp} = \pi * D_p * s_v \quad (mm^2) \qquad (2)$$

From D_p, the water flows radially to the outside of the valve. Assume that the outside diameter of the valve is called D_v (50 mm). This diameter must be chosen so large that there is some mm overlap in between the valve and the outside of the six holes. However, it must be chosen so small that the ring shaped area in between the outside of the valve and the inside of the leather cup is large enough. This ring shaped area must be at least A_{6h}. The gap area A_{gv} in between the valve and the upper disk at the outside of the valve is given by:

$$A_{gv} = \pi * D_v * s_v \quad (mm^2) \qquad (3)$$

The valve stroke s_v (4.3 mm) must be chosen so large that:

$$A_{6h} = A_{gv} \qquad (4)$$

(1) + (3) + (4) gives:

$$s_v = 1.5 * D_h^2 / D_v \quad (mm) \qquad (5)$$

For this situation, the water speed in the holes is the same as the water speed in between the gap at the outside of the valve (if the valve is in the upper position). As D_p is a lot smaller than D_v, A_{gp} will be a lot smaller than A_{gv}. The water speed at D_p will therefore be much higher than at D_v. So the average water speed in the gap will be higher than the water speed in the six holes. The law of Bernoulli says that the pressure decreases, when the velocity increases. So the increase of the water speed results in a decrease of pressure. The average under pressure multiplied by the area of the valve causes a downwards force on the valve. The valve will close when this downwards force is larger than the buoyancy force.

