Calculations executed for the 2-bladed rotor of the VIRYA-1.5 windmill (λ_d = 4.5, steel blades) with generator frame size 71 and original motor shaft

ing. A. Kragten

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It is allowed to copy this report for private use. A similar rotor of the VIRYA-1.25 has been tested for about ten years but the described VIRYA-1.5 rotor is not yet tested. The rotor should not be used on a windmill without a proper safety system!

Engineering office Kragten Design
Populierenlaan 51
5492 SG Sint-Oedenrode
The Netherlands
telephone: +31 413 475770
e-mail: info@kdwindturbines.nl
website: www.kdwindturbines.nl

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1 Introduction

The VIRYA-1.5 windmill is developed for manufacture in western countries as well as in developing countries. The VIRYA-1.5 has a 2-bladed rotor with stainless steel blades. The construction of the rotor is about similar to the construction of the VIRYA-1.25 rotor which has been tested for about ten years and which has survived rotating a measured wind speed of 26 m/s. The VIRYA-1.5 is meant for 24 V battery charging.

The generator is based on a 0.37 kW, asynchronous 3-phase motor of Indian manufacture for which the 4-pole short-circuit armature is replaced by an armature provided with permanent magnets. This generator is described in report KD 451 (ref. 1). The generator housing is of Indian manufacture and the original shaft can be used. This generator will therefore be rather cheap. The generator of the VIRYA-1.5 has been built but has not yet been measured on a test rig with which the torque can be measured. The characteristics will therefore be estimated from the measurements of the VIRYA-1.8 generator. The measurements for this generator are given in report KD 54 (ref. 2). It is assumed that the torque level is reduced by a factor 0.5 because of the shorter armature and different armature construction and because 5 mm thick magnets are used. The current VIRYA generators are described in report KD 341 (ref. 3).

The windmill is provided with the so called hinged side vane safety system with a 1 mm stainless steel vane blade. The rated wind speed with this vane blade V_{rated} is about 11 m/s. The head and the tower pipe are the same as those designed for the former VIRYA-1.46 windmill and given on drawing 1103-03. It might be possible to use the VIRYA-1.5 rotor in combination with the 34-pole PM-generator which is described in report KD 580 (ref. 4). However, this generator has a tapered shaft and this means that now the rotor hub must have a tapered central hole. The generator bracket of the head has to be modified too.

2 Description of the rotor of the VIRYA-1.5 windmill

The 2-bladed rotor of the VIRYA-1.5 windmill has a diameter D = 1.5 m and a design tip speed ratio λ_d = 4.5. Advantages of a 2-bladed rotor are that no welded spoke assembly is required and that the rotor can be balanced and transported easily.

The rotor has blades with a constant chord and is provided with a 7.14 % cambered airfoil. The rotor is made of one stainless steel strip with dimensions of 166.6 * 1500 * 2 mm and 18 strips can be made out of a standard sheet of 1.5 * 3 m. Because the blade is cambered, the chord c is a little less than the blade width, resulting in c = 164.3 mm = 0.1643 m. For cambering the blades, it might be possible to use a blade press which is derived from the blade press of the VIRYA-1.8 (but for a width of the strip of 166.6 mm in stead of 125 mm).

The camber is only made in the outer 600 mm of the blade. This part of the blade is twisted linear. The next 137.5 mm till the edge of the hub is twisted to get the correct blade angle at the blade root. It is assumed that the outer 50 mm of this 137.5 mm long part is used for the transition of camber to flat. So the inner 87.5 mm is not cambered. This non cambered part makes the blade very flexible which is necessary to prevent vibrations due to the gyroscopic moment. The central 25 mm of the blade, where it is connected to the hub, is flat.

The current VIRYA generators have a tapered shaft and the hub has a tapered hole at the centre and is pressed to the shaft by one central bolt. However, the original motor shaft will be used for the VIRYA-1.5 generator and this construction is therefore not possible. The end of the original motor shaft has a diameter of 14 mm and a length of 30 mm and is provided with a key groove.

A hub construction has been designed for which the hub is clamped to the shaft. The original key is used to prevent rotation of the hub around the shaft. The blade is clamped in between two strips of 3 * 25 * 166.6 mm using seven bolts M5. The rotor is balanced by grinding so much from the heaviest blade tip till the blade is in perfect balance. An assembly drawing and detailed drawings of the rotor components are given on drawing 1103-01.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 5). This report (KD 465) has its own formula numbering. Substitution of $\lambda_d = 4.5$ and $R = 0.75$ m in formula (5.1) of KD 35 gives:

$$\lambda_{r\,d} = 6 * r \qquad (-) \qquad (1)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \qquad (°) \qquad (2)$$

$$\phi = 2/3 \arctan 1 / \lambda_{r\,d} \qquad (°) \qquad (3)$$

Substitution of $B = 2$ and $c = 0.1643$ m in formula (5.4) of KD 35 gives:

$$C_l = 76.484\, r\, (1 - \cos\phi) \qquad (-) \qquad (4)$$

Substitution of $V = 6$ m/s and $c = 0.1643$ m in formula (5.5) of KD 35 gives:

$$R_{e\,r} = 0.658 * 10^5 * \sqrt{}\,(\lambda_{r\,d}^2 + 4/9) \qquad (-) \qquad (5)$$

