Description of the inclined hinge main vane safety system and determination of the moment equations

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1 Introduction

Windmills with fixed rotors can be protected against too high forces and too high rotational speeds by turning the rotor out of the wind. This can be done around a vertical and around a horizontal axis. All present VIRYA windmills developed by Kragten Design turn out of the wind around a vertical axis and make use of the so called hinged side vane safety system.

Safety systems for water pumping windmills are described in report R 999 D (ref. 1). Water pumping windmills normally have fixed rotors and all safety systems are working by turning the rotor out of the wind. However, electricity generating windmills can also be protected by turning the rotor out of the wind. In chapter 2 of R 999 D, the reasons are given why a safety system is necessary. These reasons are:

1 Limitation of the axial force or thrust on the rotor to limit the load on the rotor blades, the tower and the foundation.

2 Limitation of the rotational speed of the rotor to limit the centrifugal force in the blades, imbalance forces, high gyroscopic moments in the blades and the rotor shaft, to prevent flutter for blades with low torsion stiffness and to prevent too high rotational speeds of the load which is relevant for limitation of heat dissipation in a generator or for limitation of shock forces in the transmission to a piston pump.

3 Limitation of the yawing speed to limit high gyroscopic moments in the blades and the rotor shaft.

Almost all known systems are described shortly in chapters 3 and 4 of R 999 D. The three most generally used systems, the ecliptic system, the inclined hinge main vane system and the hinged side vane system are described in detail in chapter 7 of report R 999 D. Because report R 999 D is no longer available, the hinged side vane system is also described in several KD-reports. This system is described for the VIRYA-4.2 windmill in report KD 213 (ref. 2).

Every safety system has certain advantages and disadvantages. The main advantages of the hinged side vane safety system are:

1) It is simple and cheap.
2) It has a δ-V curve which is lying close to the ideal δ-V curve (see chapter 2).
3) The hinge axis is loaded only lightly and therefore simple door hinges can be used.
4) The vane blade is situated in the undisturbed wind and therefore a relatively small vane blade area is required to generate a certain aerodynamic force.
5) The moment of inertia of the head is large resulting in low yawing speeds and so large gyroscopic moments at high wind speeds are prevented.

The main disadvantages of the hinged side vane system are:

1) There must be a certain ratio in between the vane area and the vane weight if a certain rated wind speed is wanted. Therefore it appears to be difficult to make a large vane blade stiff enough. The hinged safety system is therefore limited to windmills with a maximum rotor diameter of about 5 m.
2) The system is sensible to flutter of the vane blade, if the vane blade and the vane arm is not made stiff enough. Flutter is suppressed effectively using a vane blade stop at the almost horizontal position of the vane blade
3) It is difficult to turn the head out of the wind permanently by placing the vane blade in the horizontal position because this vane blade is positioned far form the tower and far from the ground.

In report KD 377 (ref. 3) a safety system is described with which the rotor is turned out of the wind around a horizontal axis. This system has rather good characteristics which can easily be determined. However, this system needs an extra horizontal axis and it requires rather heavy balancing weights. It will be investigated if it is possible to realise about similar characteristics for the inclined hinge main vane safety system. This system will be described in chapter 3.

2 The ideal δ-V curve

Generally it is wanted that the windmill rotor is perpendicular to the wind up to the rated wind speed V_{rated}, and that the rotor turns out of the wind such that the rotational speed, the rotor thrust, the torque and the power stay constant above V_{rated}. It appears to be that the component of the wind speed perpendicular to the rotor plane determines these four quantities. The yaw angle δ is the angle in between the wind direction and the rotor axis. The component of the wind speed perpendicular to the rotor plane is therefore V cosδ. The formulas for a yawing rotor for the rotational speed n_δ, the rotor thrust $F_{t\delta}$, the torque Q_δ and the power P_δ are given in chapter 7 of report KD 35 (ref. 4). These formulas are copied as formula 1, 2, 3 and 4.

$$n_\delta = 30 * \lambda * \cos\delta * V / \pi R \qquad \text{(rpm)} \qquad (1)$$

$$F_{t\delta} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 \qquad \text{(N)} \qquad (2)$$

$$Q_\delta = C_q * \cos^2\delta * ½\rho V^2 * \pi R^3 \qquad \text{(Nm)} \qquad (3)$$

$$P_\delta = C_p * \cos^3\delta * ½\rho V^3 * \pi R^2 \qquad \text{(W)} \qquad (4)$$

These four quantities stay constant above V_{rated} if the component of the wind speed perpendicular to the rotor plane is kept constant above V_{rated}. So in formula:

$$V \cos\delta = V_{rated} \qquad (\text{for } V > V_{rated}) \qquad (5)$$

It is assumed that the rotor is loaded such that it runs at the design tip speed ratio λ_d. If the wind speed is in between 0 m/s and V_{rated}, the n-V curve is a straight line through the origin. The F_t-V and the Q-V curves are then parabolic lines and the P-n curve is a cubic line.

Formula 5 can be written as:

$$\delta = \text{arc cos}\,(V_{rated} / V) \qquad (°) \qquad (6)$$

This formula is given as a graph in figure 1 for different value of V / V_{rated}. The value of δ has been calculated for V / V_{rated} is respectively 1, 1.01, 1.05, 1.1, 1.25, 1.5, 2, 2.5, 3, 4, 5 and 6.

The rated wind speed V_{rated} is chosen on the basis of the maximum thrust and the maximum rotational speed which is allowed for a certain rotor and a certain generator. Mostly V_{rated} is chosen about 10 m/s. For the chosen value of V_{rated}, figure 1 can be transformed into the δ-V curve for which V (in m/s) is given on the x-axis. If it is chosen that V_{rated} = 10 m/s, figure 1 becomes the δ-V curve if all values on the x-axis are multiplied by a factor 10.

