Ideas about a double piston pump with a constant upwards piston speed driven by an asynchronous 4-pole, 3-phase motor or a permanent magnet DC motor frame size 71 and a reducing 2-step gear box frame size 32 of manufacture Rossi

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Contains page

1 Introduction

Water is a basic need of mankind but many places on earth suffer from water shortage. Even if the average yearly rainfall is enough, there will be dry periods when water has to be pumped. Many places on earth have no access to the electricity grid and then one has to use hand pumps or pumps driven by a motor aggregate or by renewable energy like sun or wind. As fuel prices are rising, the use of sun or wind energy becomes more and more attractive in developing countries. But because of the required investment costs, the price of energy generated by sun or wind is much higher than the price of energy supplied by the grid. So if sun or wind energy is used to power an electric pump, one has to use a pump with a high overall efficiency. This means that the ratio in between the obtained hydraulic energy in the water and the required electrical energy has to be as high as possible. However, a high efficiency should be realised at low investment costs for the pump and the pump motor. Local manufacture of the pump may result in the lowest pump price.

The overall efficiency of an electric pump depends on the efficiency of the pump motor, the efficiency of the pump, the efficiency of the transmission in between motor and pump, the losses in the cables from the energy source to the pump motor and the hydraulic losses in the piping in between the well and the pump and in between the pump and the reservoir in which the water is pumped.

The highest motor efficiency is realised for a permanent magnet DC motor. An efficiency of 0.8 is possible for a small motor. However, these motors are rather expensive and may be difficult to obtain in developing countries. The nominal rotational speed is also higher than for a 4-pole asynchronous motor so a larger reducing gearing will be needed. The cable losses will be rather high as the nominal voltage is only 24 V. An alternative is to use an asynchronous 4-pole, 3-phase motor which runs at a nominal speed of about 1400 rpm. The motor efficiency η_m of such a motor with frame size 71 is about 0.65. The cable losses are low because a high 230/400 V, 3-phase voltage is chosen. It is estimated that the cable efficiency η_c for normal currents is 0.98. The highest pump efficiency η_p is possible for a piston pump. An efficiency of maximum 0.9 has been measured for single acting piston pumps. The highest transmission efficiency is realised if there is no reducing transmission and if ball bearings are used at the crank shaft. However, if a direct drive transmission is used, the crank shaft of the pump will run at a very high rotational speed which causes severe acceleration problems. Therefore a reducing gearing is chosen using a 2 step gearbox of manufacture Rossi. The pistons will be driven by a cam shaft and two cam disks and the total efficiency of the gear box and the cam transmission is expected to be 0.92. So the maximum total efficiency η_{tot} of the pump and an asynchronous motor including cables $\eta_{tot} = \eta_c * \eta_m * \eta_t * \eta_p = 0.98 * 0.65 * 0.92 * 0.9 = 0.53$. The hydraulic losses in the piping are minimised if the flow through the pipe is not fluctuating and if smooth pipes with sufficient inner diameter are used. The resistance of the piping results in an extra dynamic head. It is assumed that the dynamic head is 10 % of the static head.

The main disadvantage of a single acting piston pump which is used in traditional water pumping windmills is that the torque and the flow vary sinusoidal during the upwards stroke and that they are zero during the downwards stroke. The peak torque is a factor π times the average torque. The main formulas for a single acting piston pump driven by a windmill are given in public report KD 294 (ref. 1). If an elastic element with a volume variation of about four times the stroke volume of the pump is added just after the pump, the flow coming out of this elastic element will be almost constant. This principle is used in the Solaflux solar pump which was tested in combination with the VIRYA-3B3 windmill already in 2003. This combination is described in chapter 5 of public report KD 490 (ref. 2).

In this report KD 544 it is investigated if it is possible to design an efficient piston pump with a constant flow driven by an electric motor. The pump is meant for an open well and will be mounted some meters above the water level. It is assumed that the total static head is 40 m.

2 Description of the motor and the transmission (see figure 5 page 15)

A 370 W asynchronous 4-pole, 3-phase motor is chosen. This motor has frame size 71 and runs at a nominal rotational speed of about n = 1400 rpm. This motor can be coupled to a 230/400 V, 3-phase grid. If only a 1-phase grid is available, one can use a 1-phase motor of the same power and frame size. A 1-phase motor in reality is a 2-phase motor and the second phase is created by a big capacitor. If a 3-phase or 1-phase grid isn't available and if the energy comes from a battery charging windmill or from a solar panel one has to use a 3-phase or 1-phase inverter. Another option is to use a permanent magnet motor if it has the same frame size and power and about the same rotational speed. Permanent magnet DC motors with the same frame size as asynchronous motors are supplied by the Dutch company Creusen (www.Creusen.nl). Motors of this frame size are supplied for 24 V in different powers and nominal rotational speeds. Type 71L-2GP with P = 350 W and n = 1500 rpm seems the best option. If this motor is used in combination with a solar panel or a windmill, the real voltage will be higher than 24 V. It is assumed that a voltage controller is used which limits the voltage up to 28 V. This means that the maximum rotational speed is 1500 * 28 / 24 = 1750 rpm. This is a factor 1.25 higher than the rotational speed of a 4-pole synchronous motor which means that the reducing gear ratio must be a factor 1.25 higher to get the same rotational speed of the cam shaft.

It might be possible to use a windmill which is directly supplying 3-phase current. As the frequency of this current must be rather high, one has to use a multi pole generator if the generator is direct drive. A 34-pole generator of this kind is described in report KD 560 (ref. 3).