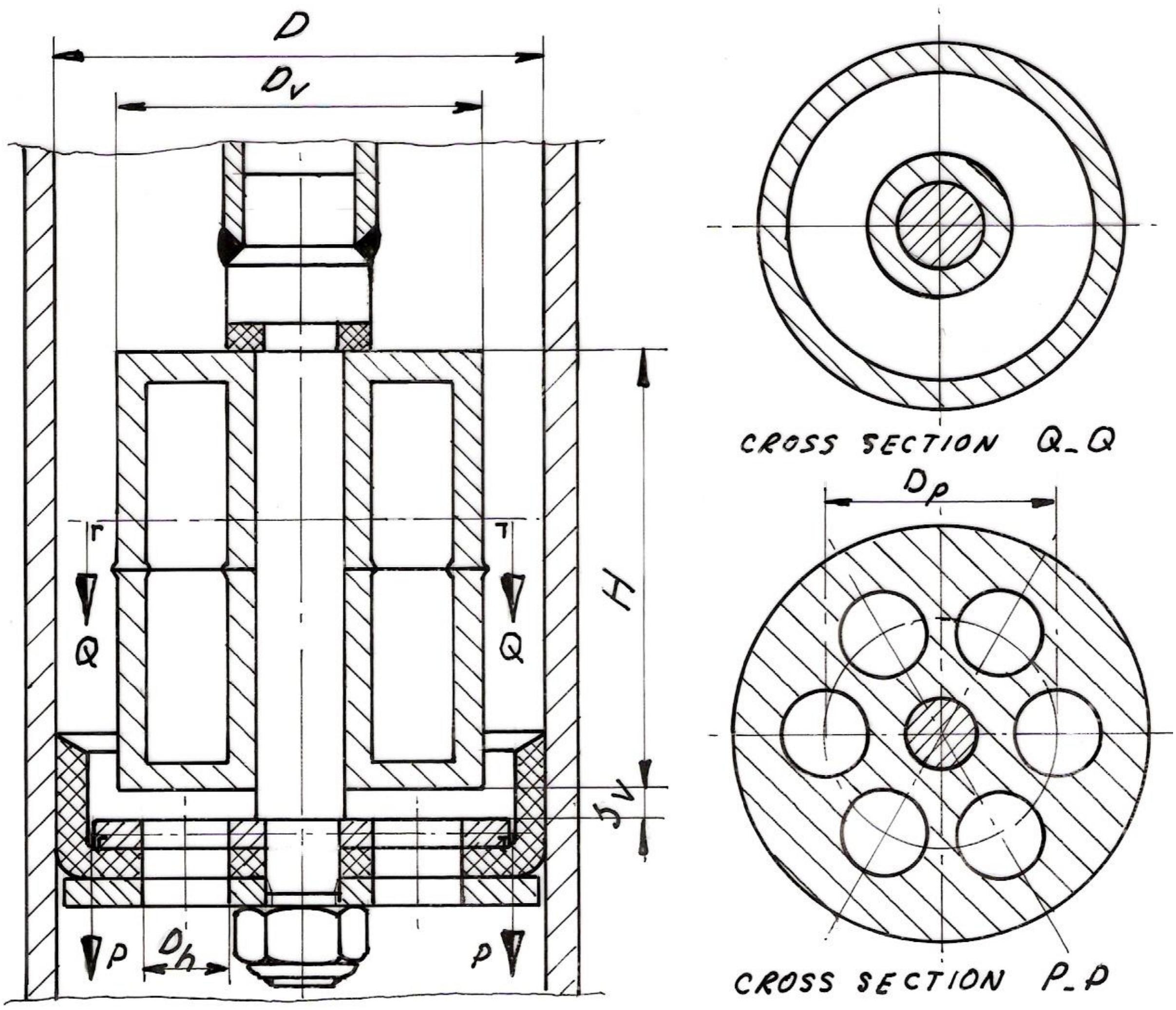

fig. 2 Cross section of piston and floating valve

The law of Bernoulli is a law which sometimes works against your feeling of common sense. So reducing the valve stroke results in increase of the downwards force on the valve! The average velocity of the water speed in the gap is proportional to the upwards speed of the piston. This upwards speed of the piston depends on the rotational speed of the crank shaft and on the crank angle α. The maximum upwards speed for a certain rotational speed is realised for $\alpha = 90°$, so half way the stroke. So the valve will close for the first time halve way the stroke and it will close at smaller angles when the rotational speed is higher. This procedure is explained in detail in chapter 4. Once the valve is closed, there will be a pressure drop over the valve due to the static water height and the valve will only open during the downwards stroke.

If the rotor is rotating slowly, the piston velocity for which the valve closes will not be reached, even not for $\alpha = 90°$. This means that the water is flowing through the piston during the whole upwards stroke. If the friction of the leather cup and the pressure drop over the valve is neglected and if the weight of the pump rod is balanced, no upwards pump force is required and this means that the rotor starts unloaded.

At a certain critical rotational speed n_{crit}, the upwards pump velocity will become so high that the valve closes for the first time. This will happen for $\alpha = 90°$. Water will be pumped for the remaining halve stroke and therefore the average torque will be halve the value of the same pump with no floating valve. The volumetric efficiency for this point will be 0.5. If the rotational speed increases, the required upwards piston velocity will be realised at a smaller angle α which means that water will be pumped during a larger part of the stroke.

This results in increase of the average pump torque and in increase of the volumetric efficiency. At high rotational speeds, the valve will close at a small angle α and now the average torque will be almost the same as for the same pump with a normal piston valve.

A disadvantage of a floating valve is that the valve always closes when the piston has a rather high upwards speed. So the water column behind the piston has to be accelerated suddenly and this results in a shock force. The shock force is damped somewhat by the elasticity of the pump rod but it is rather large and the whole transmission has to be made strong enough for this shock force. It is assumed that the shock force works only for a very short time and that it can be neglected in terms absorbed energy.

4 Derivation of the formulas

If the rotor shaft is placed eccentric with respect to the tower axis because of the safety system, a crank rod with rocker arm is necessary to bridge this eccentricity and to transform the rotating movement of the crank into an oscillating movement of the pump rod. However, for the description of the system, it is easier to assume that there is no eccentricity and that the pomp rod is driven directly by a crank on the rotor shaft. It is assumed that the pump rod is long with respect to the length of the crank and therefore it is always vertical and the movement of the piston is sinusoidal. The transmission is given in figure 3.