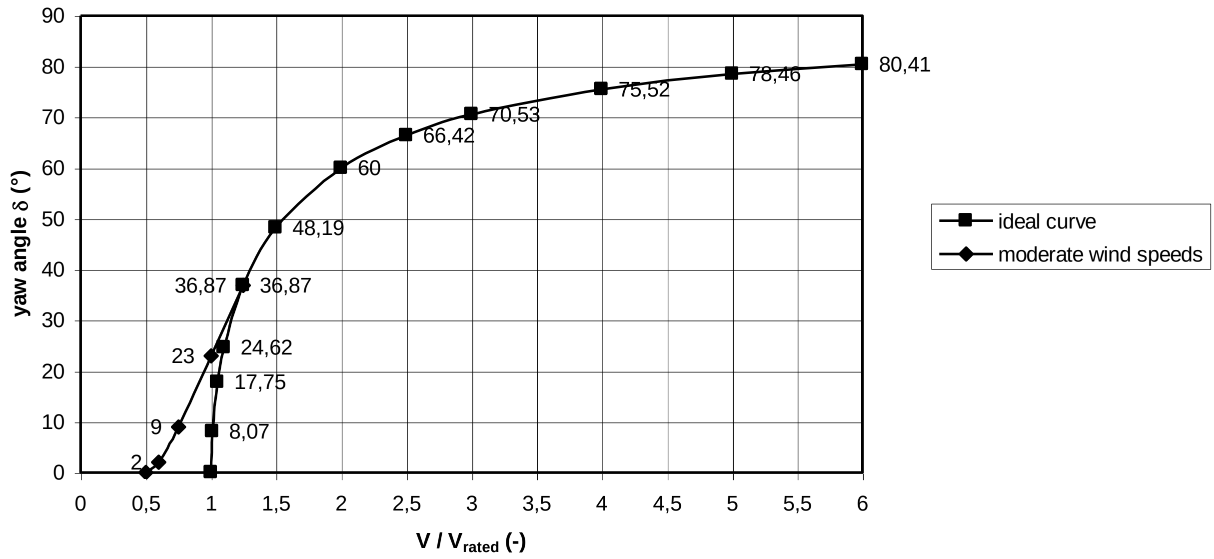

figure 1 the δ-V/V_{rated} curve for the ideal safety system

In figure 1 it can be seen that the rotor is perpendicular to the wind for (V / V_{rated}) < 1 but that the required change in δ is very sudden if V / V_{rated} is a very little higher than 1. So even if one would have a safety system which theoretically has the ideal δ-V curve, in practice this curve will not be followed because the inertia of the system prevents sudden changes of δ around V / V_{rated} = 1. So the system will turn out of the wind less than according to the ideal δ-V curve. This will result in a certain overshoot of the rotational speed and the thrust.

For high values of V / V_{rated}, a certain increase of V, and therefore a certain increase of V / V_{rated}, requires only a relatively small increase of δ. It is therefore much easier to follow the theoretical δ-V curve at high wind speeds.

Because of this effect, it is practically impossible to follow the ideal δ-V curve for wind speeds lower than about 1.25 * V_{rated} and the practical δ-V curve must therefore start increasing already at a much lower wind speed than the theoretical rated wind speed. An example of a practical δ-V curve for moderate wind speeds is also given in figure 1. Even for this practical curve for moderate wind speeds and for the ideal curve for values of V / V_{rated} > 1.25, there will be a certain overshoot of n and F_t because of inertia effects.

3 Description of the inclined hinge main vane safety system (see fig. 2, 6 and 7)

The inclined hinge mane vane system is used in traditional water pumping windmills like windmills of manufacture Southern Cross. It was also used in some of the water pumping windmills which were developed by the former CWD (Consultancy Services Wind Energy Developing Countries). It is also used in several electricity generating wind turbines like the wind turbines of the Dutch manufacture Fortis. The inclined hinge mane vane system is described in general in chapter 4.6 and in detail in chapter 7.5 of report R999D (ref. 1). It is also described in chapter 11.2 of report CWD 82-1 ”Introduction to Wind Energy” (ref. 5).

The inclined hinge mane vane system can be used in combination with an eccentrically placed rotor or with a centrally placed rotor and an auxiliary vane. Only the use in combination with an eccentrically placed rotor will be taken into account. For electricity generation wind turbines, the eccentricity e has to be taken rather large with respect to the rotor diameter D because the so called self orientating moment M_{so} has to be over powered. The ratio e / D must preferably be taken not smaller than 0.08 (8 %). For windmills with a low design tip speed ratio, the ratio e / D can be taken smaller because M_{so} is almost zero .

The main feature of the inclined hinge main vane safety system is that the main vane is turning around a vane axis which makes a small angle with the tower axis. The vertical tower axis is called the z-axis. The inclined vane axis is called the s-axis. It is assumed that both axes intersect. The angle in between both axes is called ε. Provisionally it is assumed that $\varepsilon = 15°$. The plane through the s-axis and the z-axis is making an angel ϕ_1 with the rotor axis. Provisionally it is chosen that $\phi_1 = 25°$. This angle is necessary to realise that the rotor is about perpendicular to the wind direction at low wind speeds.

For traditional water pumping windmills, the vane arm is about horizontal and the vane blade is therefore normally positioned in the rotor shadow. However, it comes out of the rotor shadow at high wind speeds. The wind speed at the vane blade is reduced by the rotor shadow and it is very difficult to describe the system for this vane orientation because the wind speed at the vane blade is not known. For electricity generating wind turbines, the vane arm is normally making an angle of about 45° with the horizon and therefore the vane blade juts above the rotor and is positioned in the undisturbed wind speed V. For this vane orientation, the system can be described much easier and this vane orientation is therefore used in this report KD 431 to describe the inclined hinge main vane system.

When the wind speed is zero, the vane is hanging in the lowest position and is therefore in the plane through the s-axis and the z-axis. As soon as a certain aerodynamic force F_v is exerted perpendicular to the vane blade, the vane will turn away right hand if seen from above. F_v is lying in an inclined plane which is perpendicular to the s-axis. The angle over which the vane moves in this plane from the lowest position, is called γ. The lowest position is called the zero line in figure 2. The aerodynamic force acting on the vane arm is neglected. F_v is working on a distance R_v from the s-axis. The plane in which F_v moves has a certain point of intersection with the s-axis. The distance in between this point and the z-axis is called h. The total weight G of vane arm and vane blade is acting in the centre of gravity which is lying at a distance R_G from the s-axis.

The rotor turns out of the wind left hand if seen from above because of the rotor moment M_{rotor}. This rotor moment is mainly determined by the rotor thrust $F_{t\delta}$ and the eccentricity e but the side force on the rotor $F_{s\delta}$ in combination with the distance f in between the rotor plane and the tower axis and the so called self orientating moment M_{so} also have a certain influence. The angle in between the rotor axis and the wind direction is called δ. For very low wind speeds, δ is negative.

The vane arm is making an angle ϕ_2 with the vane axis. Provisionally it is assumed that $\phi_2 = 30°$. Because it was assumed that the vane axis is making an angle $\varepsilon = 15°$ with the z-axis, the angle in between the vane arm and the z-axis is 45° when the vane arm is in its lowest position. If the vane arm is chosen long enough, the vane blade will therefore jut above the rotor plane and will be streamed by the undisturbed wind speed V. The vane blade is square and will be connected to the vane arm such that two sides are vertical and two sides are horizontal when the vane arm is in its lowest position.

The geometry of rotor, head and vane will be chosen such that the vane can rotate 360° without touching the rotor. Therefore no stops will be necessary. This geometry has been chosen for a small 1 m diameter battery charging windmill called Wesp (Wasp in English) which was developed at the Wind Energy Group of the UT-Eindhoven in about 1980 by a student who I have guided.

In figure 2 it can be seen that:

$$\delta = \gamma + \alpha - \phi_1 \quad (°) \tag{7}$$

This formula is not exact because δ, α and ϕ_1 are measured in a horizontal plane but γ is measured in a plane perpendicular to the s-axis. However, as ε is only 15°, this formula is accurate enough to calculate δ.