The advantage of an asynchronous motor is that the outside dimensions are standardised and that therefore any brand can be chosen. Motors of frame size 71 are normally supplied with a foot B3 but can also be supplied by a flange. A gear box is chosen which has a flange at both sides which means that the motor must have a flange too. The smallest gear box of Rossi has been chosen. This gear box has size 32. I have an old catalogue type E94 of Rossi in which coaxial gear motors are given. An on line catalogue can be found on the website of Rossi: www.rossi.com by following the path: products – coaxial gear motors Cat. E – pdf Coaxial gear reducers and gear motors Catalogue E04 (1.68 Mb). A picture of the gearbox plus motor is given on page 50 of the online catalogue. The rotational speeds, torques, safety factors and gear box and motor specifications for a 0.37 kW motor as a function of the gear ratio i are given on page 30 and 31 of the on line catalogue. A combination of a 2-steps gear box size 32 and a 4-pole motor size 71 is available for eight different gear ratios i being 5.06, 6.33, 7.29, 8.12, 9.57, 10.8, 13.5 and 16.5. The safety factor fs decreases at increasing gear ratio i and is smaller than 1 for the two largest gear ratios. This means that it isn't allowed to load the shaft higher than the nominal torque for these two largest gear ratios.

In the first instance it is chosen to use the gear ratio i = 10.8 for the 4-pole asynchronous motor and i = 13.5 for the permanent magnet DC motor. These ratios differ a factor 13.5 : 10.8 = 1.25. The nominal torque isn't reached for the DC motor because the nominal power of this motor is supplied at a much higher rotational speed than 1400 rpm.

The slow shaft of the gear box has a diameter of 16 mm and a length of 30 mm. It is provided with a 5 mm key which juts out 2 mm out of the shaft. The front bearing of the gear box is a double row ball bearing of probably size 17 * 40 * 16 mm. The dynamic load factor of this bearing C = 14800 N. The static load factor C_0 = 9000 N.

The camshaft has a diameter of 25 mm. The front side runs in a sealed ball bearing size 20 * 52 * 15 mm. For this bearing it is valid that C = 15900 N and C_0 = 7800 N. The back side is shifted over the gear box shaft and makes use of the gear box bearing. The cams are welded to the camshaft.

3 Description of the pump (see figure 5 page 15)

A normal single acting piston pump lifts only water during the upwards stroke. If a crank mechanism is used to transform the rotation of the crank shaft into an oscillation of the piston, this results in a sinusoidal variation of the flow and the torque during the upwards stroke and in no torque and flow during the downwards stroke. The peak flow is a factor π times the average flow. This very strong fluctuation of the flow in the suction and the pressure pipes is only acceptable for very low rotational speeds. A high rotational speed results in strong dynamic forces on the piston, the pump rod and the piping.

Already in 1984, when I worked at the Wind Energy Group of the University of Eindhoven, I got the idea of a double piston pump for which every piston moves upwards with a constant speed during 50 % of the time. Every piston moves downwards in the remaining time with a varying speed. This idea is described for two in line pistons and two concentric pump rods by two students in the TUE-report R675D (ref. 4).

The working of a double piston pump can be explained by the following analogy. Assume a big boat is floating in the centre of a large pond and a rope connects the boat to a man on the edge. If the man wants to pull the boat to the edge, he pulls hand over hand. It will take some time because of the large mass of the boat, but at a certain time the boat will move with a constant speed. Therefore the speed of the pulling hand will be constant during half the time. During the remaining time, this hand has to be brought forwards with an accelerating and decelerating speed.

A big problem for use of this pump by a windmill is that both pistons must be in line and that therefore two concentric pump rods are required. Coupling of these pump rods is difficult and this also counts for the transmission from the cam shaft to the pump rods. Another problem is that, because of the elasticity of long pump rods, the movement of the pistons is not the same as the movement at the top of the pump rod. That’s why this pump has never been built.

These problems can be eliminated for a pump driven by an electric motor because for this use, the pump rods can be very short and it isn’t necessary to mount both pistons in the same cylinder. If the pistons move in two different cylinders which are positioned besides each other, one needs two cams on one camshaft. The shape of cam 1 must be rotated 180° with respect to the shape of cam 2.

The pump will be placed in an open well and will have a low suction head and a high pressure head. The pump rod has a cam roller at the end which runs directly against the cam. So no rocker arm is used which simplifies the construction. The consequence is that a certain side force will act on the pump rod guide and a good material has to be chosen for this guide. It is chosen to use a Powerslide rod guide type EPWS supplied by Eriks Alkmaar. The smallest guide is available for a rod diameter of 25 mm so this pomp rod diameter is chosen.

The cam shaft is placed under the pistons. Each piston has a valve at the upper side. The valve is pushed downwards by a small stainless steel spring. The upwards force on the piston is supplied by the cam. The downwards force on the piston is supplied by a spring which is positioned around the pump rod. The valve is open during the downwards movement and water is flowing through the valve. This flow of water creates a certain pressure drop over the valve and so a certain upwards force. A downwards force is created by the weight of the piston and the pump rod. An upwards force is created and by the inertia during downwards acceleration of the piston. The piston spring must be that strong that the supplied downwards force is larger than the total upwards force acting on the piston.

Each cam has two regions. For one region in between 180° and 360° or in between π rad and 2π rad, the cam roller has to move upwards from its starting position at $\varphi = \pi$ with a constant speed. For the second region in between 0° and 180° or in between 0 rad and π rad, the cam roller has to be brought back to its starting position.

The shape of the curve can be seen as a displacement of the hart of the roller, unrolled on a circle. The shape of the displacement curve as a function of the rotational angle φ is given in figure 1 for S = 1.