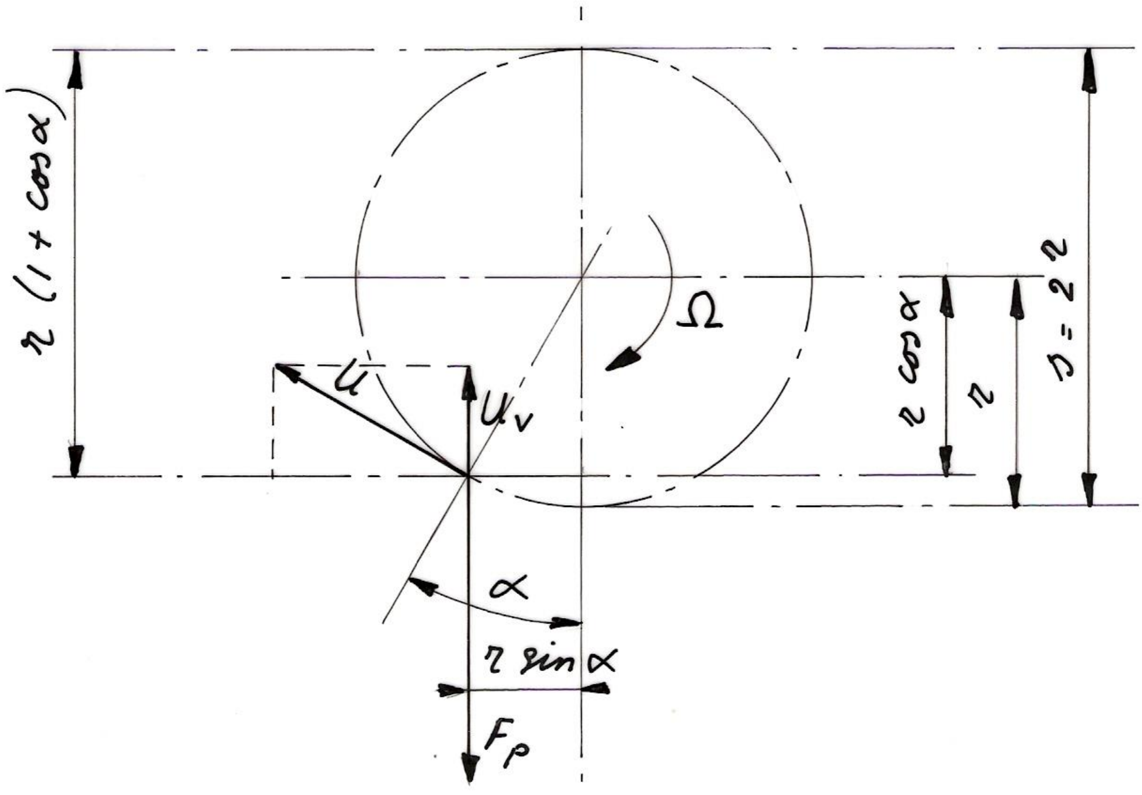

fig. 3 Pump rod directly driven by a crank on the rotor shaft

The rotor is rotating right hand with an angular velocity Ω (rad/s). The crank radius is called r (m), so the stroke s is 2 * r. The downwards piston force is called F_p (N). The velocity of the crank is called U (m/s). The vertical component of U is called U_v (m/s). The crank angle from the bottom dead centre is called α (°). The crank velocity U is given by:

$U = \Omega * r$ (m/s) (6)

U_v is given by:

$U_v = U * \sin\alpha$ (m/s) (7)

Ω is given by:

$\Omega = \pi * n / 30$ (rad/s, if n is given in rpm) (8)

(6) + (7) + (8) gives:

$U_v = \pi/30 * n * r \sin\alpha$ (m/s) (9)

The rotational speed for which the valve closes at $\alpha = 90°$ is called n_{crit} (rpm). Substitution of $n = n_{crit}$ and $\alpha = 90°$ in formula 9 gives:

$U_v = \pi/30 * n_{crit} * r$ (m/s) (10)

Now it is assumed that the valve closes always at the same speed U_v for rotational speeds larger than n_{crit}. This assumption and (9) + (10) gives:

$\pi/30 * n * r \sin\alpha = \pi/30 * n_{crit} * r$ or

$n \sin\alpha = n_{crit}$ or

$\alpha = \arcsin(n_{crit} / n)$ (°) (for $n \geq n_{crit}$) (11)

In figure 3 it can be seen that if the valve closes for the angle α, water will be pumped for the remaining stroke which is $r (1 + \cos\alpha)$. The average torque for a pump with a floating valve $Q_{av\ fv}$ is therefore given by:

$Q_{av\ fv} = Q_{av} * r (1 + \cos\alpha) / (2 * r)$ or

$Q_{av\ fv} = 0.5\ Q_{av} * (1 + \cos\alpha)$ (Nm) (for $n \geq n_{crit}$) (12)

(11) + (12) gives:

$Q_{av\ fv} / Q_{av} = 0.5 * \{1 + \cos[\arcsin(n_{crit} / n)]\}$ (Nm) (for $n \geq n_{crit}$) (13)

If this formula is compared to formula 11 out of report R 1192 S it is completely different. However, the shape of the curve is the same and may be it is possible to transfer one formula into the other.