The blade is calculated for six stations A till F which have a distance of 0.12 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 6). The Reynolds values for the stations are calculated for a wind speed of 6 m/s because this is a reasonable wind speed for a windmill which is designed for a rated wind speed of 11 m/s. Those airfoil Reynolds numbers are used which are laying closest to the calculated values.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 6 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	0.75	4.5	8.4	0.1643	0.61	0.62	2.67	2.5	-0.7	-0.6	9.1	9	0.05
B	0.63	3.78	9.9	0.1643	0.71	0.71	2.25	2.5	-0.1	-0.1	10.0	10	0.04
C	0.51	3.06	12.1	0.1643	0.86	0.77	1.84	1.7	2.1	1.1	10.0	11	0.042
D	0.39	2.34	15.4	0.1643	1.07	1.03	1.44	1.2	3.7	3.4	11.7	12	0.03
E	0.27	1.62	21.1	0.1643	1.39	1.36	1.04	1.2	9.7	8.1	11.4	13	0.063
F	0.15	0.9	32.0	0.1643	1.74	1.27	0.69	1.2	-	18.0	-	14	0.3

table 1 Calculation of the blade geometry of the VIRYA-1.5 rotor

No value for α_{th} and therefore for β_{th} is found for station F because the required C_l value can not be generated. The theoretical blade angle β_{th} varies in between 9.1° and 11.7°. If a blade angle of 9° taken at the blade tip and of 14° at the blade root, the linearised blade angles are laying close to the theoretical values. The non cambered part of the strip is twisted 14° to get the correct blade angle at the blade root.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.6 of KD 35 (for B = 2) en $\lambda_{opt} = 4.5$ and $C_d/C_l = 0.04$ gives $C_{p\ th} = 0.41$. The blade is stalling in between station E and F so only the part of the blade till 0.05 m outside station F is taken for the calculation of C_p. This gives an effective blade length k' = 0.55 m.
Substitution of $C_{p\ th} = 0.41$, R = 0.75 m and blade length k = k' = 0.55 m in formula 6.3 of KD 35 gives $C_{p\ max} = 0.38$. $C_{q\ opt} = C_{p\ max} / \lambda_{opt} = 0.38 / 4.5 = 0.0844$.
Substitution of $\lambda_{opt} = \lambda_d = 4.5$ in formula 6.4 of KD 35 gives $\lambda_{unl} = 7.2$.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R – ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

The average blade angle is 11.5°. For a non rotating rotor, the average angle of attack α is therefore 90° - 11.5° = 78.5°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 78.5° it can be read that $C_l = 0.39$. During starting, the whole blade is stalling. So now the real blade length k = 0.6 m is taken.
Substitution of B = 2, R = 0.75 m, k = 0.6 m, $C_l = 0.39$ en c = 0.1643 m in formula 6 gives that $C_{q\ start} = 0.020$. For the ratio in between the starting torque and the optimum torque we find that it is 0.020 / 0.0844 = 0.24. This is rather high for a rotor met a design tip speed ratio of 4.5.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \qquad (m/s) \qquad (7)$$

The peak sticking torque Q_s of the VIRYA-1.5 generator has been measured for a prototype of the VIRYA-1.5 generator and it was found that Q_s is rather high and that $Q_s = 0.33$ Nm. Substitution of $Q_s = 0.33$ Nm, $C_{q\ start} = 0.020$, ρ = 1.2 kg/m^3 and R = 0.75 m in formula 7 gives that $V_{start} = 4.6$ m/s. This is rather high for a rotor with a design tip speed ratio of 4.5 and a rated wind speed of 11 m/s. However, the average torque at very low rotational speeds is lower than the peak torque so once the rotor is rotating a little, the fly wheel effect makes that only the average torque has to be supplied. So the real starting wind speed will be about 4 m/s which is acceptable for a moderate wind regime. The generator is rectified in delta and the unloaded Q-n curve is rising rather fast at increasing rotational speed. However, the Q-n curve of the rotor for V = 4 m/s will rise even faster and the real starting wind speed will therefore be about 4 m/s.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing a S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 7). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-1.5 rotor are given in figure 1 and 2. .