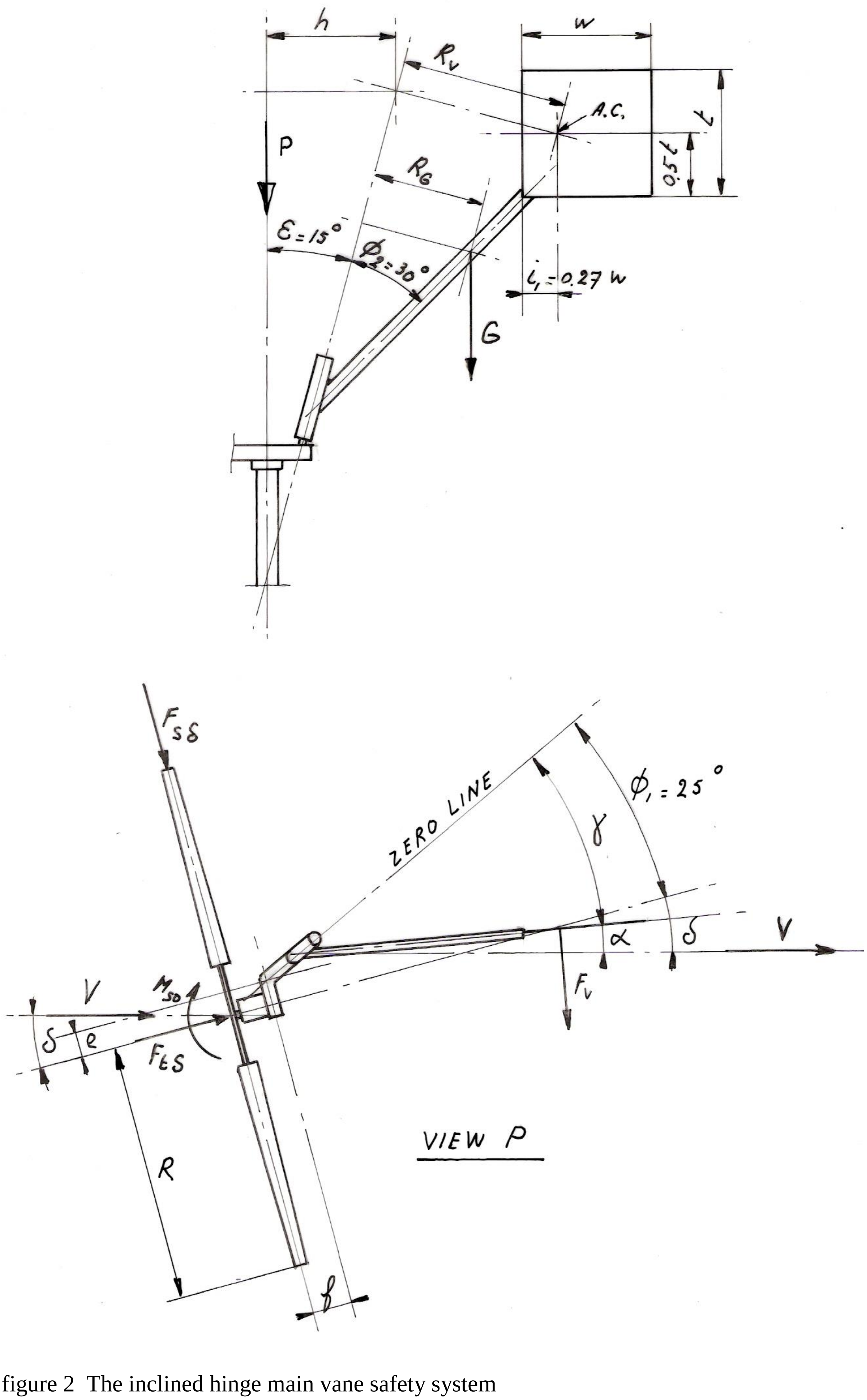

figure 2 The inclined hinge main vane safety system

4 Determination of the moment equations

The moment produced by the horizontal component of the aerodynamic normal force F_v around the z-axis is called $M_{vane\ z\text{-}axis}$. Now it is assumed that the system is functioning quasi-stationary. So no moments are needed for acceleration of the system from one position into the other. Balance of moments gives that:

$$M_{rotor} = M_{vane\ z\text{-}axis} \qquad (8)$$

The moment produced by the aerodynamic force F_v around the s-axis, is called $M_{vane\ s\text{-}axis}$. The moment produced by the component of the weight G of vane arm plus vane blade around the s-axis, is called M_G. Balance of moments gives that:

$$M_G = M_{vane\ s\text{-}axis} \qquad (9)$$

For the determination of both moment equations, it is necessary to determine the formulas for M_{rotor}, $M_{vane\ s\text{-}axis}$, $M_{vane\ z\text{-}axis}$ and M_G.

4.1 Determination of M_{rotor}

M_{rotor} is caused by the influence of the thrust force $F_{t\delta}$, the side force $F_{s\delta}$ and the so called self orientating moment M_{so}. $F_{t\delta}$ results in a moment $M_{Ft\delta}$. $F_{s\delta}$ results in a moment $M_{Fs\delta}$. Both $M_{Ft\delta}$ and $M_{Fs\delta}$ have a right hand direction which tends to increase δ. However, M_{so} has a left hand direction such that δ is decreased. M_{rotor} is therefore given by:

$$M_{rotor} = M_{Ft\delta} + M_{Fs\delta} - M_{so} \quad \text{(Nm)} \qquad (10)$$

$$M_{Ft\delta} = F_{t\delta} * e \quad \text{(Nm)} \qquad (11)$$

(2) + (11) gives:

$$M_{Ft\delta} = C_t * \cos^2\delta * \tfrac{1}{2}\rho V^2 * \pi R^2 * e \quad \text{(Nm)} \qquad (12)$$

$$M_{Fs\delta} = F_{s\delta} * f \quad \text{(Nm)} \qquad (13)$$

The distance in between the rotor plane and the tower centre is called f. For the side force on the rotor $F_{s\delta}$, no formula is given in report KD 35 (ref. 4). If one would calculate $F_{s\delta}$ with the component of the wind speed in the rotor plane, V sinδ, the side force would be proportional to $\sin^2\delta$. However, from measurements (see figure 23, report R 999 D) it is found that $F_{s\delta}$ increases much faster than a $\sin^2\delta$ function for small values of δ. A sinδ function gives a better approximation.

For very large angles δ, the tip speed of the rotor is only little with respect to the wind speed. The side area of the rotor A_s, then can be seen as a drag area with a drag coefficient C_d. The ratio i in be between A_s and the swept rotor area $\pi * R^2$ depends on the type of rotor. For fast running rotors as used in the VIRYA windmills, A_s is very small with respect to the swept rotor area because the chord, the airfoil thickness and the blade angles are small. In report KD 213 (ref. 2), the two bladed VIRYA-4.2 rotor is taken which has a design tip speed ratio of 8. For this rotor it is determined that $i = A_s / (\pi * R^2) = 0.01$. The drag coefficient C_d depends on the airfoil and is rather low if an aerodynamic airfoil is used.

