Determination of the C_m-α curves for different positions of the axis of rotation for a 10 %, 5 % and 7.14 % cambered airfoil with an aspect ratio of 5.

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1 Introduction

In report KD 35 (ref. 1) a method is given for the design of a windmill rotor. The blade of the rotor must be provided with an aerodynamic airfoil. Kragten Design has developed some windmills for which the 7.14 % cambered airfoil is used. In report KD 398 (ref. 2), the aerodynamic characteristics are given for the 7.14 %, 10 % and 12.5 % cambered airfoil with an infinitive aspect ratio. An infinitive aspect ratio means that the blade has the same length as the width of the wind tunnel and so no wake is created at the blade ends. These characteristics are derived from measurements given in report R443D (ref. 3). The measurements for the 7.14 %, 10 % and 12.5 % airfoils have originally been performed by Buering of Imperial Collage but access to the original source is no longer possible. The 10 % cambered airfoil with an infinitive aspect ratio has also been measured by other researchers like Volkers and Bruining (see R443D). For all these measurements, the lift coefficient C_l and the drag coefficient C_d is given as a function of the angle of attack α. However, the moment coefficient C_m is not measured.

The moment coefficient C_m is only measured by Flachsbart but not for a blade with an infinitive aspect ratio but for a blade with an aspect ratio of 5 and for 5 %, 10 % and 15 % camber. An aspect ratio of 5 means that the blade length is five times the chord and that the width of the wind tunnel must have been larger than the blade length. So a wake is created at both sides of the blade and this results in reduction of the lift coefficient and increase of the drag coefficient.

At this moment I am investigating if it is possible to design a windmill rotor with a safety system based on pitch control activated by the aerodynamic moment acting on the blade. The blades will have a 7.14 % cambered airfoil. To investigate if this is possible, the C_m-α curve around a certain axis of rotation is needed. However, Flachsbart hasn’t measured a 7.14 % cambered airfoil and therefore the measurements for 10 % and 5 % camber are investigated. The values for 7.14 % camber will lie somewhere in the middle.

2 Determination of the C_m-α curves for 10 % camber and different turning points

The measurements of Flachsbart for 10 % camber are given on page 3-5 of report R443D (ref. 3). Only a picture with the C_l-C_d and the C_m-C_d curves is given but fortunately the measuring points are also given in a table. It is not defined around which point the moment coefficient is taken. This can be the quart chord point or the airfoil nose. For $\alpha = 90°$ it is found that the moment coefficient C_m is just halve the drag coefficient C_d and this means that the moment coefficient is take around the airfoil nose. As C_d and C_m are both positive for $\alpha = 90°$, it means that the left hand direction of the aerodynamic moment is taken positive. For most aerodynamic airfoils the right hand direction is taken positive but I will follow the definition of Flachsbart, so the left hand direction is taken positive. The table out of R443D has been copied as table 1. The measurements are performed for a Reynolds value $Re = 4 * 10^5$. The variation of C_m as a function of α is given in figure 2. It can be seen that C_m is increasing strongly for $-20° < \alpha < 12.5°$. This is not nice for a pitch control system because the system will react very strongly for small variations of α. It would be better if C_m would be about constant because in this case the aerodynamic moment is only depending on the rotational speed of the rotor. It might be possible to realise this by taking another position of the axis of rotation. For normal airfoils, the moment coefficient is about constant for small angles of α if the moment is taken around the quart chord point. It will be investigated if an axis can be found for the 10 % cambered airfoil for which this is also the case.