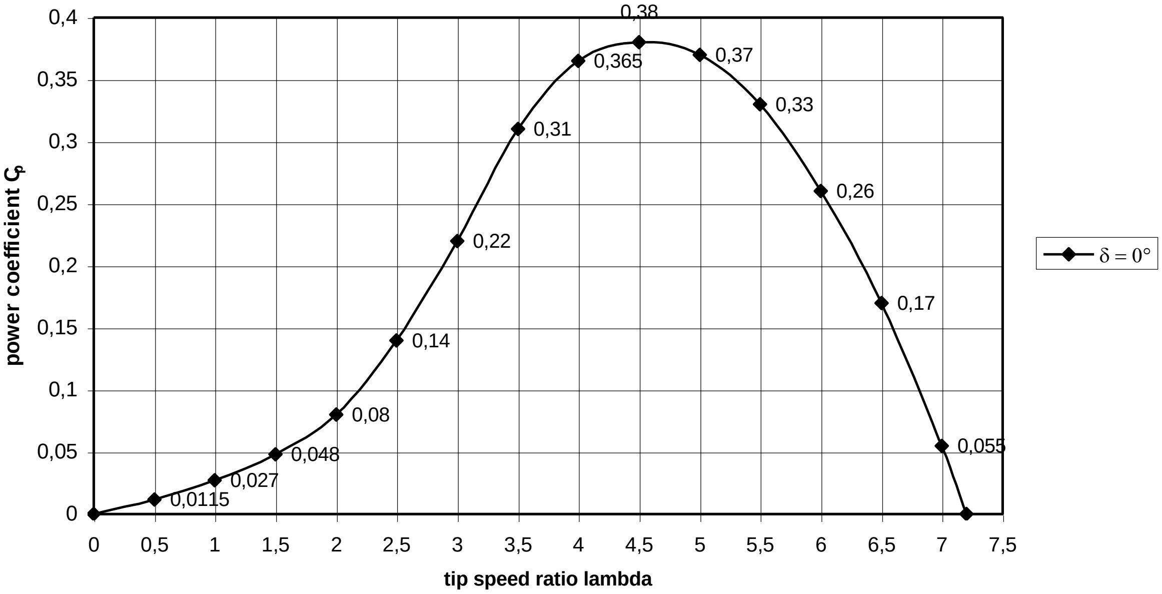

fig. 1 Estimated C_p-λ curve for the VIRYA-1.5 rotor for the wind direction perpendicular to the rotor (δ = 0°)

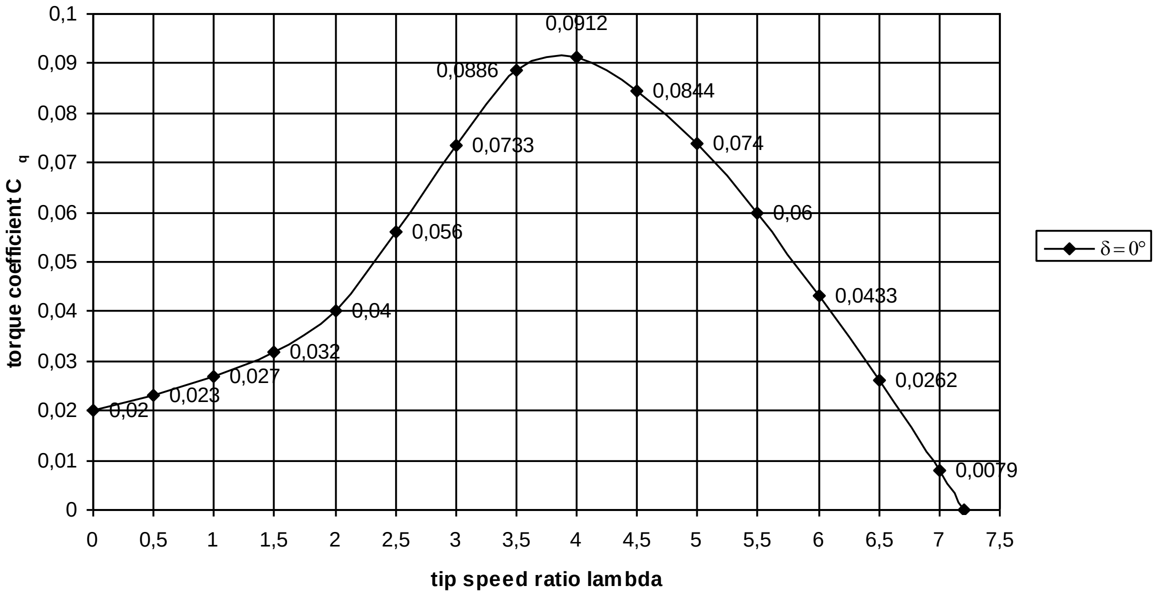

fig. 2 Estimated C_q-λ curve for the VIRYA-1.5 rotor for the wind direction perpendicular to the rotor (δ = 0°)

5 Determination of the P-n curves and the P_{el}-V curves for 26 V delta for a modified 115/200 V winding

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The C_p-λ curve is given in figure 1. The δ-V curve of the safety system depends on the vane blade mass per area. The vane blade is made of 1 mm stainless steel. The rated wind speed for this vane blade is about 11 m/s. The estimated δ-V curve is given in figure 3.

The head starts to turn away at a wind speed of about 6 m/s. For wind speeds above 11 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 11 m/s will therefore also be valid for wind speeds higher than 11 m/s.