The yaw angel δ is large at very high wind speeds and the lower blade sees a much larger relative wind speed than the upper blade. It is assumed that the average C_d value for the whole rotor is 1. The side force $F_{s\delta}$ for a yawing rotor is now given by:

$$F_{s\delta} = C_d * \sin\delta * \tfrac{1}{2}\rho V^2 * i * \pi R^2 \quad \text{(N)} \qquad (14)$$

(13) + (14) gives:

$$M_{Fs\delta} = C_d * f * i * \sin\delta * \tfrac{1}{2}\rho V^2 * \pi R^2 \quad \text{(Nm)} \qquad (15)$$

In KD 35 (ref. 4) no formula is given for the self orientating moment M_{so}. M_{so} is created because the exertion point of the thrust doesn't coincide with the hart of the rotor. There is only little known about M_{so} and only some very rough measurements have been performed which are given in report R 344 D (ref. 6, in Dutch). For these measurement an unloaded two bladed rotor was used with a design tip speed ratio of 5 and provided with a curved sheet airfoil. Practical experience with the VIRYA windmills using a Gö 623 airfoil indicate that M_{so} is much lower for this airfoil. Recently I have made a model of a two bladed rotor with a diameter of 0.8 m with a design tip speed ratio of about 6.5 and using a Gö 623 airfoil. The maximum eccentricity which was possible for which the rotor doesn't turn out of the wind completely, was about 0.027 m. From this measurement it is derived that the maximum self orientating moment for a certain wind speed is about half the value as for the same diameter rotor with a curved sheet airfoil.

M_{so} is given by:

$$M_{so} = C_{so} * \tfrac{1}{2}\rho V^2 * \pi R^3 \quad \text{(Nm)} \qquad (16)$$

C_{so} depends on the yaw angle δ and appears to have a maximum for $\delta = 30°$. The estimated C_{so}-δ curve for a rotor with a Gö 623 or similar airfoil can be approximated by two goniometric functions, one function for $0° \leq \delta \leq 40°$ and one function for $40° \leq \delta \leq 90°$. These functions are:

$$C_{so} = 0.0225 \sin 3\delta \quad (-) \quad (\text{for } 0° \leq \delta \leq 40°) \qquad (17)$$

$$C_{so} = 0.0332 \cos^2\delta \quad (-) \quad (\text{for } 40° \leq \delta \leq 90°) \qquad (18)$$

If the direction of the moment for a negative value of δ is taken the same as for a positive value of δ, formula 17 can also be used for $-40° \leq \delta \leq 0°$. The path of both curves is given in figure 3.

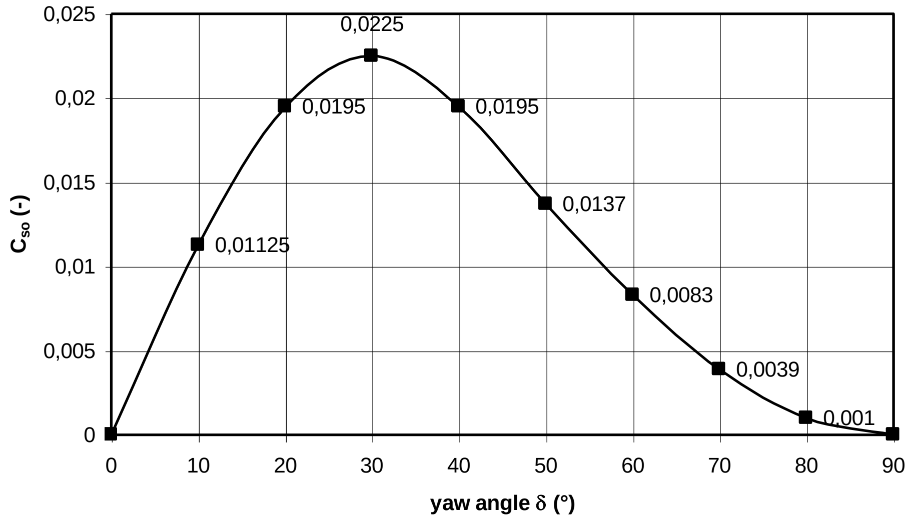

figure 3 Path of C_{so} as a function of the yaw angle δ

(16) + (17) gives:

$M_{so} = 0.0225 \sin 3\delta * ½\rho V^2 * \pi R^3$ (Nm) (for $0° \leq \delta \leq 40°$) (19)

(16) + (18) gives:

$M_{so} = 0.0332 \cos^2\delta * ½\rho V^2 * \pi R^3$ (Nm) (for $40° \leq \delta \leq 90°$) (20)

(10) + (12) + (15) + (19) gives:

$M_{rotor} = C_t * e * \cos^2\delta * ½\rho V^2 * \pi R^2 + C_d * f * i * \sin\delta * ½\rho V^2 * \pi R^2 -$
$0.0225 \sin 3\delta * ½\rho V^2 * \pi R^3$ or

$M_{rotor} = ½\rho V^2 * \pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0225 * \sin 3\delta)$
(Nm) (for $0° \leq \delta \leq 40°$) (21)

(10) + (12) + (15) + (20) gives:

$M_{rotor} = C_t * e * \cos^2\delta * ½\rho V^2 * \pi R^2 + C_d * f * i * \sin\delta * ½\rho V^2 * \pi R^2 -$
$0.0332 \cos^2\delta * ½\rho V^2 * \pi R^3$ or

$M_{rotor} = ½\rho V^2 * \pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0332 * \cos^2\delta)$
(Nm) (for $40° \leq \delta \leq 90°$) (22)

To get an impression of the contribution of $M_{Ft\delta}$, $M_{Fs\delta}$ and M_{so} to M_{rotor}, the moments are made dimensionless by dividing by $½\rho V^2 * \pi R^3$. The formulas 10, 12, 15, 19 and 20 for M_{rotor}, $M_{Ft\delta}$, $M_{Fs\delta}$ and M_{so} change into 23, 24, 25, 26 and 27 for C_{Mrotor}, $C_{MFt\delta}$, $C_{MFs\delta}$, and C_{Mso}.

$C_{Mrotor} = C_{MFt\delta} + C_{MFs\delta} - C_{Mso}$ (-) (23)

$C_{MFt\delta} = C_t * e/R * \cos^2\delta$ (-) (24)

$C_{MFs\delta} = C_d * f/R * i * \sin\delta$ (-) (25)

$C_{Mso} = 0.0225 \sin 3\delta$ (-) (for $0° \le \delta \le 40°$) (26)

$C_{Mso} = 0.0332 \cos^2\delta$ (-) (for $40° \le \delta \le 90°$) (27)

Now the path of $C_{MFt\delta}$, $C_{MFs\delta}$, C_{Mso} and C_{Mrotor} is determined as a function of δ for the VIRYA-4.2 rotor and it is assumed that this rotor is now combined with the inclined hinge main vane safety system. For this rotor it is valid that R = 2.1 m. It is assumed that e = 0.42 m and that f = 0.48 m. So e/R = 0.2 and f/R = 0.2286. It was assumed earlier that i = 0.01. The theoretical thrust coefficient is 8/9 = 0.89 for $\lambda = \lambda_d$. However in practice it is a lot lower because the inner part of the rotor is not effective and because a part of the thrust is lost by tip and root losses. Assume $C_t = 0.7$. For the drag coefficient it was earlier assumed that $C_d = 1$. Substitution of these values in formula 24 and 25 gives:

$C_{MFt\delta} = 0.14 \cos^2\delta$ (-) (28)

$C_{MFs\delta} = 0.00229 \sin\delta$ (-) (29)

The moment coefficients are calculated for values of δ in between δ = -40° and δ = 90° rising with 10°. The results of the calculations are given in table 1 and figure 4. If the direction of moments for negative values of δ is taken the same as for positive values of δ, formulas 26, 28 and 29 can also be used for negative values of δ.