α (°)	C_l (-)	C_d (-)	C_m (-)	$C_{m0.25}$ (-)	$C_{m0.333}$(-)	$C_{m0.4}$ (-)	$C_{m0.5}$ (-)
-19.9	-0.340	0.237	-0.087	0.013	0.046	0.073	0.113
-10.0	-0.131	0.118	0.021	0.058	0.071	0.081	0.096
-5.0	0.111	0.0670	0.127	0.101	0.092	0.085	0.075
-0.2	0.580	0.0680	0.351	0.206	0.160	0.119	0.061
4.6	1.037	0.0965	0.510	0.250	0.163	0.093	-0.011
9.5	1.313	0.147	0.558	0.228	0.119	0.030	-0.102
12.5	1.459	0.194	0.592	0.225	0.104	0.005	-0.141
14.4	1.520	0.239	0.595	0.212	0.085	-0.018	-0.171
16.5	1.462	0.308	0.605	0.233	0.109	0.009	-0.140
19.6	1.181	0.368	0.547	0.238	0.135	0.053	-0.071
24.6	1.200	0.478	0.569	0.246	0.139	0.053	-0.076
29.6	1.156	0.588	0.575	0.251	0.144	0.057	-0.073
34.6	1.040	0.659	0.565	0.257	0.155	0.073	-0.050
39.7	0.945	0.705	0.540	0.246	0.148	0.069	-0.049
49.7	0.812	0.853	0.537	0.243	0.145	0.067	-0.051
59.8	0.671	0.984	0.549	0.252	0.153	0.074	-0.045
69.8	0.497	1.112	0.573	0.269	0.168	0.087	-0.035
79.9	0.299	1.185	0.585	0.280	0.179	0.097	-0.025
90.0	0	1.219	0.609	0.304	0.203	0.121	-0.001

table 1 C_l, C_d, C_m, $C_{m0.25}$, $C_{m0.333}$, $C_{m0.4}$ and $C_{m0.5}$ as a function of α for 10 % camber. C_m is taken around the nose. The left hand direction is taken positive. Re = $4 * 10^5$

The 10 % cambered airfoil and the lift L, the drag D and the moment M acting on it, is given in figure 1.

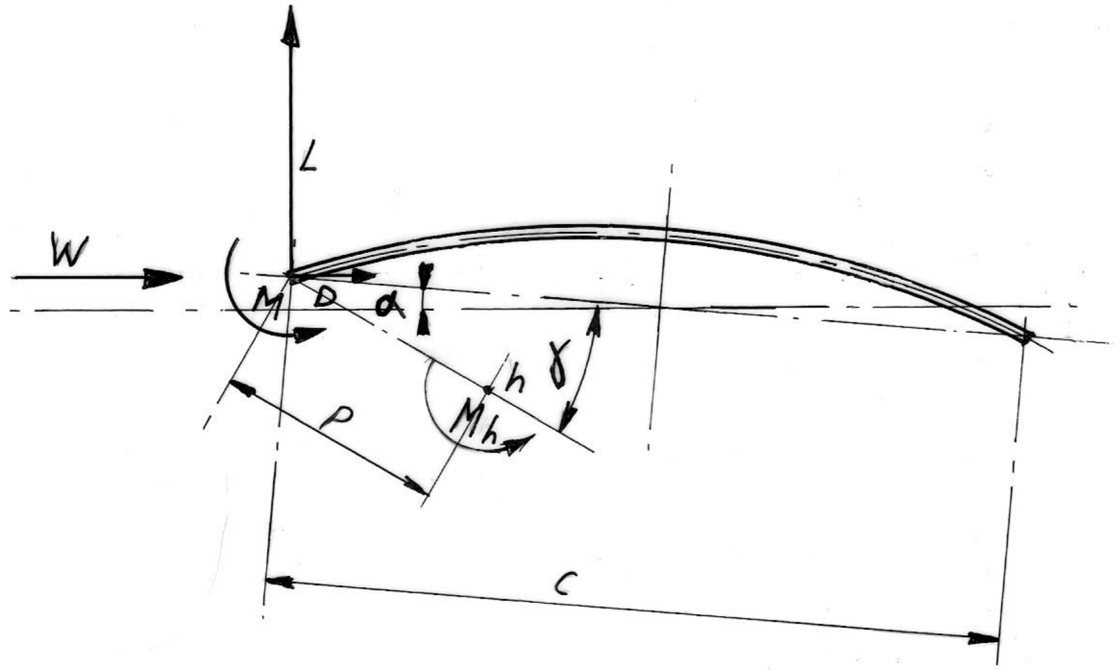

fig. 1 Lift L, drag D and moment M acting on the 10 % cambered airfoil

So the moment M and the moment coefficient C_m are taken around the airfoil nose by Flachsbart. If the moment is taken around another point h, lying at a distance p from the nose and if the line through the nose and point h makes a right hand angle γ with the zero line of the airfoil, it can be proven that C_{mh} is given by:

$$C_{mh} = C_m - C_l * p/c * \cos(\alpha + \gamma) - C_d * p/c * \sin(\alpha + \gamma) \qquad (1)$$