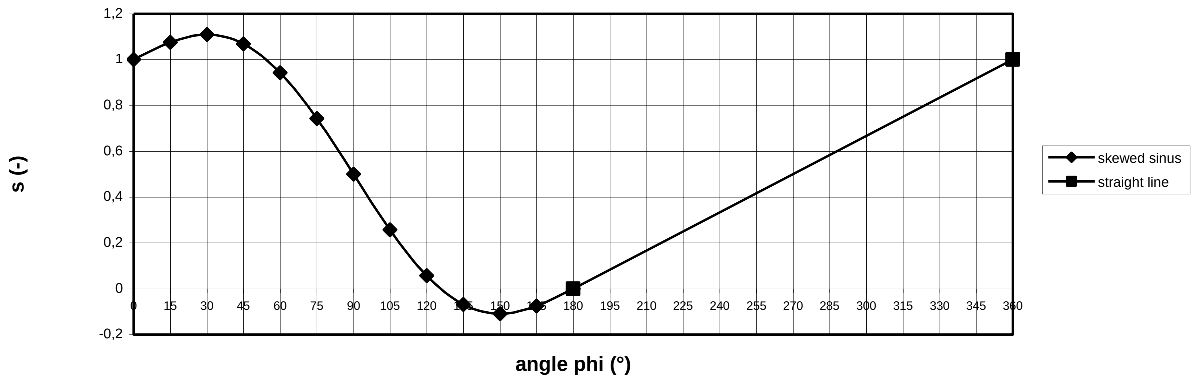

fig 1 Displacement curve of the piston as a function of φ for S = 1

The curve is a straight line for $\pi < \varphi < 2\pi$. It is chosen to take a skewed sinus for $0 < \varphi < \pi$. The mathematical functions for a straight line and for a skewed sinus are given in chapter 2 of report R 675 D. The displacement is called s. The displacement at the end of the straight line is called S. The mathematical function for the straight line is given by:

$$s = f(\varphi) = S/\pi * \varphi - S \qquad (\pi \leq \varphi \leq 2\pi) \qquad (1)$$

The mathematical function for the skewed sinus is given by:

$$s = f(\varphi) = - S/\pi * \varphi + S/\pi * \sin 2\varphi + S \qquad (0 \leq \varphi \leq \pi) \qquad (2)$$

The first and the second derivatives to φ are:

$$f'(\varphi) = -S/\pi + 2\, S/\pi * \cos 2\varphi \qquad (3)$$

$$f''(\varphi) = -4\, S/\pi * \sin 2\varphi \qquad (4)$$

$f'(\varphi)$ is proportional to the speed v. $f''(\varphi)$ is proportional to the acceleration a. However, to find the real speed and the real acceleration, the angular velocity ω has to be taken into account and therefore it is necessary to take the derivatives to t in stead of to φ. For the determination of the shape of the cam one has to take the derivative to φ. The relation in between the angle φ (in rad), the angular velocity ω (in rad/s) and the time t (s) is given by:

$$\varphi = \omega * t \qquad \text{(rad)} \qquad (5)$$

(2) + (5) gives:

$$s = f(t) = - S/\pi * \omega * t + S/\pi * \sin 2\omega * t + S \qquad \text{(m)} \qquad (0 \leq \varphi \leq \pi) \qquad (6)$$

$$v = f'(t) = -S/\pi * \omega + 2\, S/\pi * \omega * \cos 2\omega * t \qquad \text{(m/s)} \qquad (7)$$

$$a = f''(t) = -4\, S/\pi * \omega^2 * \sin 2\omega * t \qquad (\text{m/s}^2) \qquad (8)$$

The downwards speed of the piston is maximal for the angle φ for which the acceleration a is zero. Using formula 4, it is found that this is the case for φ = ½ π. Substitution of φ = ½ π in formula 3 gives that f'(φ) = - 3 * S/π. The maximum upwards speed is realised for φ = 0 and for φ = π. Substitution of φ = 0 in formula 4 gives that f'(φ) = S/π. So the maximum speed half way the downwards stroke is three times the upwards speed. The variation of the speed during one revolution is given in figure 2 for S/π = 1.

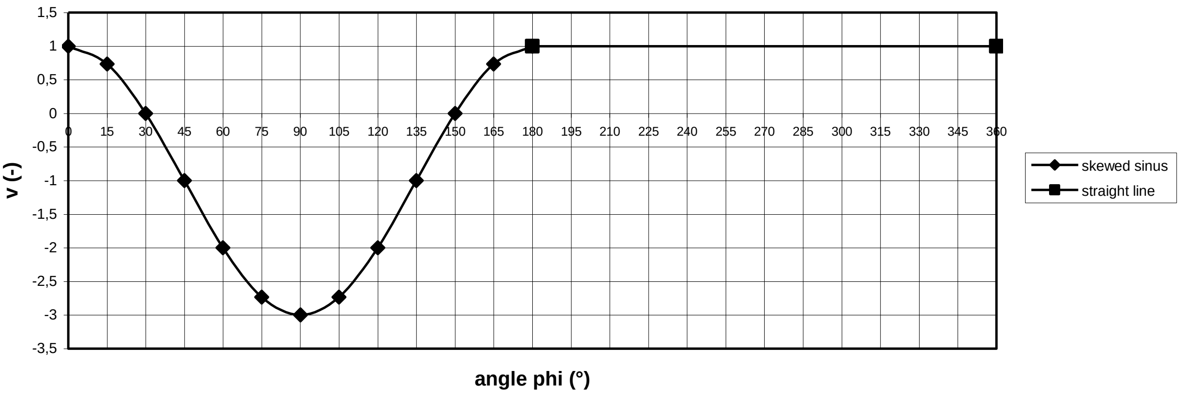

fig 2 Speed of the piston as a function of φ for S/π = 1

The relation in between the angular velocity ω (rad/s) and the rotational speed of the cam shaft n_c (rpm) is given by:

$$\omega = \pi * n_c / 30 \qquad \text{(rad/s)} \qquad (9)$$

For the determination of the shape of the cam it is necessary to choose a certain value for S. The flow q depends on S, on the piston diameter D and on the rotational speed of the cam shaft n_c. These objects have to be chosen such that the required power can be supplied by the electric motor for the given head.