δ (°)	$C_{MFt\delta}$ (-)	$C_{MFs\delta}$ (-)	C_{Mso} (-)	C_{Mrotor} (-)
-40	0.08216	-0.00147	-0.01949	0.10018
-30	0.10500	-0.00115	-0.02250	0.12635
-20	0.12362	-0.00078	-0.01949	0.14233
-10	0.13578	-0.00040	-0.01125	0.14663
0	0.14	0	0	0.14
10	0.13578	0.00040	0.01125	0.12493
20	0.12362	0.00078	0.01949	0.10491
30	0.10500	0.00115	0.02250	0.08365
40	0.08216	0.00147	0.01949	0.06414
50	0.05784	0.00175	0.01372	0.04587
60	0.03500	0.00198	0.00830	0.02868
70	0.01638	0.00215	0.00388	0.01465
80	0.00422	0.00226	0.00100	0.00548
90	0	0.00229	0	0.00229

table 1 Calculated values for $C_{MFt\delta}$, $C_{MFs\delta}$, C_{Mso} and C_{Mrotor} for the VIRYA-4.2 rotor

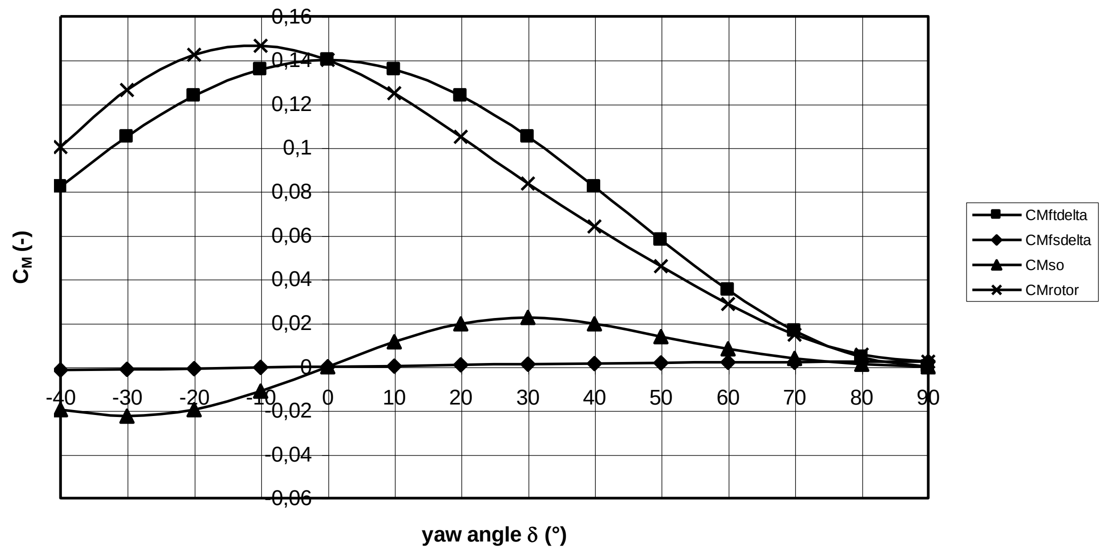

figure 4 Path of $C_{MFt\delta}$, $C_{MFs\delta}$, C_{Mso} and C_{Mrotor} for the VIRYA-4.2 rotor and head geometry

In figure 4 it can be seen that the contribution of $C_{MFs\delta}$ to C_{Mrotor} can be neglected except for very large angles δ. The contribution of C_{Mso} to C_{Mrotor} can not be neglected and causes that the decrease of the C_{Mrotor}-δ curve at increasing δ is much faster than for the $C_{MFt\delta}$-δ curve. For angles δ in between 25° and 60°, C_{Mrotor} is about a factor 0.8 van $C_{MFt\delta}$. C_{Mrotor} has a maximum at about δ = -13°.

The path found in figure 4 for the dimensionless moment coefficients is also valid for the real moments for a certain wind speed.

4.2 Determination of $M_{vane\ s\text{-}axis}$

$$M_{vane\ s\text{-}axis} = F_v * R_v \qquad \text{(Nm)} \qquad (30)$$

F_v is the aerodynamic force perpendicular to the vane blade. R_v is the distance in between the aerodynamic centre (a. c.) where F_v applies and the s-axis. The position of the a. c. point depends on the angle α in between the wind direction and the vane blade. The a. c. point is lying at a distance i_1 from the leading edge of the vane blade. The vane blade has a width w and a height t. The ratio i_1 / w as a function of α is given in figure 7 report KD 213 (ref. 2). The angel α will be rather small for the inclined hinge main vane system and will lie in between about 15° and 5°. The average ratio i_1 /w is about 0.27 for this α ranged. To simplify the procedure, the calculations are made for a fixed a. c. point lying at i_1 = 0.27 w.

Exact determination of α and so of F_v is very difficult because the vane blade is only perpendicular to the earth surface for the vane blade in the zero position. If the vane arm has rotated over γ = 90°, the vane blade will make an angle of 75° with the earth surface because of the angle ε = 15°. The upper and lower sides of the vane blade will make an angle of 15° with the horizon. This results in a small change of α and the direction of the wind with respect to the sides of the vane blade but this effect is neglected for the determination of α and it is assumed that formula 7 can be used. However, the angle ε can't be neglected for the determination of the active component of G and for the horizontal component of F_v. F_v is given by:

$$F_v = C_n * ½\rho V^2 * t * w \qquad \text{(N)} \qquad (31)$$

C_n is the normal force coefficient (-). ρ is the air density which is about 1.2 kg/m^3 for air of 20 °C at sea level. t * w is the vane blade area (m^2). The C_n-α curve for a square plate is given in figure 6 of KD 213. This figure is copied as figure 5.