For most airfoils the moment coefficient is given around the quart chord point which is lying on the zero line at a distance of 1 / 4 * c from the nose. As the quart chord point is lying on the zero line it means that $\gamma = 0°$. For the quart chord point it is valid that p = 0.25 c. Substitution of these values in formula 1 and taking $C_{mh} = C_{m0.25}$ gives:

$$C_{m0.25} = C_m - 0.25\,(C_l * \cos\alpha + C_d * \sin\alpha) \qquad (2)$$

$C_{m0.25}$ has been calculated for all values of α out of table 1 and the result is mentioned in table 1. The $C_{m0.25}$-α curve is also given in figure 2. In figure 2 it can be seen that the $C_{m0.25}$-α curve is still strongly rising for small angles of α. So a larger value p = 0.333 c is chosen and the moment coefficient around this point is called $C_{m0.333}$. Substitution of these values in formula 1 gives:

$$C_{m0.333} = C_m - 0.333\,(C_l * \cos\alpha + C_d * \sin\alpha) \qquad (3)$$

$C_{m0.333}$ has been calculated for all values of α out of table 1 and the result is mentioned in table 1. The $C_{m0.333}$-α curve is also given in figure 2. The same procedure is followed for p = 0.4 c and the moment coefficient around this point is called $C_{m0.4}$. Substitution of these values in formula 1 gives:

$$C_{m0.4} = C_m - 0.4\,(C_l * \cos\alpha + C_d * \sin\alpha) \qquad (4)$$

$C_{m0.4}$ has been calculated for all values of α out of table 1 and the result is mentioned in table 1. The $C_{m0.4}$-α curve is also given in figure 2. The same procedure is followed for p = 0.5 c and the moment coefficient around this point is called $C_{m0.5}$. Substitution of these values in formula 1 gives:

$$C_{m0.5} = C_m - 0.5\,(C_l * \cos\alpha + C_d * \sin\alpha) \qquad (5)$$

Comparing the curves of figure 2 learns that, for angles α larger than 0°, The $C_{m0.25}$-α curve is most constant and has an average C_m value of about 0.24 which is rather high. The $C_{m0.333}$-α curve has a dip at $\alpha = 15°$ and an average C_m value of about 0.14. The $C_{m0.4}$-α curve and the $C_{m0.5}$-α curve both have a dip at $\alpha = 15°$. The $C_{m0.5}$-α curve is negative for $\alpha > 4°$.

The angle α depends on the blade design. For the blade with a constant chord, α is small at the blade tip and large at the blade root if the rotor runs at the design tip speed ratio. α increases suddenly at wind gusts. α is very large during starting of the rotor.