In figure 3 it can be seen that the torque Q is given by:

$Q = r * \sin\alpha * F_p$ (Nm) (14)

The value of α and $Q_{av\ fv} / Q_{av}$ is now calculated for different values of n / n_{crit} using formula 11 and 13. The result of the calculation is given in table 1.

n / n_{crit} (-)	α (°)	$Q_{av\ fv} / Q_{av}$ (-)
1	90	0.5
1.01	81.931	0.570
1.05	72.247	0.652
1.1	65.380	0.708
1.25	53.130	0.8
1.5	41.810	0.873
2	30	0.933
3	19.471	0.971
5	11.537	0.990
10	5.739	0.997

table 1 Calculated values of α and $Q_{av\ fv} / Q_{av}$ as a function of n / n_{crit}

The calculated values of $Q_{av\ fv} / Q_{av}$ as a function of n / n_{crit} are given in figure 4.

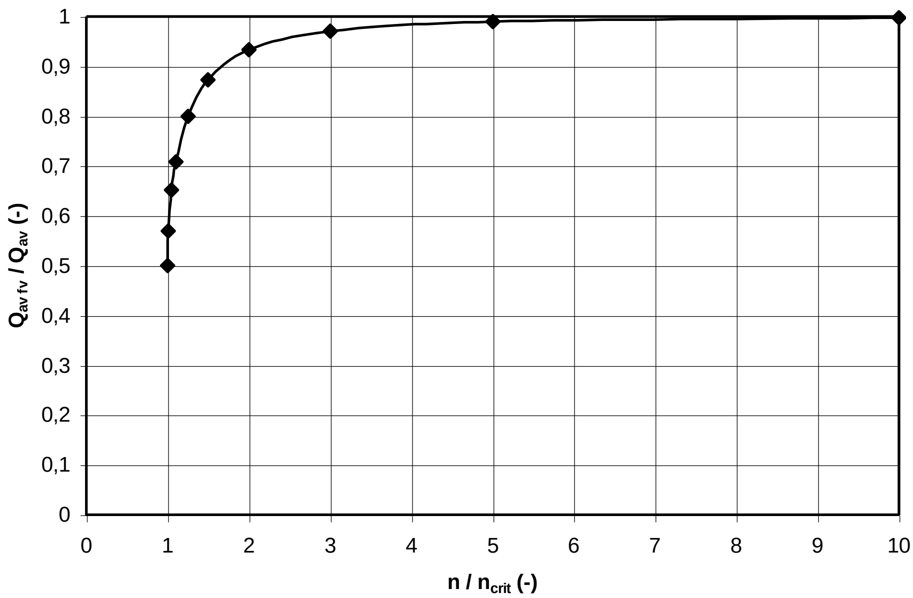

fig. 4 Values of $Q_{av\ fv} / Q_{av}$ as a function of n / n_{crit}

The curve which gives the volumetric efficiency η_{vol} as a function of n / n_{crit} is the same as figure 4 but now on the y-axis one can read η_{vol} as a factor of 1.

5 Choice of n_{crit}

The rotational speed n_{crit} where the valve closes for the first time depends on the buoyancy force of the valve and on the under pressure which is created in between the valve seat and the valve. I know no direct way to calculate the under pressure. The buoyancy force depends on the average density of the valve and the valve volume. For a certain valve diameter, the volume can only be increased by increase of the valve height. The required buoyancy force for a certain value of n_{crit} is therefore determined by try and error and it was found that it is difficult to get n_{crit} high enough. If the valve is made according to the method as given in chapter 2 it must have a height of at least the outside diameter.