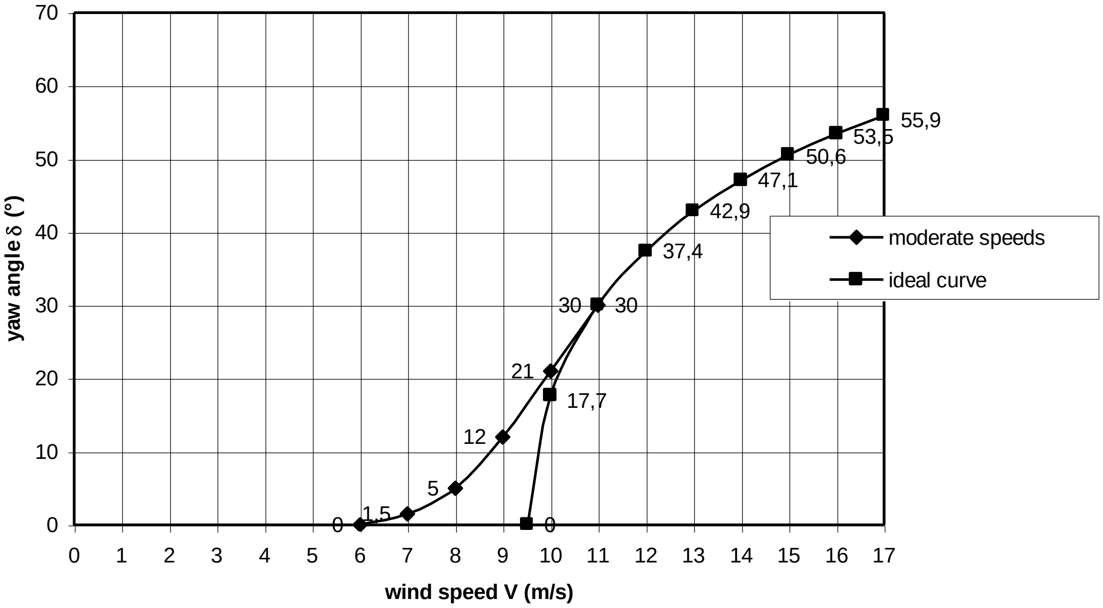

fig. 3 Estimated δ-V curve VIRYA-1.5 for 1 mm stainless steel vane blade

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator for a certain gear ratio i (the VIRYA-1.5 has no gearing so i = 1). Because we are especially interested in the domain around the optimal cubic line and because the P-n curves for low values of λ appear to lie very close to each other, the P-n curves are not determined for low values of λ. The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9, 10 and 11 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of R = 0.75 m in formula 7.1 of KD 35 gives:

$$n_\delta = 12.7324 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of $\rho = 1.2\ kg / m^3$ en R = 0.75 m in formula 7.10 of KD 35 gives:

$$P_\delta = 1.0603 * C_p * \cos^3\delta * V^3 \qquad \text{(W)} \qquad (9)$$

The P-n curves are determined for C_p values belonging to λ is 2.5, 3.5, 4.5, 5.5, 6.5 and 7.2. (see figure 1). For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 3, is taken into account. The result of the calculations is given in table 2.

		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 0°		V = 7 m/s δ = 1.5°		V = 8 m/s δ = 5°		V = 9 m/s δ = 12°		V = 10 m/s δ = 21°		V = 11 m/s δ = 30°	
λ (-)	C_p (-)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)
2.5	0.14	95.5	4.01	127.3	9.50	159.2	18.56	191.0	32.06	222.7	50.9	253.7	75.1	280.2	101.3	297.2	120.8	303.2	128.3
3.5	0.31	133.7	8.87	178.3	21.04	222.8	41.09	267.4	71.00	311.8	112.6	355.2	166.4	392.3	224.2	416.0	267.5	424.5	284.2
4.5	0.38	171.9	10.88	229.2	25.79	286.5	50.36	343.8	87.03	400.9	138.1	456.6	203.9	504.4	274.9	534.9	327.8	545.8	348.3
5.5	0.33	210.1	9.45	280.1	22.39	350.1	43.74	420.2	75.58	490.0	119.9	558.1	177.1	616.5	238.7	653.8	284.7	667.1	302.5
6.5	0.17	248.3	4.87	331.0	11.52	413.8	22.53	496.6	38.93	579.1	61.8	659.6	91.2	728.6	123.0	772.6	146.7	788.4	155.8
7.2	0	275.0	0	366.7	0	458.4	0	550.0	0	641.5	0	730.6	0	807.0	0	855.8	0	873.3	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-1.5 rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which can be drawn through the maximum of all P-n curves is also given in figure 4.