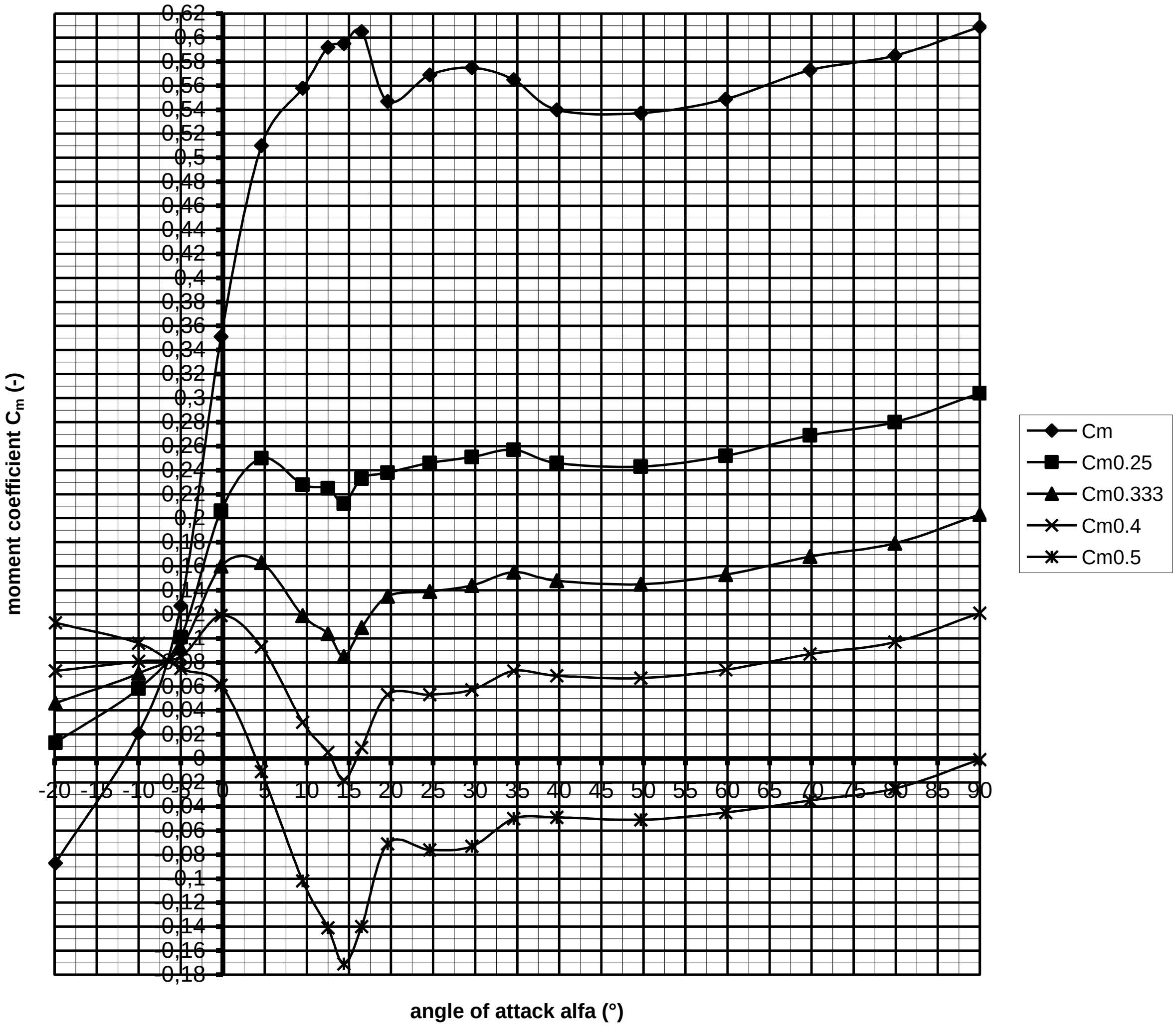

fig. 2 C_m-α curves for different positions of the axis of rotation for a 10 % cambered plate for Re = $4 * 10^5$

3 Determination of the C_m-α curves for 5 % camber and different turning points

The measurements of Flachsbart for 5 % camber are also given on page 3-5 of report R443D (ref. 3). The table out of R443D has been copied as table 2. The measurements are performed for a Reynolds value Re = 4 * 10^5. The variation of C_m as a function of α is given in figure 3.

The $C_{m0.25}$, $C_{m0.333}$, $C_{m0.4}$ and $C_{m0.5}$ values are determined in the same way as done in chapter 2 for 10 % camber. The result of the calculations is given in table 2. The C_m-α curves for different turning points are given in figure 3.