A criteria for n_{crit} is that there is enough kinetic energy in the rotor to finish at least the upwards stroke when the valve closes at n_{crit}. So the kinetic energy in the rotor must be at least the energy which is required for half a pump stroke. But in figure 4 it can be seen that the curve rises very sharply if n is only a little higher than n_{crit} so the most difficult point may lay a little above n_{crit}. As the kinetic energy in the rotor increases proportional to n^2, there will always be enough kinetic energy in the rotor for higher rotational speeds

Another aspect is that the matching in between the pump and the windmill rotor must be good for low wind speeds. A windmill rotor has an optimum parabola in the Q-n graph for which the power coefficient is maximum. A normal piston pump with no floating valve has an average torque which is an horizontal line. The point of intersection of this line and the parabola is called the design point and the wind speed belonging to this point is called the design wind speed V_d.

The average Q-n curve of a pump with a floating valve will have a shape which is similar to the curve given in figure 4. This curve can have no point of intersection, a point of contact, two points of intersection or one point of intersection with the optimum parabola of the rotor. It seems a good choice to chose for two points of intersection because then there is a rather large wind speed interval for which the C_p of the rotor is almost maximum. But this means that now there are two design wind speeds. A good first choice may be to take n_{crit} such that the starting point of the $Q_{av\ fv}$–n curve is laying just on the optimum parabola of the rotor. An example of this choice is given in figure 5 for which the rotational speed and the torque are also made dimensionless.

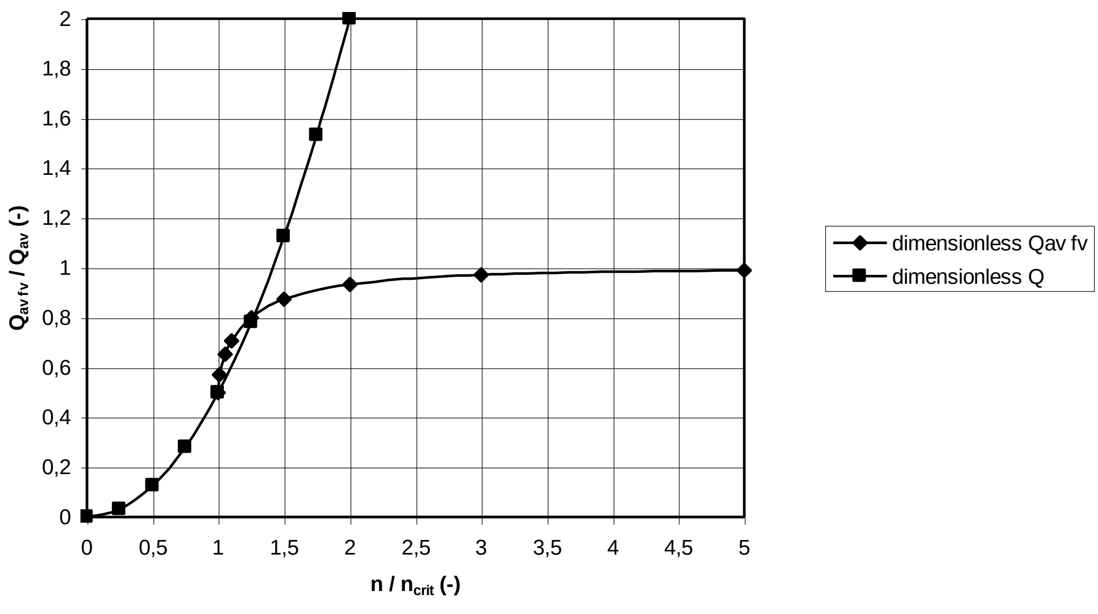

fig. 5 Values of $Q_{av\ fv}$ / Q_{av} and optimum rotor torque Q / Q_{av} as a function of n / n_{crit}

In figure 5 it can be seen that if the first point of intersection is laying at n / n_{crit} = 1 and $Q_{av\ fv}$ / Q_{av} = 0.5. The second point of intersection is laying at n / n_{crit} = 1.272 and $Q_{av\ fv}$ / Q_{av} = 0.809. I call this second point of intersection the real design point and the belonging wind speed the real design wind speed V_d because of similarity with the design point for a pump with no floating valve. The volumetric efficiency for the real design point is 0.809.

More elaboration about this subject is without the scope of this report.

6 References

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