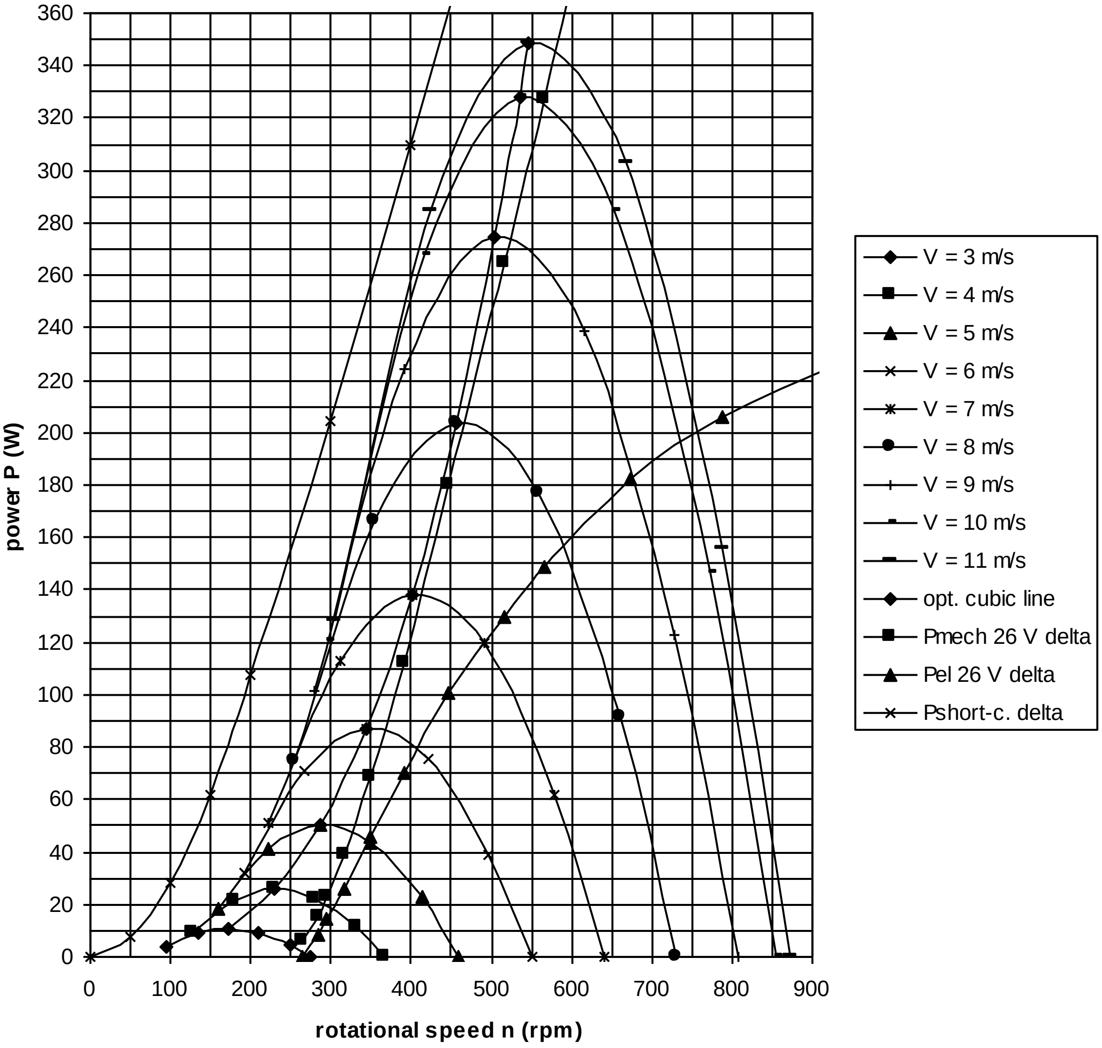

fig. 4 P-n curves of the VIRYA-1.5 rotor, optimum cubic line, P_{mech}-n and P_{el}-n curves for 26 V delta and P-n curve for short-circuit in delta for a modified 115/200 V winding

The generator is made of a 0.37 kW motor with a 230/400 V 3-phase winding. The standard 230/400 V winding is modified into a 115/200 V winding by connecting the first and the second layer in parallel in stead of in series. This procedure is described in report KD 341 (ref. 3). This modification means that the measured characteristics for 52 V delta for the original 230/400 V winding will be identical to the characteristics for 26 V delta for the modified 115/200 V winding. It is assumed that the torque level is a factor 0.5 of the torque level of the VIRYA-1.8 generator and that the P_{mech}–n curve is shifted 25 rpm to the right. This assumption must be checked if a prototype of the generator is available.

The estimated P_{mech}-n and P_{el}-n curves for 26 V delta for the modified 115/200 V winding are also given in figure 4. The estimated P-n curve for short-circuit in delta is determined by multiplying the curve of the VIRYA-1.8 generator by a factor 0.5.

The point of intersection of the P_{mech}-n curve for 26 V delta of the generator with the P-n curve of the rotor for a certain wind speed, gives the working point for that wind speed. The electrical power P_{el} for that wind speed is found by going down vertically from the working point up to the point of intersection with the P_{el}-n curve. The values of P_{el} found this way for all wind speeds, are plotted in the P_{el}-V curve (see figure 5). For high powers, the voltage is higher than 26 V and therefore the generator efficiency will be higher too. This results in a little higher electrical power. The P_{el}-V curve is corrected for this effect.

The matching of rotor and generator is good for wind speeds in between 5 and 11 m/s because the P_{mech}-n curve of the generator is lying close to the optimum cubic line. The supply of power starts already at a wind speed of 3.2 m/s ($V_{cut\ in}$ = 3.2 m/s). This is low and therefore the windmill can be used in regions with low wind speeds. In chapter 4 it was calculated that V_{start} = 4 m/s so there is some hysteresis in the P_{el}-V curve for wind speeds in between 3.2 and 4 m/s. The maximum power is 160 W which is acceptable for a rotor with 1.5 m diameter.

The P-n curve for short-circuit in delta is lying left from the P-n curve of the rotor for V = 11 m/s and higher. This means that the rotor will slow down to almost stand still for every wind speed if short-circuit is made.