α (°)	C_l (-)	C_d (-)	C_m (-)	$C_{m0.25}$ (-)	$C_{m0.333}$(-)	$C_{m0.4}$ (-)	$C_{m0.5}$ (-)
-19.8	-0.580	0.262	-0.210	-0.133	-0.001	0.044	0.107
-9.8	-0.420	0.116	-0.118	-0.010	0.026	0.055	0.099
-4.9	-0.173	0.0600	0.009	0.053	0.068	0.080	0.098
-0.1	0.302	0.0248	0.186	0.111	0.086	0.065	0.035
4.8	0.660	0.037	0.282	0.117	0.062	0.018	-0.048
9.6	0.984	0.110	0.367	0.120	0.038	-0.028	-0.127
12.1	1.083	0.175	0.425	0.151	0.060	-0.013	-0.123
14.6	1.070	0.238	0.460	0.186	0.095	0.022	-0.088
22.1	1.050	0.355	0.463	0.186	0.095	0.020	-0.090
29.6	0.986	0.513	0.481	0.203	0.111	0.037	-0.074
39.7	0.867	0.665	0.465	0.192	0.101	0.028	-0.081
49.7	0.750	0.804	0.480	0.205	0.114	0.041	-0.069
59.8	0.614	0.964	0.515	0.229	0.135	0.058	-0.066
69.8	0.473	1.080	0.542	0.248	0.150	0.071	-0.046
79.9	0.290	1.160	0.573	0.275	0.176	0.096	-0.023
90.0	0	1.210	0.605	0.303	0.202	0.121	0.000

table 1 C_l, C_d, C_m, $C_{m0.25}$, $C_{m0.333}$, $C_{m0.4}$ and $C_{m0.5}$ as a function of α for 5 % camber.
C_m is taken around the nose. The left hand direction is taken positive. Re = 4 * 10^5

Comparing the curves of figure 3, for angles α larger than 0°, learns that $C_{m0.25}$ is about constant for $0° < \alpha < 10°$ for which $C_{m0.25}$ is about 0.12. $C_{m0.25}$ rises from 0.12 to 0.19 for $10° < \alpha < 15°$. $C_{m0.25}$ is again about constant and 0.19 for $15° < \alpha < 25°$. The $C_{m0.333}$-α, the $C_{m0.4}$-α and the $C_{m0.5}$-α curves all have a dip at $\alpha = 10°$. The $C_{m0.5}$-α curve is negative for $\alpha > 2°$.