4 Calculation of the stroke volume, the flow, the power and the pump rod force

The effective stroke of one piston is S. So the stroke volume ∇ of the two pistons, which both have a piston diameter D and a pump rod diameter d, is given by:

$$\nabla = \pi/2 * S * (D^2 – d^2) \qquad (m^3) \qquad (10)$$

The flow of both pistons q is given by:

$$q = \nabla * n_c / 60 \qquad (m^3/s) \qquad (11)$$

(10) + (11) gives:

$$q = \pi * S * n_c (D^2 – d^2) / 120 \qquad (m^3/s) \qquad (12)$$

The diameter of a piston D is chosen 80 mm = 0.08 m. The diameter of a pump rod d is chosen 25 mm = 0.025 m. The stroke S is chosen 30 mm = 0.03 m. The rotational speed of the gear box shaft is 130 rpm for i = 10.8 and a rotational speed of the motor of 1400 rpm.

Substitution of D = 0.08 m, d = 0.025 m, S = 0.03 m and n_c = 130 rpm in formula 13 gives that q = 0.000590 m^3/s. This is about 2.12 m^3/hour or about 51 m^3/day. This is a substantial amount of water which indicates that the pump can be used for drinking water and even for small irrigation. The hydraulic power P_{hyd} is given by:

$$P_{hyd} = \rho_w * g * H * q \qquad (W) \qquad (13)$$

In this formula H is the total head. H is the sum of the static head H_{stat} and the dynamic head H_{dyn}. The static head is the height in between the water level in the well and the height of the outlet opening of the pressure pipe. The dynamic head is caused by the pipe losses in the suction and the pressure pipe. The dynamic head depends on the flow, the length of the pipes and very much on the inside pipe diameter. Calculation of the dynamic head is out of the scope of this report. It is assumed that the dynamic head is 10 % of the static head. Assume H_{stat} = 40 m and H_{dyn} = 4 m so H = 44 m. Substitution of ρ_w = 1000 kg/m^3, g = 9.81 m/s^2, H = 44 m and q = 0.000590 m^3/s in formula 13 gives P_{hyd} = 255 W. The required mechanical power P at the pump shaft is given by:

$$P = P_{hyd} / (\eta_t * \eta_p) \qquad (W) \qquad (14)$$

Substitution of P_{hyd} = 255 W, η_t = 0.92 and η_p = 0.9 in formula 14 gives P = 308 W. So a 370 W motor seems a good choice and has even some reserve if the efficiencies are lower than the values as used for the calculation. The required electrical power P_{el} = 308 / (0.98 * 0.65) = 483 W.
Another aspect is the bearing load and the bending stress in the cam shaft due to the force in the pump rod. It is assumed that total height H = 44 m. The pressure drop Δp over the piston during the upwards stroke is given by:

$$\Delta p = \rho_w * g * H \qquad (N/m^2) \qquad (15)$$

Substitution of ρ_w = 1000 kg/m^3, g = 9.81 m/s^2 and H = 44 m in formula 15 gives that Δp = 431640 N/m^2. The piston area A for a piston with a piston diameter D and a pump rod diameter d is given by:

$$A = \pi/4 * (D^2 - d^2) \qquad (m^2) \qquad (16)$$

Substitution of D = 0.08 m and d = 0.025 m in formula 16 gives that A = 0.004536 m^2. The force F acting on a piston is given by:

$$F = \Delta p * A \qquad (N) \qquad (17)$$

Substitution of Δp = 431640 N/m^2 and A = 0.004536 m^2 in formula 17 gives F = 1958 N. This force is working on each piston during the upwards stroke. This force is not working at the same time on both pistons. The cam shaft bearings are about symmetrical positioned around the cams. It is assumed that 80 % of the pump rod force is taken by the camshaft bearing closest to the cam which is active. So 20 % is taken by the other cam shaft bearing. So the maximum reaction force on one cam shaft bearing F_r = 0.8 * 1958 = 1566 N. The front camshaft bearing has size 20 * 52 * 15 mm with a dynamic load factor C = 15900 N and a static load factor C_0 = 7800 N. The dynamic load factor is used to calculate the life time. The back cam shaft bearing is the double row bearing of the gear box size 17 * 40 * 16 mm.

The life time in hours L_h is calculated with the formula:

$$L_h = 10^6 * (C/P)^3 / (60 * n) \qquad (h) \qquad (18)$$

For the double row gear box bearing it is valid that C = 14800 N and C_0 = 9000 N. So the gear box bearing is critical. Substitution of C = 14800 N, P = F_r = 1566 N and n = 130 rpm in formula 18 gives L_h = 108222 h = 12.35 year. This is certainly enough because the pump won't be used continuously.

Each cam roller is loaded by the normal force F_N = 1989 N (see chapter 8). In the first instance it is chosen to use an INA cam roller type NUTR17. This cam roller has an inner diameter of 17 mm, an outer diameter of 40 mm and a width of 21 mm. It has C = 18400 N and C_0 = 22600 N. The formula for calculation of the life time of a roller bearing differs from formula 18 on the point that the exponent is 3.3333 in stead of 3. The rotational speed of the cam roller for the cam as chosen in chapter 8 is about a factor 2.2 higher than the rotational speed of the cam shaft. Assume n = 286 rpm. Substitution of C = 18400 N, P = F = 1989 N, n = 286 rpm and 3 = 3.3333 in formula 18 gives L_h = 96841 h = 11.1 year. This is certainly enough because the pump won't be used continuously. The cam roller has only a dust seal so it must be prevented that some water which may leak along the pump rod comes into contact with the roller. This can be realised by covering the cam roller by a bush. This bush can also be used to connect the piston spring.