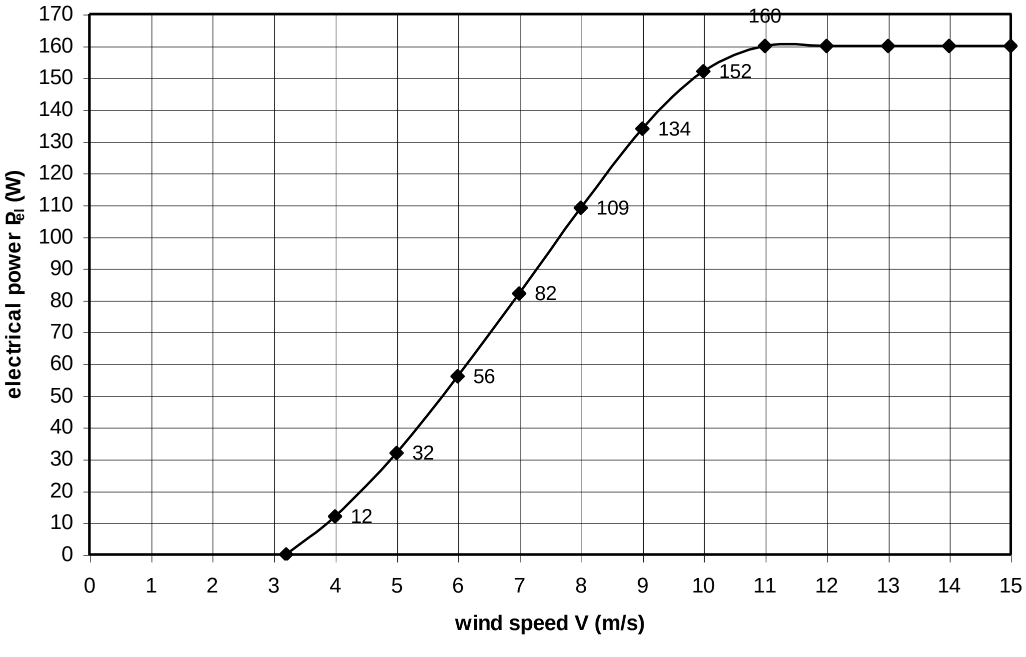

fig. 5 P_{el}-V curve of the VIRYA-1.5 windmill with V_{rated} = 11 m/s for 24 V battery charging and rectification in delta for a 0.37 kW housing with a modified 115/200 V winding

6 Calculation of the strength of the blade

The cambered part of the blade is connected to the hub by the transition part and the non cambered part. The non cambered part has about a length l = 82.5 mm, a width b = 166.6 mm and a height h = 2 mm. The strip is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The strip is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because the strip is very thin it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1^e The load which appears for a rotating rotor at V_{rated} = 11 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 30° for V_{rated} = 11 m/s.

2^e The load which appears for a slowed down rotor. The rotor is slowed down by making short-circuit in the generator winding. A graph has been made in which the Q-n curve of the rotor for V = 11 m/s has been plotted together with the Q-n curve of the generator for short-circuit in delta. For the working point it is found that the rotor rotates at a rotational speed of about 35 rpm and has a tip speed ratio of about 0.3. For this very low rotational speed the effect of compensation by the centrifugal moment is negligible and a tip speed ratio of 0.3 is very low. Therefore it is assumed that the rotor stands still.

6.1 Bending stress in the strip for a rotating rotor and V = 11 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * \tfrac{1}{2}\rho V^2 * \pi R^2 / B \qquad (N) \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The effective blade length k' of the VIRYA-1.5 rotor is only 0.55 m but the rotor radius R = 0.75 m. Therefore there is a disk in the centre with an area of about 0.071 of the rotor area on which almost no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.929 = 0.826. Because of the tip losses the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75. It is assumed that the thrust coefficient is constant for values of λ in between λ_d and $\lambda_{unloaded}$.

Substitution of C_t = 0.75, δ = 30°, ρ = 1.2 kg/m^3, V = 11 m/s, R = 0.75 m and B = 2 in formula 10 gives $F_{t\,\delta\,bl}$ = 36.1 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is laying at 2/3 R = 0.5 m. Because the effective blade length is only k’, there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k’ falls off. The centre of gravity of the trapezium has been determined graphically and is laying at about $r_1 = 0.53$ m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is laying at $r_2 = 0.0125$ m. At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_1 - r_2) \qquad \text{(Nm)} \qquad (11)$$

Substitution of $F_{t\,\delta\,bl} = 36.1$ N, $r_1 = 0.53$ m and $r_2 = 0.0125$ m gives $M_{b\,t} = 18.7$ Nm = 18700 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$$\sigma_b = M / W \qquad (\text{N/mm}^2) \qquad (12)$$

The moment of resistance W of a strip is given by:

$$W = 1/6\ bh^2 \qquad (\text{mm}^3) \qquad (13)$$

(12) + (13) gives:

$$\sigma_b = 6\ M / bh^2 \qquad (\text{N/mm}^2) \qquad (\text{M in Nmm}) \qquad (14)$$

Substitution of M = 18700 Nmm, b = 166.6 mm and h = 2 mm in formula 14 gives $\sigma_b = 168$ N/mm^2. For this stress the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the strip is bending only over the non cambered part. So it is assumed that the cambered 600 mm of the blade and the 50 mm of the transition part are not bending. The point where the transition part starts, is laying at $r_3 = 0.1$ m = 100 mm. So the length of the strip l which is loaded by bending is given by:

$$l = r_3 - r_2 \qquad \text{(mm)} \qquad (15)$$

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$$M = F * (r_1 - r_3) \qquad \text{(Nmm)} \qquad (16)$$

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$$\phi = l * (M + \tfrac{1}{2} Fl) / EI \qquad \text{(rad)} \qquad (17)$$