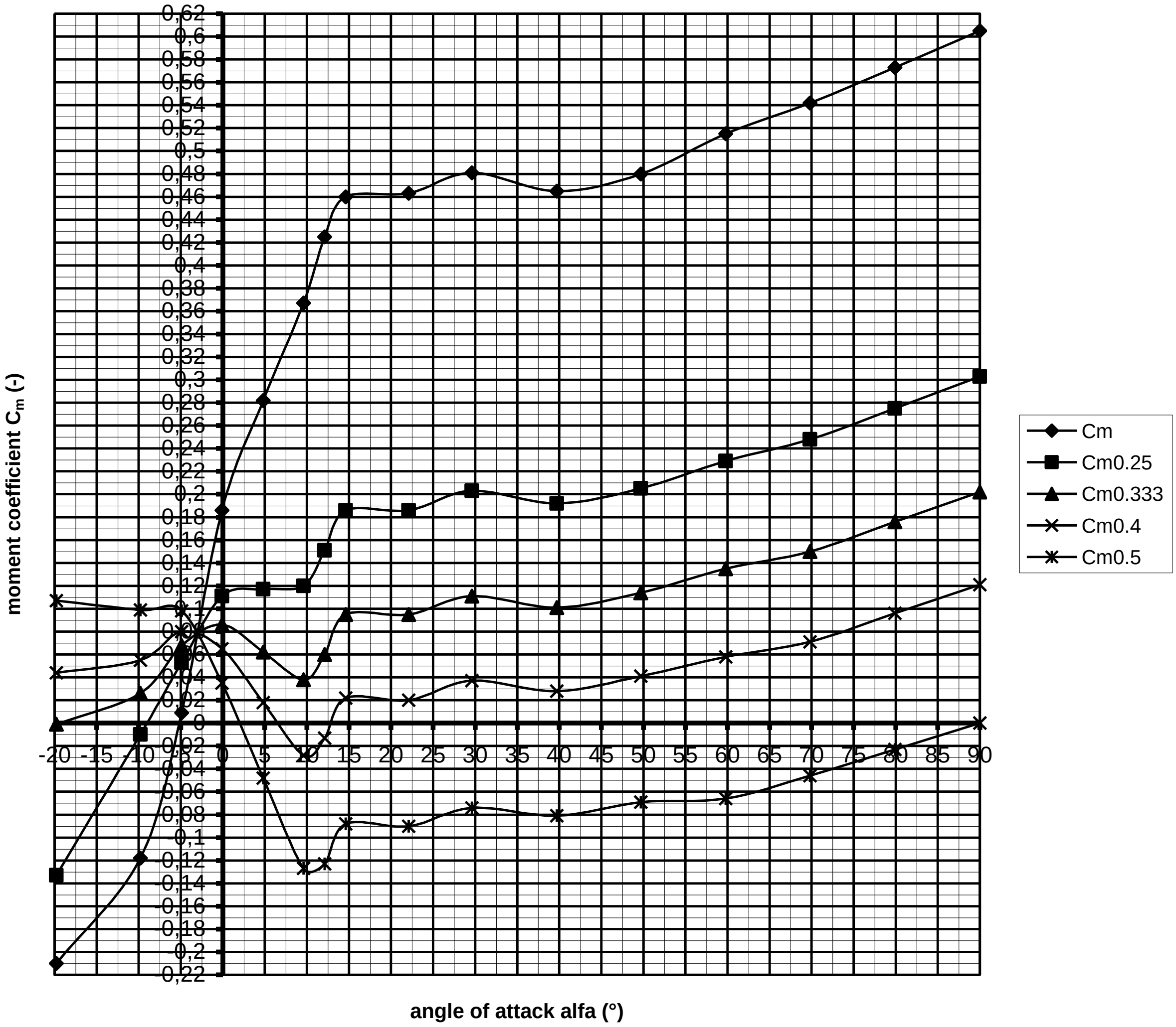

fig. 3 C_m-α curves for different positions of the axis of rotation for a 5 % cambered plate for Re = 4 * 10^5

4 Estimation of the $C_{m0.25}$-α, the $C_{m0.333}$-α and the $C_{m0.5}$-α curves for 7.14 % camber

The $C_{m0.25}$-α and $C_{m0.33}$-α curves seem useful for a positive pitch control system for which the blade turns to larger blade angles. The $C_{m0.5}$-α seems useful for a negative pitch control system for which the blade turns to smaller blade angles resulting in stalling. It is assumed that the curve for 7.14 % camber is the average of the curves of 10 % and 5 % camber.

The $C_{m0.25}$-α curve of figure 2 for 10 % camber and the $C_{m0.25}$-α curve of figure 3 for 5 % camber are put together in figure 4. The estimated $C_{m0.25}$-α curve for 7.14 % camber is also given in figure 4.

The $C_{m0.333}$-α curve of figure 2 for 10 % camber and the $C_{m0.333}$-α curve of figure 3 for 5 % camber are put together in figure 5. The estimated $C_{m0.333}$-α curve for 7.14 % camber is also given in figure 5.

The $C_{m0.5}$-α curve of figure 2 for 10 % camber and the $C_{m0.5}$-α curve of figure 3 for 5 % camber are put together in figure 6. The estimated $C_{m0.333}$-α curve for 7.14 % camber is also given in figure 6.