The static force acting on a cam gives a bending moment M in the cam shaft. This bending moment is maximal at the hart of the cam. The distance l in between the hart of the front cam shaft bearing and the hart of the front cam is about 43 mm. The bending moment is given by:

$$M = F_{net} * l \qquad \text{(Nm)} \qquad (19)$$

The bending stress σ in a round shaft with a diameter d is given by:

$$\sigma = 32\ M / (\pi * d^3) \qquad (20)$$

(19) + (20) gives:

$$\sigma = 32\ F * l / (\pi * d^3) \qquad (21)$$

Substitution of F = F_r = 1566 N, l = 43 mm and d = 25 mm in formula 21 gives σ = 44 N/mm^2. The shaft is made of steel for which the allowable bending stress for a fatigue load will be about 160 N/mm^2. So a bending stress of 44 N/mm^2 is certainly acceptable.

5 Calculation of the piston acceleration

The piston is accelerated when it is following the skewed sinus, so for $0 \leq \varphi \leq \pi$. The acceleration of an skewed sinus is maximal for ¼ and for ¾ of the curve, so for φ = ¼ π and for φ = ¾ π. The acceleration for φ = ¼ π is critical because this acceleration has to be supplied by the weight of the piston and by the piston spring. The acceleration a is given by formula 8. Substitution of φ = ¼ π in formula 5 gives that ω * t = ¼ π and this gives that sin2ω * t = 1. So the maximum acceleration a_{max} is given by:

$$a_{max} = -4\ S/\pi * \omega^2 \qquad (\text{m/s}^2) \qquad (22)$$

Substitution of n_c = 130 rpm in formula 9 gives that ω = 13.6 rad/s. Substitution of S = 0.03 m and ω = 13.6 rad/s in formula 22 gives that a_{max} = - 7.06 m/s^2. The absolute value of a_{max} is smaller than the acceleration of gravity g for which it is valid that g = 9.81 m/s^2. This means that the cam roller will follow the cam if no piston spring is used. This is the advantage of positioning the pistons above the cam shaft.

In reality there will be a pressure drop over the piston due to the valve resistance and this will require a spring. It is difficult to determine this pressure drop and the geometry of the spring therefore has to be determined by try and error.

6 Determination of the water speed and the acceleration of the valves

If the pump is running at a rotational speed of 130 rpm, so at 2.17 revolutions per second, the valves have to open and close with a frequency of 2.17 Hz. This is rather fast and a valve will only be able to follow this frequency if it is very light and if the spring force which closes the valve is strong enough. The valves therefore have a stroke of only 7 mm and are made from a light plastic like POM which has a density ρ_v of about 1.4 * 10^3 kg/m^3 or 1.4 * 10^{-6} kg/mm^3. A valve has a thickness of 5 mm, an outside diameter of 68 mm and an inside diameter of 18 mm. So the valve volume $\nabla = \pi/4\ (68^2 - 18^2) * 5 = 16886$ mm^3. So the valve mass m_v is given by $m_v = 16886 * 1.4 * 10^{-6} = 23.6 * 10^{-3}$ kg, so 23.6 gram which is very light.

Six holes with a diameter of 17 mm are drilled in the piston at a pitch circle of 43 mm. The six holes in the piston sheet have a total area $A_h = 6 * \pi/4 * 17^2 = 1362$ mm^2. The water has to pass these holes but it also has to pass the circular gap in between the valve and the piston. This circular gap has a diameter of 60 mm at the outside of the holes. So the gap area $A_g = \pi * 60 * 7 = 1319$ mm^2 = 0.001319 m^2. The water also has to pass through the ring shaped area A_r in between the valve and the cylinder. This area $A_r = \pi/4\ (80^2 - 68^2) = 1395$ mm^2. So $A_g < A_h$ and $A_g < A_r$ and therefore A_g is critical and the highest water speed will be realised in the gap.

The flow through the valves is maximal half way the downwards stroke, so for $\varphi = ½\ \pi$ (see figure 2). Substitution of $\varphi = ½\ \pi$ in formula 5 gives $\omega * t = ½\ \pi$. So $\cos 2\omega * t = -1$. The piston speed v is given by formula 7. Substitution of $\cos 2\omega * t = -1$ in formula 7 gives:

$$v = -3 * S/\pi * \omega \qquad \text{(m/s)} \qquad (23)$$

Formula 23 gives the maximum downwards speed of the piston half way the downwards stroke. However, the water is moving upwards with a speed $S/\pi * \omega$ (see figure 2). So the relative speed in between the water and the piston v_{rel} is given by:

$$v_{rel} = 4 * S/\pi * \omega \qquad \text{(m/s)} \qquad (24)$$

Substitution of S = 0.03 m and ω = 13.6 rad/s in formula 24 gives that v_{rel} = 0.52 m/s.

As water is incompressible, the product of the relative piston velocity v_{rel} times the piston area A must be equal to the product of the water speed in the gap v_g times the gap area A_g. This results in:

$$v_g = v_{rel} * A / A_g \qquad \text{(m/s)} \qquad (25)$$

Substitution of v_{rel} = 0.52 m/s, A = 0.004825 m^2 and A_g = 0.001319 m^2 in formula 25 gives that v_g = 1.9 m/s. This is a rather high speed and at this moment I don't know if this speed is allowed and what pressure drop it will cause over the piston. How the valve moves as the result of this varying water speed and the influence of the valve mass and the spring, is difficult to determine.