The bending moment of inertia I of a strip is given by:

$$I = 1/12\ bh^3 \qquad (mm^4) \qquad (18)$$

(15) + (16) + (17) + (18) gives:

$$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½\,(r_3 - r_2)\} / (E * bh^3) \qquad (rad) \qquad (19)$$

Substitution of $F = 36.1$ N, $r_3 = 100$ mm, $r_2 = 12.5$ mm, $r_1 = 530$ mm, $E = 2.1 * 10^5$ N/mm^2, $b = 166.6$ mm and $h = 2$ mm in formula 19 gives: $\phi = 0.06416$ rad = 3.68°. This is an angle which can not be neglected. In report R409D (ref. 8) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-1.5 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin\left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2}\right) \qquad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 166.6 mm and a plate thickness of 2 mm it is found that $A_{pr} = 0.000333$ m^2. The blade is made of steel sheet with a density ρ_{pr} of about $\rho_{pr} = 7.8 * 10^3$ kg/m^3. In figure 4 it can be seen that for high wind speeds, the rotor is running at a tip speed ratio of about $\lambda = 4.75$. Substitution of $C_t = 0.75$, $\rho = 1.2$ kg/m^3, $R = 0.75$ m, $B = 2$, $A_{pr} = 0.000333$ m^2, $\rho_{pr} = 7.8 * 10^3$ kg/m^3 and $\lambda = 4.75$ in formula 20 gives: $\varepsilon = 0.78°$.

This angle is much smaller than the calculated angle of 3.68° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 0.78°.

The real bending angle ε is determined a follows. A thrust moment $M_t = 18.7$ Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. $M_b = 18.7$ Nm for $\varepsilon = 3.68°$. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. $M_c = 18.7$ Nm for $\varepsilon = 0.78°$. The path of these three moments is given in figure 6. The sum total of $M_b + M_c$ is determined and the line $M_b + M_c$ is also given in figure 6.

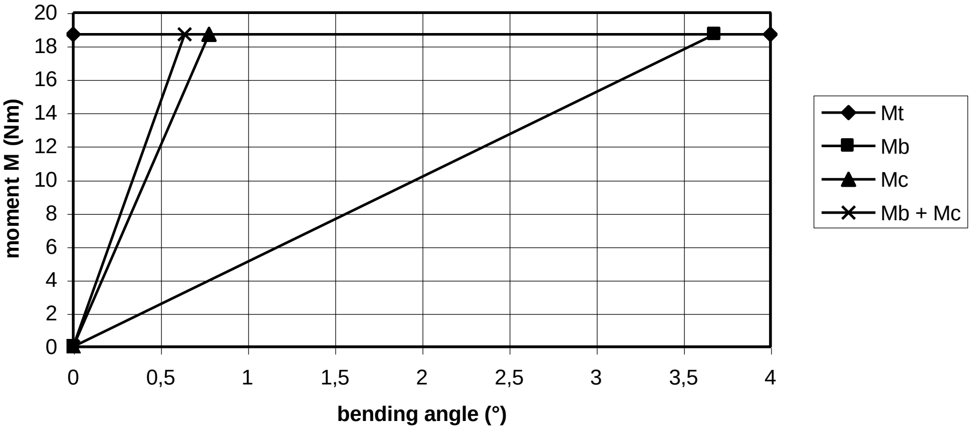

fig. 6 Path of M_t, M_b, M_c, and $M_b + M_c$ as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 6 it can be seen that ε = 0.64°. This is a factor 0.174 of the calculated angle of 3.68°. Because the bending stress is proportional to the bending angle it will also be a factor 0.174 of the calculated stress of 168 N/mm^2 resulting in a stress of about 29 N/mm^2. This is a very low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with sinα with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment dF * r which is exerted by this force with respect to the blade is therefore varying sinusoidal too.

However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \qquad \text{(Nm)} \qquad (21)$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three $\sin^2\alpha$ functions which make an angle of 120° which each other, appear to result in a constant value. The resulting moment for a two bladed rotor fluctuates just as it is does for one blade because the moments of both blades are strengthening each other. Formula 21 is still valid for the average value of the moment but the momentary value is given by:

$$M_{gyr\ x\text{-}as} = 2\sin^2\alpha * I_{rot} * \Omega_{rot} * \Omega_{head} \qquad \text{(Nm)} \qquad (22)$$

This function is given in figure 7.

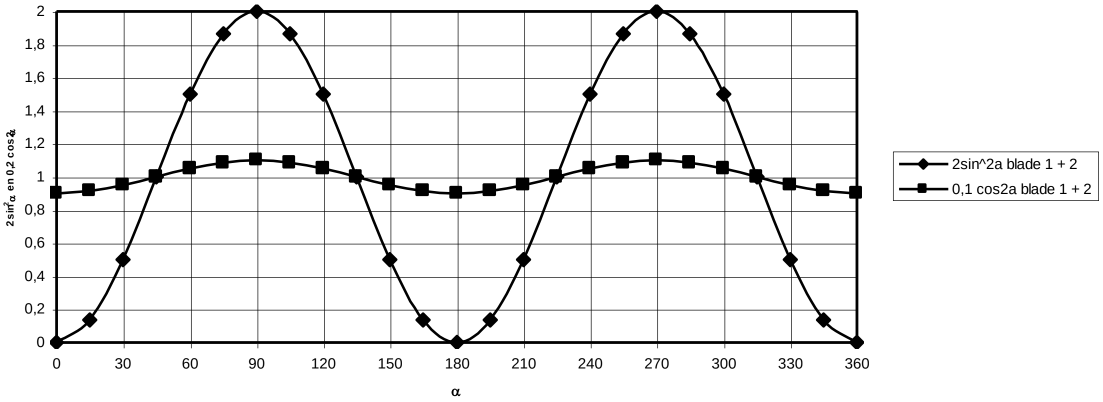

fig. 7 Path of 2 $\sin^2\alpha$ and (1 – 0.1 cos2α) for a two bladed rotor

Formula 22 can also be written as:

$$M_{gyr\ x\text{-}as} = (1 - \cos 2\alpha) * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (23)$$