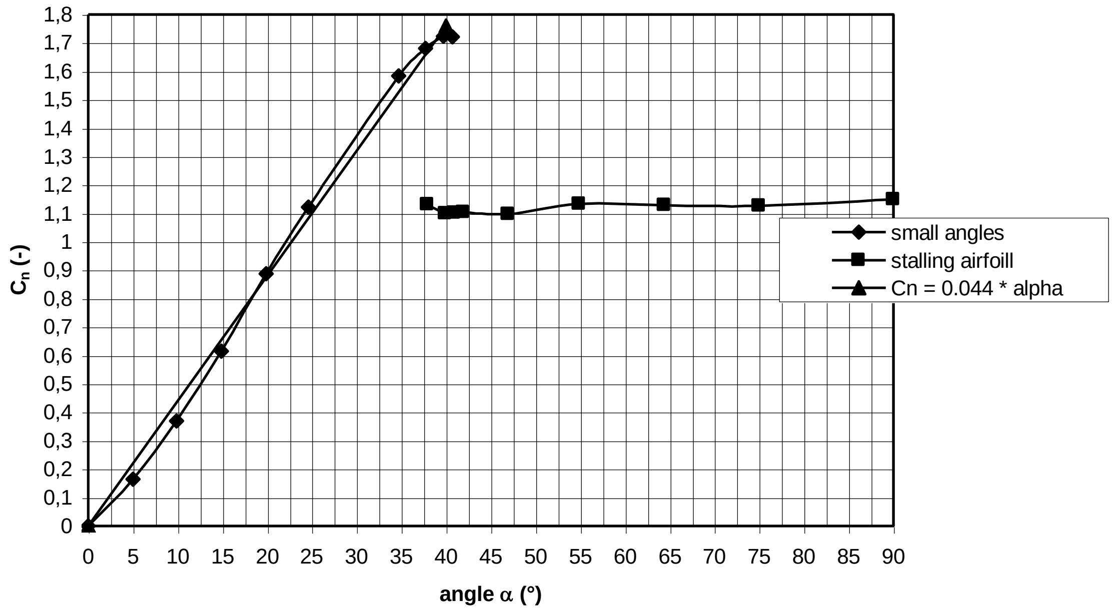

figure 5 C_n-α curve for a square plate

The C_n-α curve is about a straight line for 0° < α < 40°. The C_n-α curve for 0° < α < 40° can be replaced by the function:

$C_n = 0.044 * \alpha$ (-) $(0° \le \alpha \le 40°)$ (32)

This function is also given in figure 5. As ϕ_1 is chosen 25° and γ has a certain value even for low wind speeds, the angle α is normally smaller than 20°. This means that sudden variations of the wind direction with a maximum deviation of 20° in both directions result in variation of α in between 0° and 40° with corresponding values of C_n varying in between 0 and 1.76. So the vane will work properly to keep the rotor perpendicular to the wind.

(31) + (32) gives:

$F_v = 0.044 * \alpha * ½\rho V^2 * t * w$ (N) (33)

(30) + (33) gives:

$M_{vane\ s\text{-}axis} = 0.044 * \alpha * ½\rho V^2 * t * w * R_v$ (Nm) (34)

4.3 Determination of $M_{vane\ z\text{-}axis}$ (see figure 6)

To determine $M_{vane\ z\text{-}axis}$, the horizontal component of F_v and the distance of this horizontal component with respect to the z-axis have to be determined. For this determination, a special picture is required which is given as figure 11.9 in report CWD 82-1 (ref. 5). This picture (which I have made already in 1983 for the given report when I was working at the University of Technology Eindhoven) is copied as figure 6.

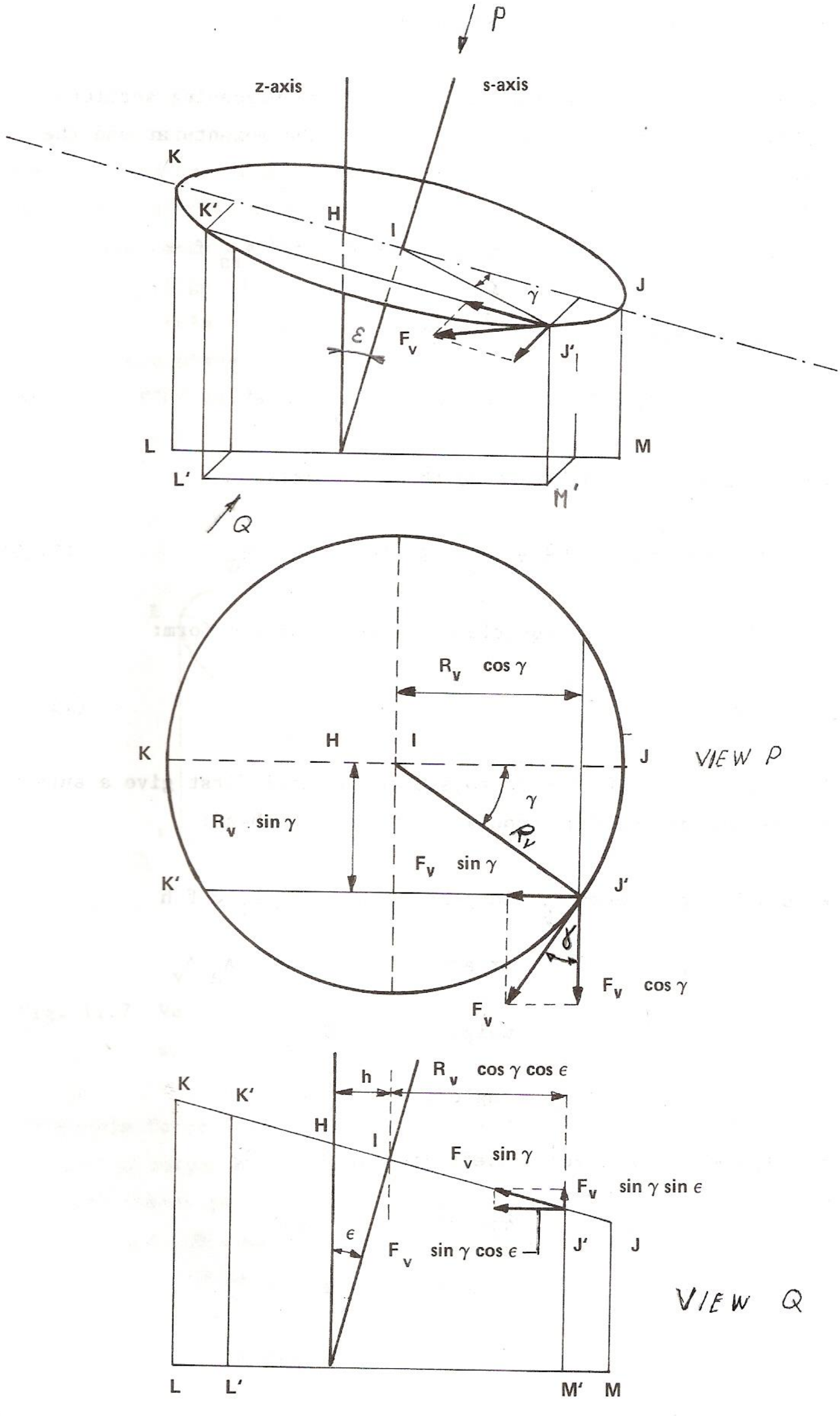

figure 6 Resolution of the aerodynamic force on the vane blade F_v

F_v is first resolved into a component F_v sinγ and a component F_v cosγ (see view P) both lying in a plane perpendicular to the s-axis. The component F_v sinγ is lying in the plane K' L' M' J' which is parallel to the plane K L M J through the s-axis and the z-axis. The component F_v cosγ is perpendicular to the plane K L M J. The component F_v cosγ is lying horizontal because the plane K L M J is perpendicular to the horizontal plane.