To get a rough impression, it is first assumed that the valve moves sinusoidal. This means that the water speed in the gap is constant if the water speed varies sinusoidal. The gap of a valve is becoming wider during the first halve downwards stroke, so during a quart revolution. The gap is becoming narrower during the second halve downwards stroke.

The sinusoidal movement of the valve is only valid for the time that water is flowing through the valve, so during a halve revolution of the crank shaft. The valve is closed during the other halve revolution. The valve stroke is 7 mm = 0.007 m. The maximum acceleration for a sinusoidal movement is given by:

$a_{max} = \omega^2 * r$ (m/s^2) (26)

The radius r is identical to the valve stroke because the valve follows a half sinus. Substitution of ω = 13.6 rad/s and r = 0.007 m in formula 26 gives a_{max} = 1.29 m/s^2.

The real movement of the valve will be not sinusoidal. It can be expected that the valve opens in a short time, then rest against the stop for a certain time and then again closes in a short time. This will result in much higher acceleration forces than for a sinusoidal movement. Assume the real maximum acceleration a is a factor 5 higher that the calculated force for a sinusoidal movement. So a = 6.5 m/s^2. This is lower than the acceleration of gravity g = 9.81 m/s^2 so the valve will close by its own weight and no spring would be required to close the valve if the force to accelerate the valve would be the only force. However, the water flowing in between the valve and the top of the piston will cause a certain pressure difference over the valve and to overcome this pressure difference a spring might be needed.

The water speed in between the valve and the piston will decrease strongly as the down going piston is reaching the bottom dead centre. For the up going part of the stroke in between $5/6\ \pi < \varphi < \pi$ the valve is still open but the water speed decreases very smoothly to zero at $\varphi = \pi$ when the piston speed becomes equal to the speed of the other piston. So the valve will close very smoothly at $\varphi = \pi$. Practical tests with a prototype have to prove if a valve spring is needed or not.

The water enters and leaves the pump through 1” hose pillars. A 1” hose pillar has an inner diameter of 25 mm. It is expected that the used hoses for the suction and the pressure piping also have an inner diameter of 25 mm. So the hose area $A_h = \pi/4 * 25^2$ = 491 mm^2 = 0.000491 m^2. In chapter 4 it was calculated that q = 0.000590 m^3/s. So the water speed in the hose $v_h = q / A_h$ = 0.000590 / 0.000491 = 1.2 m/s. This seems an acceptable speed.

7 Determination of the real maximum stroke

The real maximum stroke of a piston is larger than the effective stroke S = 30 mm. The extreme values of s for the skewed sinus are reached for values of φ for which the second derivative is zero. Using formula 3 it can be proven that this is the case for φ = 30° or 1/6 π rad and for φ = 150° or 5/6 π rad (see figure 1).
Substitution of $\varphi = 1/6\ \pi$ and S = 30 mm in formula 2 gives that s_{max} = 33.2699
Substitution of $\varphi = 5/6\ \pi$ and S = 30 mm in formula 2 gives that s_{min} = - 3.2699
The distance in between the maximum and the minimum value is the real stroke of the piston S_{real}. It is found that S_{real} = 33.2699 + 3.2699 = 36.5398 mm. The cylinder in which the piston moves has to be long enough for this real stroke.

8 Determination of the shape of the cam

The curve given in figure 1 is the translation of the hart of the roller on a flat line. This curve has to be unrolled on a basic circle with a basic radius r_b. This basic circle should be taken not too small otherwise the pressure angle in between the roller and the cam may become too large. A large pressure angle results in a large side force in between the pump rod and the pump rod guide. As an analogy for the cam it is assumed that the roller runs against a hill. It is chosen that the horizontal translation is five times the vertical translation which is S at the end of the hill. So the horizontal translation is 150 mm for S = 30 mm. This situation is given in figure 3.

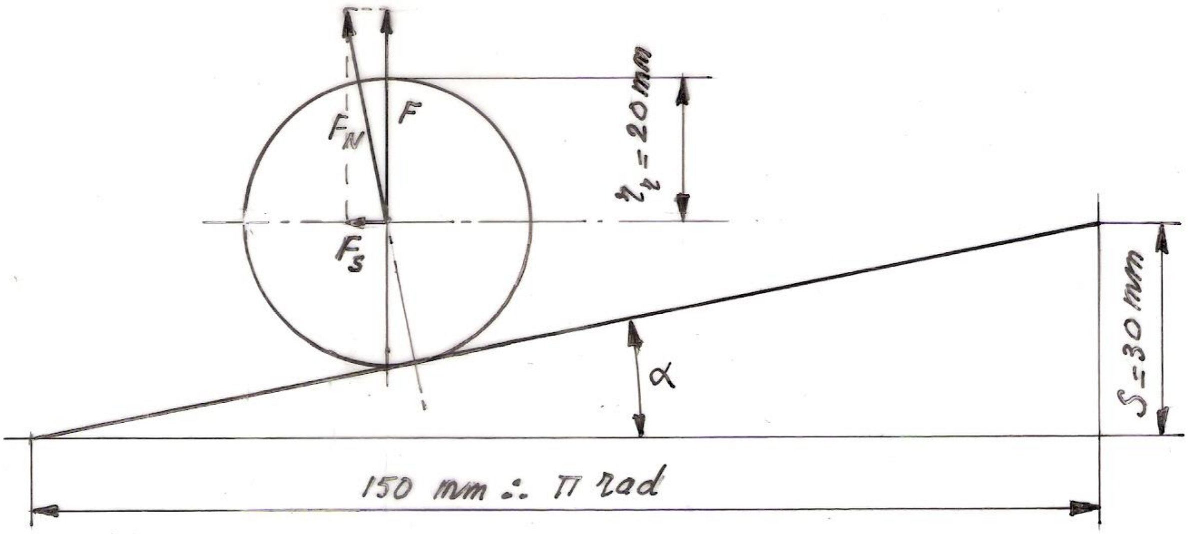

fig. 3 Translation of the roller against a hill

The angle of the hill is α. It is valid that tan α = 30 / 150 = 0.2. This gives α = 11.3°. The pump rod force F has a normal component F_N perpendicular to the hill and a side component F_S perpendicular to the pump rod. It is valid that:

$F_N = F / \cos \alpha$ (N) (27)

$F_S = F * \tan \alpha$ (N) (28)

Substitution of F = 1958 N and α = 11.3° in formula 27 gives F_N = 1989 N.
Substitution of F = 1958 N and α = 11.3° in formula 28 gives F_S = 391 N.