Because the average of $\cos 2\alpha$ is zero, the average of formula 23 is the same as formula 21.

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are very flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 22 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $2 \sin^2\alpha$ function is almost flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened curve can be given by:

$$M_{gyr\ x\text{-}as\ flattened} = (1 - 0.1 \cos 2\alpha) * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (24)$$

The function $(1 - 0.1 \cos 2\alpha)$ is also plotted in figure 7. This function has a maximum for $\alpha = 90°$ and for $\alpha = 270°$. The maximum is $1.1 * I_{rot} * \Omega_{rot} * \Omega_{head}$.

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then halve the value of the moment for the whole rotor.

Therefore the maximum moment in one blade is given by:

$$M_{gyr\ bl\ max} = 0.55 * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (25)$$

For a two bladed rotor, the moment of inertia of the whole rotor I_{rot} is twice the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$$M_{gyr\ bl\ max} = 1.1\ I_{bl} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (26)$$

For the chosen blade geometry it is calculated that $I_{bl} = 0.365$ kgm^2. The maximum loaded rotational speed of the rotor can be read in figure 4 and it is found that $n_{max} = 575$ rpm. This gives $\Omega_{rot\ max} = 60.2$ rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-1.5 is provided with the hinged side vane safety system which has a light van blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be 0.4 rad/s at very high wind speeds.

Substitution of $I_{bl} = 0.365$ kgm^2, $\Omega_{rot\ max} = 60.2$ rad/s en $\Omega_{head\ max} = 0.4$ rad/s in formula 26 gives: $M_{gyr\ bl\ max} = 9.67$ Nm = 9670 Nmm.

Substitution of M = 9670 Nmm, b = 166.6 mm and h = 2 mm in formula 14 gives $\sigma_{b\ max} = 87$ N/mm^2. This value has to be added to the bending stress of 29 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max} = 116$ N/mm^2. The minimum stress is 29 – 87 = - 58 N/mm^2. So the stress is becoming negative and therefore it is necessary to take the load as a fatigue load.

For the strip material stainless steel AISI 304 (MCB quality 1.4301) is chosen. The 0.2 % deformation limit for this steel is 230 N/mm^2. However this is for a pulling stress. The deformation limit for a bending stress is much higher and it is expected that it is 350 N/mm^2.

The allowable fatigue stress is much lower than the 0.2 % deformation stress for bending because stainless steel is sensible to fatigue. The value is not given in the MCB catalogue but it is assumed that the allowable fatigue stress for bending is 160 N/mm^2. It is assumed that the allowable bending stress for a non fatigue stress is 240 N/mm^2. The calculated stress is lower than the allowable fatigue stress so the strip is strong enough.

In reality the blade is not extremely stiff and will also bend somewhat. This reduces the bending of the strip and therefore the stress is the strip will be somewhat lower.

6.2 Bending stress in the strip for a slowed down rotor

The rotational speed for a rotor which is slowed down by making short-circuit of the generator is very low. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 8.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 36.1 N for $V = V_{rated} = 11$ m/s and $\delta = 30°$. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 30° if the force on one blade is 36.1 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at $r_4 = 0.375$ m because the relative wind speed is almost constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 11 changes into:

$$M_{b\ t} = F_{t\ \delta\ bl} * (r_4 - r_2) \qquad \text{(Nm)} \qquad (27)$$

Substitution of $F_{t\ \delta\ bl} = 36.1$ N, $r_4 = 0.375$ m en $r_2 = 0.0125$ m in formula 27 gives $M_{b\ t} = 13.1$ Nm = 13100 Nmm. Substitution of M = 13100 Nmm, b = 166.6 mm and h = 2 mm in formula 14 gives $\sigma_b = 118$ N/mm^2. This is a higher than the calculated stress for a rotating rotor. However, this load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable stress is 240 N/mm^2 for stainless steel sheet, so the strip is strong enough.

Because the strip and the blade are rather flexible it has to be checked if a slowed down rotor can’t hit the tower. In chapter 6.1 it has been calculated, for no compensation of the gyroscopic moment, that the bending angle is 3.68° for a stress of 168 N/mm^2. So for a stress of 118 N/mm^2 the bending angle will be 3.68 * 118 / 168 = 2.58°. For a rotor radius of R = 0.75 m this results in a movement at the tip of about 0.034 m. Because the blade itself will bend too, the movement will be larger and it is expected that it will be about 0.06 m. The minimum distance in between the blade tip and the tower pipe is much larger if the blade is not bending. So there is no chance that the blade hits the tower for a slowed down rotor.

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