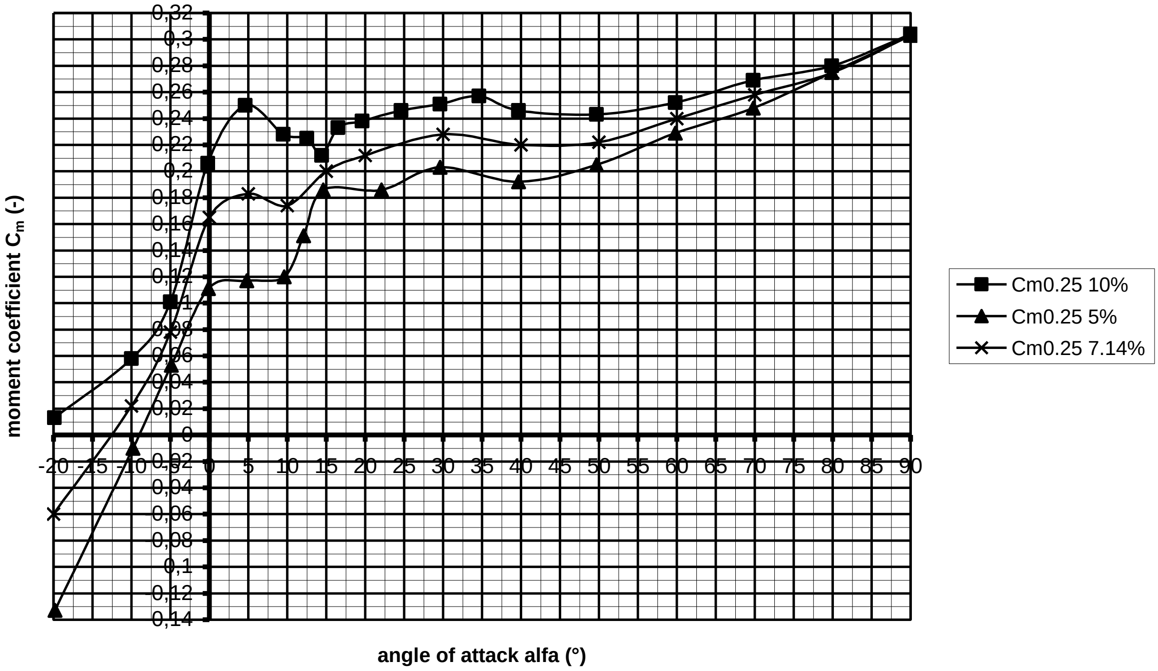

fig. 4 Measured $C_{m0.25}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.25}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

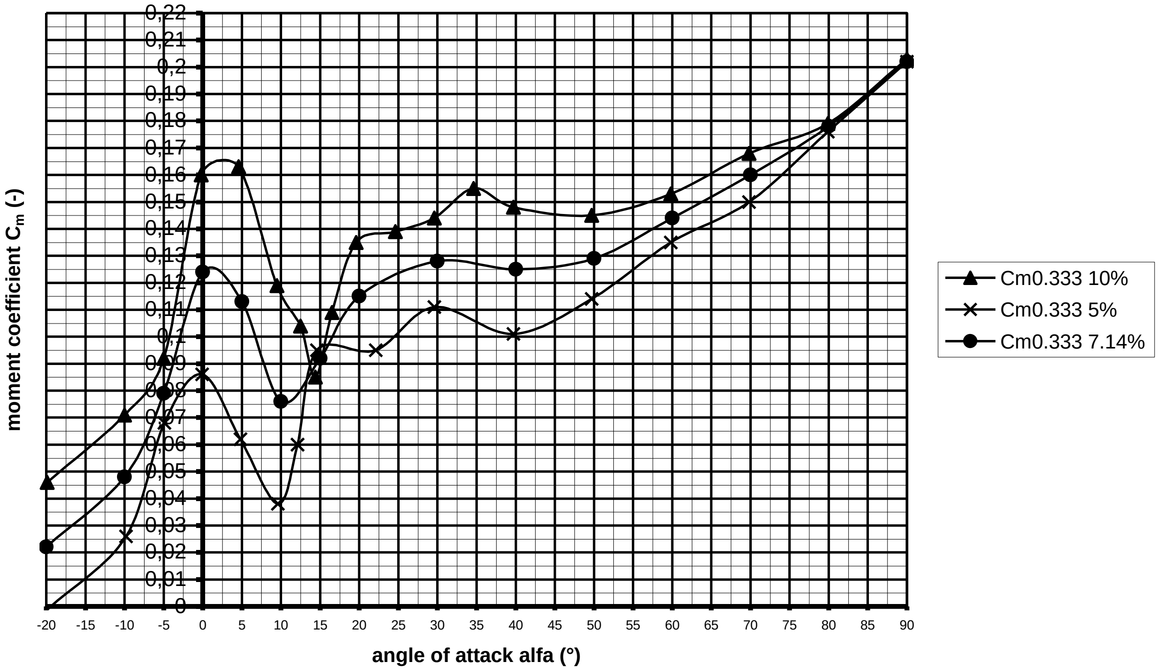

fig. 5 Measured $C_{m0.333}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.333}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

It can be seen that the estimated $C_{m0.25}$-α curve for 7.14 % camber is about constant for $2° < \alpha < 12°$. The $C_{m0.25}$ value is about 0.18 which is rather high. The $C_{m0.333}$-α curve is decreasing from 0.125 to 0.075 for $2° < \alpha < 15°$. So it is difficult to make a stable pitch control system for this blade shaft position if the aerodynamic moment is the only steering moment. But if centrifugal weights are added and if the moment created by the centrifugal moment is increasing at increasing α, a stable working pitch control system might also be possible for this blade shaft position.