The normal force is the load on the roller and this force is a little higher than F. The side force is the load on the pump rod guide. For the pump rod guide it is chosen to use a Powerslide rod guide type EPWS which is supplied by the Dutch company Eriks from Alkmaar. The smallest guide is available for a rod diameter of 25 mm and this is the reason why this pump rod diameter is chosen. The pump rod guide has an inner diameter d = 25 mm, an outer diameter D = 30 mm and a width B = 5.5 mm The effective guide area A_{eff} is given by:

$A_{eff} = 0.9 * d * B$ (mm^2) (29)

Substitution of d = 25 mm and B = 5.5 mm in formula 29 gives that A_{eff} = 124 mm^2.

The surface pressure σ is given by:

$\sigma = F_S / A_{eff}$ (N/mm^2) (30)

Substitution of F_S = 391 N and A_{eff} = 124 mm^2 in formula 30 gives that σ = 3.2 N/mm^2. The allowable surface pressure is in between 50 and 60 N/mm^2 so the guide seems strong enough. However, the normal use of this guide is for oil cylinders. For use with water the allowable surface pressure may be much lower. If one guide is wearing too fast one can take two guides.

If the curve of figure 3 is unrolled on a basic circle, this basic circle will have a basic radius r_b = 150 / π = 47.75 mm. It is chosen to take r_b = 47 mm. The curve of the hart of the roller is drawn from this point for ever 15°. Every 15° an arc with r_r = 20 is drawn.

The curve which touches these arcs at the inside gives the final curve of the cam. The cam be made out a disk with a diameter of 100 mm if the 25 mm hole for the shaft is drilled 11 mm outside the centre of the disk at an angle $\varphi = 120°$.

The cams are welded to a 25 mm shaft which has a thickness of 28 mm in between the cams. This creates a 1.5 mm collar and therefore it is easier to weld the disks perpendicular to the shaft. It is very important that both disks have the same right hand direction of rotation seen from the front side of the gear box. The cam pattern is rotated exactly 180° in between both disks. A sketch of the cam is given in figure 4.