Next F_v sinγ is resolved in the vertical component F_v sinγ * sinε and the horizontal component F_v sinγ * cosε.

The arm for which F cosγ applies with respect to the z-axis is h + R_v cosγ cosε. The plane in which F_v moves, intersects with the s-axis at point I. The distance in between point I and the z-axis is called h. The arm for which F_v sinγ cosε applies with respect to the z-axis is R_v sinγ. For the moment $M_{vane\ z\text{-}axis}$ which is produced by both forces around the z-axis it is found that:

$$M_{vane\ z\text{-}axis} = F_v \cos\gamma\ (h + R_v \cos\gamma * \cos\varepsilon) + F_v \sin\gamma * \cos\varepsilon * R_v \sin\gamma \quad \text{or}$$

$$M_{vane\ z\text{-}axis} = F_v \{\cos\gamma\ (h + R_v \cos\gamma * \cos\varepsilon) + \sin\gamma * \cos\varepsilon * R_v \sin\gamma\} \quad \text{or}$$

$$M_{vane\ z\text{-}axis} = F_v\ (h \cos\gamma + R_v \cos^2\gamma * \cos\varepsilon + R_v \sin^2\gamma * \cos\varepsilon) \quad \text{or}$$

$$M_{vane\ z\text{-}axis} = F_v \{h \cos\gamma + R_v \cos\varepsilon * (\cos^2\gamma + \sin^2\gamma)\} \quad \text{or}$$

$$M_{vane\ z\text{-}axis} = F_v\ (h \cos\gamma + R_v \cos\varepsilon) \quad \text{(Nm)} \qquad (35)$$

(33) + (35) gives:

$$M_{vane\ z\text{-}axis} = 0.044 * \alpha * \tfrac{1}{2}\rho V^2 * t * w\ (h \cos\gamma + R_v \cos\varepsilon) \quad \text{(Nm)} \qquad (36)$$

4.4 Determination of M_G (see figure 7)

The total weight G of the vane arm and the vane blade applies in the centre of gravity which lies at a distance R_G from the s-axis (see figure 2). This centre of gravity moves also in a plane which is perpendicular to the s-axis. To determine M_G, the component of G in the plane of rotation and the distance of this component with respect to the s-axis, have to be determined. For this determination, a special picture is required which is given as figure 11.8 in report CWD 82-1 (ref. 5). This picture is copied as figure 7.

G is first resolved into a component G sinε and a component G cosε (see view Q). The component G sinε is lying in the inclined plane E' E D D'. The component G cosε is perpendicular to this plane. The component G cosε is lying parallel to the s-axis and will therefore give no moment around the s-axis.

Next G sinε is resolved in a component G sinγ * cosγ which points in the direction of R_G and in a component G sinε * sinγ perpendicular to R_G (see view P). The component in the direction of R_G will give no moment around the s-axis. The component perpendicular to R_G supplies M_G which is given by:

$$M_G = G \sin\varepsilon * \sin\gamma * R_G \quad \text{(Nm)} \qquad (37)$$

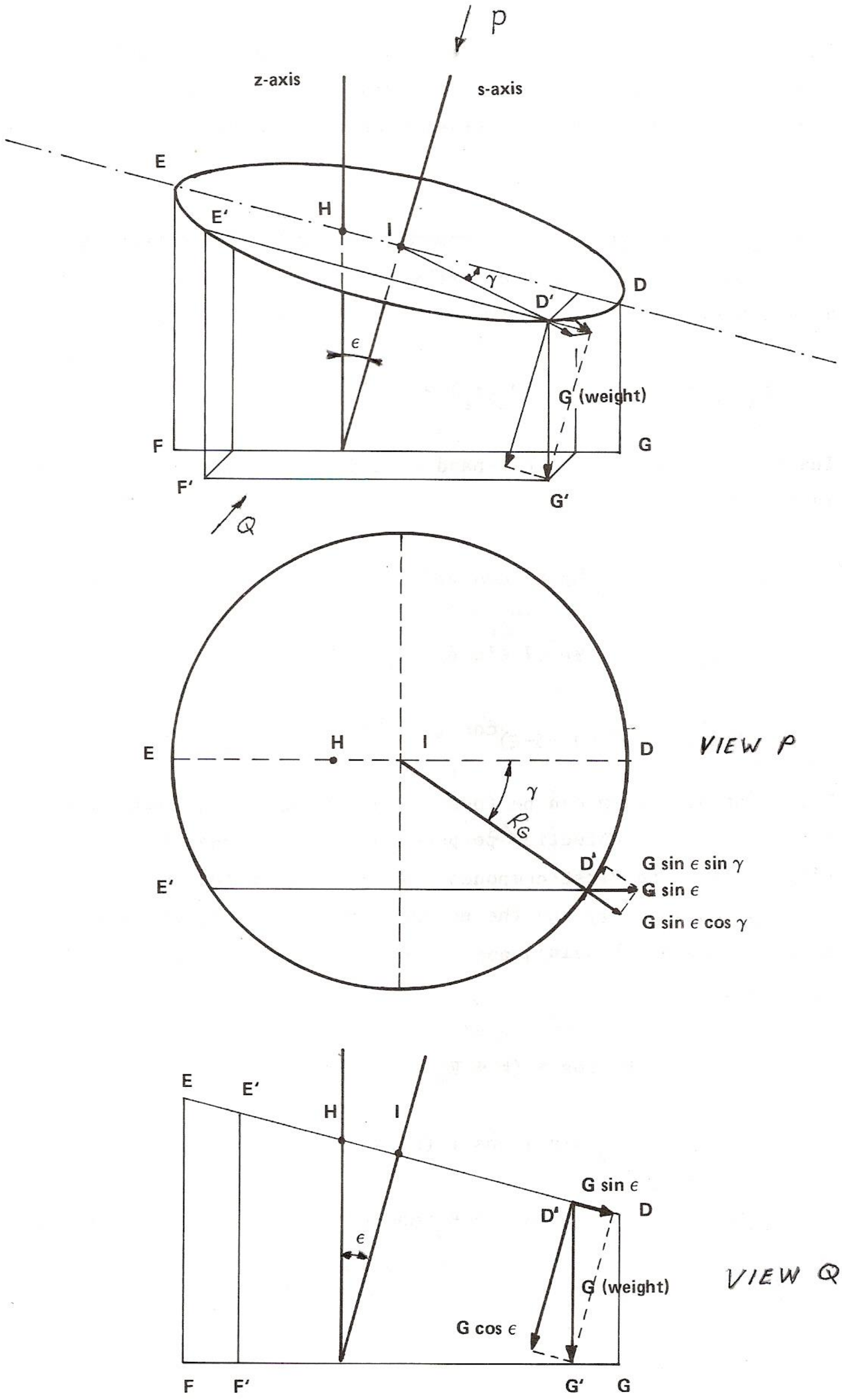

figure 7 Resolution of the weight G

4.5 Determination of the moment equations

(8) + (21) + (36) and $\varepsilon = 15°$ gives:

$\pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0225 * \sin 3\delta) =$
$0.044 * \alpha * t * w (h \cos\gamma + R_v \cos 15°)$
(for $0° \leq \delta \leq 40°$) (38)

(8) + (22) + (36) and $\varepsilon = 15°$ gives:

$\pi R^3 (C_t * e/R * \cos^2\delta + C_d * f/R * i * \sin\delta - 0.0332 * \cos^2\delta) =$
$0.044 * \alpha * t * w (h \cos\gamma + R_v \cos 15°)$
(for $40° \leq \delta \leq 90°$) (39)

Formula 7 can be written as:

$\gamma = \phi_1 + \delta - \alpha \quad (°)$ (40)

(9) + (37) + (34) + (40) and $\phi_1 = 25°$ and $\varepsilon = 15°$ gives:

$G \sin 15° * \sin (25° + \delta - \alpha) * R_G = 0.044 * \alpha * ½\rho V^2 * t * w * R_v$ (41)

Formula 41 can be written as:

$V = \sqrt{\{G \sin 15° * \sin (25° + \delta - \alpha) * R_G / 0.044 * \alpha * ½\rho * t * w * R_v\}}$ (m/s) (42)

Formulas 38 and 42 are the moment equations for $0° \leq \delta \leq 40°$. Formulas 39 and 42 are the moment equations for $40° \leq \delta \leq 90°$. Formula 42 gives the relation in between δ and V for a certain vane geometry. However, formula 42 contains α and α is a function of V. So the δ-V curve can't be simply derived from one formula like it could be done for the pendulum safety system as described in report KD 377 (ref. 3).

Kragten Design has described the ecliptic safety system in report KD 409 (ref. 7). For this safety system, the vane arm is pulled against a stop by a torsion spring. The geometry of rotor and head are chosen such that the rotor is perpendicular to the wind direction as long as the vane arm makes contact with the stop. The wind speed for which the force in between vane arm and stop is just zero, is called the design wind speed V_d. In report KD 409 it was chosen that $V_d = 7$ m/s. For higher wind speeds, the vane arm rotates right hand over an angle γ and the rotor axis turns out of the wind left hand over an angle δ. For the ecliptic system it is found that the angle α in between the wind direction and the vane blade is decreasing at increasing wind speed. This means that the variation in γ is larger than the variation in δ.

For the inclined hinge main vane system, the same will happen. So α will be rather large at low wind speeds but will be very small at high wind speeds and the variation of γ will be larger than the variation of δ. For a wind speed V = 0 m/s, the vane arm will be in the zero position but the rotor can stand in any position. For a very low wind speed of e. g. V = 2 m/s and a rotating rotor, a certain small rotor moment M_{rotor} will be produced. Because there must be balance in between M_{rotor} and $M_{vane\ z\text{-}axis}$, a certain small vane force F_v is needed and this force pushes the vane arm some degrees γ out of the zero position. However, the zero position of the vane arm makes a rather large angle $\phi_1 = 25°$ with the rotor axis and the rotor axis will therefore make a negative angle δ with the wind direction at V = 2 m/s. At a certain wind speed, the rotor axis will be perpendicular to the wind direction. This wind speed is called the design wind speed V_d.

The design wind speed for the inclined hinge main vane system must be chosen lower than for the ecliptic system because M_G and so $M_{vane\ s\text{-}axis}$ and $M_{vane\ z\text{-}axis}$ are increasing strongly at increasing γ. So if V_d is chosen too high, the rotor will turn not far enough out of the wind at high wind speeds. Suppose that it is chosen that $V_d = 4$ m/s. So for this wind speed the rotor is perpendicular to the wind direction and so $\delta = 0°$. Formula 38 and 41 become much simpler for this condition. The air density $\rho = 1.2$ kg/m^3 for a temperature of 20 °C at sea level.

For these conditions, formula 38 changes into:

$\pi R^2 * C_t * e = 0.044 * \alpha * t * w (h \cos\gamma + R_v \cos 15°)$
(for $V_d = 4$ m/s) (43)

For these conditions, formula 41 changes into:

$G \sin 15° * \sin (25° - \alpha) * R_G = 0.4224 * \alpha * t * w * R_v$
(for $V_d = 4$ m/s) (44)

For a certain vane geometry, with certain values of G, R_G, t, w and R_v, formula 44 can be used to calculate the value of α for which the equation is in balance. However, this is not easy because α is not given explicitly. It is easier to chose a certain value of α and then to determine which conditions for the vane parameters have to be realised. Earlier it has been explained that α is large for low wind speeds and low for high wind speeds. Assume $\alpha = 15°$ for V = 4 m/s.

Substitution of $\phi_1 = 25°$, $\delta = 0°$ and $\alpha = 15°$ in formula 40 gives $\gamma = 10°$.
Substitution of $\alpha = 15°$ in formula 44 gives:

$G \sin 15° * \sin (25° - 15°) * R_G = 0.4224 * 15 * t * w * R_v$ or
$G \sin 15° * \sin 10° * R_G = 6.336 * t * w * R_v$ or
$0.04494\ G * R_G = 6.336 * t * w * R_v$ or
$G * R_G / (t * w * R_v) = 140.99$ (valid if $\alpha = 15°$ for $V_d = 4$ m/s) (45)

Assume that the vane geometry is chosen such that formula 45 is fulfilled.
Formula 41 can be written as:

$G * R_G / t * w * R_v = 0.044 * \alpha * ½\rho V^2 / \{\sin 15° * \sin (25° + \delta - \alpha)\}$ (46)

(45) + (46) and $\rho = 1.2$ kg/m^3 gives:

$140.99 = 0.044 * \alpha * 0.6 * V^2 / \{\sin 15° * \sin (25° + \delta - \alpha)\}$ or
$0.0264 * \alpha * V^2 = 140.99 * 0.2588 * \sin (25° + \delta - \alpha)$ or
$\alpha * V^2 = 1382.13 \sin (25° + \delta - \alpha)$ or
$V = 37.18 \sqrt{\{ 1/\alpha * \sin (25° + \delta - \alpha)\}}$ (m/s) (47)

Suppose we take the rotor and head geometry of the VIRYA-4.2 rotor which is also used in figure 4. For this rotor it is assumed that R = 2.1 m, e = 0.42 m, f = 0.48 m, $C_t = 0.7$, $C_d = 1$, f/e = 0.48 / 0.42 = 1.1429, i = 0.01 and R/e = 2.1 / 0.42 = 5. Substitution of R = 2.1 m, $C_t = 0.7$ and e = 0.42 m in formula 43 gives:

$\alpha * t * w (h \cos\gamma + R_v \cos 15°) = 92.573$ (m^3)
(for VIRYA-4.2 rotor and $V_d = 4$ m/s) (48)

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