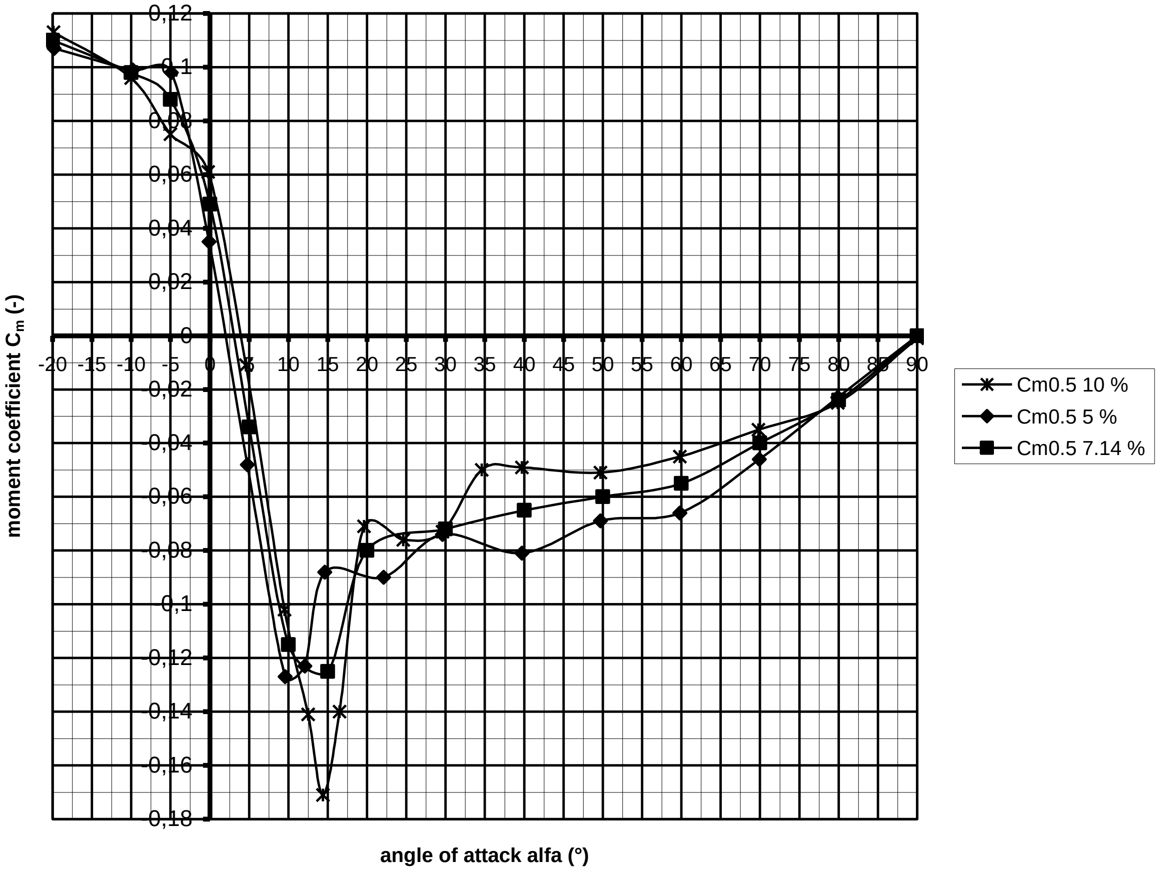

fig. 6 Measured $C_{m0.5}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.5}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

It can be seen that the estimated $C_{m0.5}$-α curve for 7.14 % camber is negative and decreasing strongly for about $3° < \alpha < 12°$. So a negative pitch control system will react such that suddenly the blade angel is decreased a lot.

5 References

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