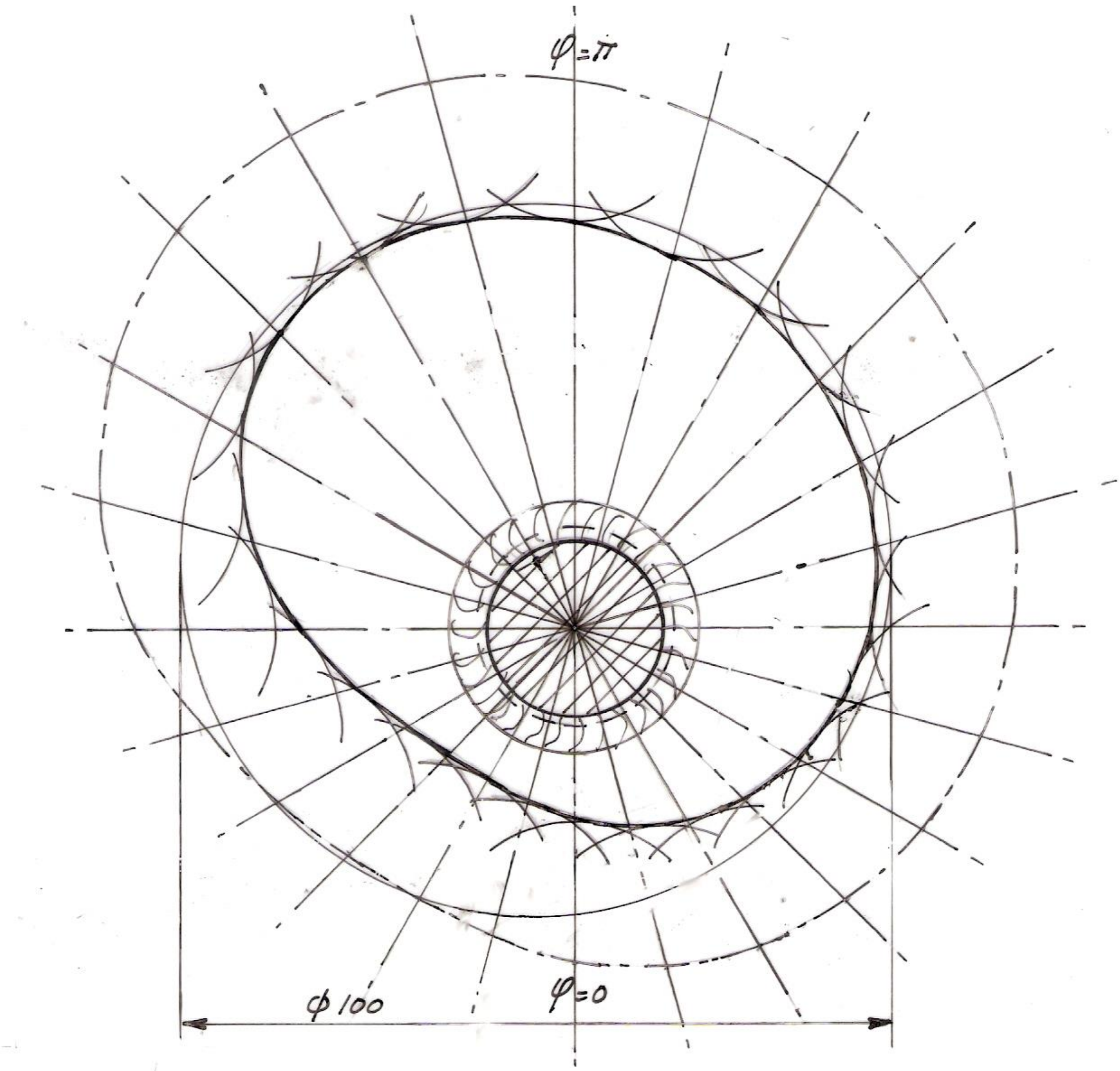

fig. 4 Sketch of the side view of a cam

To know if the given pump will work nicely, a prototype has to be built and tested. For testing one needs a motor for which the rotational speed can be varied which makes it possible to start with a low rotational speed. One can use a frequency modulator if the original 4-pole, 3-phase motor is used. One also can use the prescribed permanent magnet DC motor but one has to limit the voltage up to maximal 22.4 V if a gear box with i = 10.8 is used! If the permanent magnet DC motor is the final motor, one has to order a gear box with i = 13.5 and in this case the maximum voltage can be 28 V.

The gear motors mentioned in the Rossi catalogue already contain an asynchronous motor. But it is possible to order a gear box without a motor but including the gear wheel which has to be mounted to the motor shaft.

9 References

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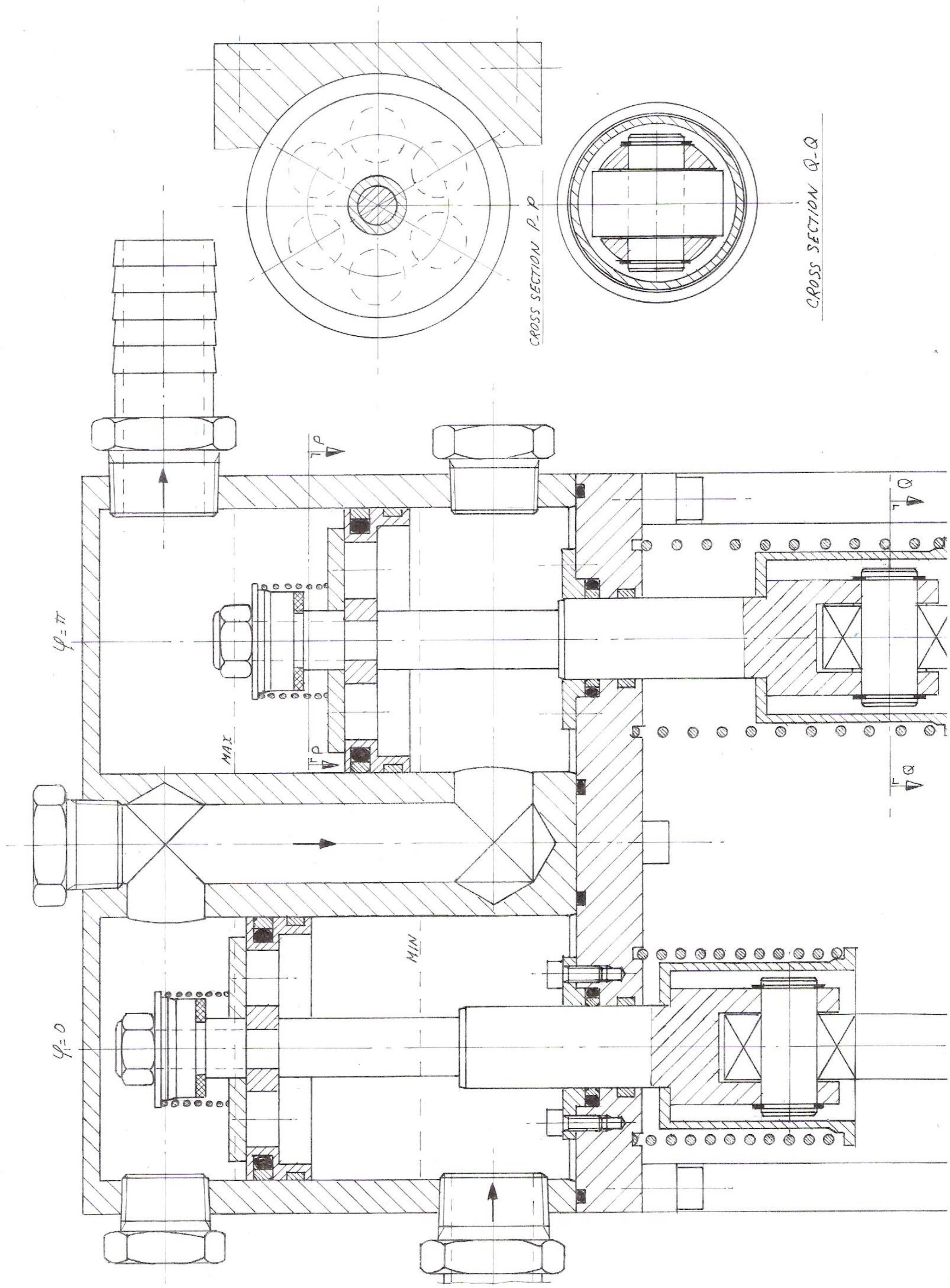

fig. 5 Cam driven double piston pump