Calculations executed for the 3-bladed rotor of the VIRYA-3.4 windmill (λ_d = 4.5, galvanised steel blades) driving a centrifugal pump through a Polycord transmission with i = 2.5 and a vertical shaft in the tower centre

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 The VIRYA-3.4 is not yet tested.

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1 Introduction

The VIRYA-3.4 windmill is designed to drive a centrifugal pump using a Polycord transmission with a round string and an accelerating gear ratio i = 2.5 in the windmill head and a vertical shaft in the tower. The main advantages of this transmission are that it bridges the eccentricity in between the rotor shaft and the vertical shaft and that it contains no oil which may leak into the water. The windmill can therefore be used in nature areas. A centrifugal pump can be coupled directly to the vertical shaft or one can use a second accelerating transmission at ground level. It is also possible to connect a generator in stead of a pump to the vertical shaft and even a pump plus a generator seems possible.

The Polycord transmission with a 12 mm string is first described in report KD 320 (ref. 1) for the VIRYA-2.8B4 windmill which is primary meant to drive a rope pump. It has a big wheel on the rotor shaft, a small wheel on the vertical shaft and one small auxiliary wheel. A 15 mm string will be used for the VIRYA-3.4 and all dimensions of the transmission and the head will be scaled up with about a factor 5/4. The transmission is described in chapter 7.

The VIRYA-3.4 is meant for use in developing countries and western countries. The use of stainless steel is limited as much as possible. The VIRYA-3.4 windmill has a 3-bladed rotor with galvanised steel blades but if available, stainless steel can also be used.

The windmill will be provided with the hinged side vane safety system which is used in all VIRYA windmills but the eccentricity e will be chosen at the left side of the tower. This safety system will be equipped with a relatively light vane blade resulting in a rated wind speed of about 8 m/s. The tower is probably about the same as the tower of the VIRYA-3, the VIRYA-3B3, the VIRYA-3.8 and the VIRYA-4.1 windmills. It has a 7 m long lowest part made of angle iron and strip and a 2 m or 3 m long upper part made of pipe.

2 Description of the rotor of the VIRYA-3.4 windmill

The 3-bladed rotor of the VIRYA-3.4 windmill has a diameter D = 3.4 m and a design tip speed ratio $\lambda_d = 4.5$. The rotor has blades with a constant chord and is provided with a 7.14 % cambered airfoil. Aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 2). A blade is made of a steel strip with dimensions of 208 * 1250 * 3 mm and 12 blades can be made from standard galvanised sheet size 1.25 * 2.5 m. Because the blade is cambered, the chord c is a little less than the blade width, resulting in c = 205 mm = 0.205 m.

A design tip speed ratio $\lambda_d = 4.5$ is rather high for a rotor with 7.14 % cambered blades and the rotor therefore can be rather noisy, especially if it is running at a higher tip speed ratio than λ_d. So the VIRYA-3.4 windmill should only be used at places where some noise production isn't a problem.

The blades are connected to each other by a welded spoke assembly which is built up from three 0.5 m long spokes which are welded together at 120° at the centre. Each spoke has a width of 60 mm and a thickness of 10 mm. A cambered strip size 210 * 50 * 6 mm is welded square to the ends of each spoke and the blade is connected to this strip by three M10 bolts. The overlap in between a blade and a spoke is 0.05 m which results in a free blade length of 1.2 m. This blade length in combination with a design tip speed ratio $\lambda_d = 4.5$ and a blade thickness of 3 mm is expected to be enough to prevent flutter of the blade at high rotational speeds.

The hub is made of 70 mm round stainless steel bar with a tapered hole in the centre for connection to the rotor shaft. The spoke assembly is clamped in between the hub and a thick 70 mm disk by means of three bolts M10 and that is why the spoke assembly isn't loaded by a bending moment at the welds and at the position of the holes. The hub is pulled on the tapered shaft end by one central bolt M16. The mass of the whole rotor including the hub is about 29 kg which seems acceptable for a steel rotor with a diameter of 3.4 m. The rotor is balanced by adding weights under the M10 bolts. A sketch of the rotor is given in figure 1.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 634) has its own formula numbering. Substitution of λ_d = 4.5 and R = 1.7 m in formula (5.1) of KD 35 gives:

$$\lambda_{r\,d} = 2.6471 * r \qquad (-) \qquad (1)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \qquad (°) \qquad (2)$$

$$\phi = 2/3 \arctan 1 / \lambda_{r\,d} \qquad (°) \qquad (3)$$

Substitution of B = 3 and c = 0.205 m in formula (5.4) of KD 35 gives:

$$C_l = 40.866\ r\ (1 - \cos\phi) \qquad (-) \qquad (4)$$

Substitution of V = 5 m/s and c = 0.205 m in formula (5.5) of KD 35 gives:

$$R_{e\,r} = 0.684 * 10^5 * \sqrt{(\lambda_{r\,d}^2 + 4/9)} \qquad (-) \qquad (5)$$

The blade is calculated for six stations A till F which have a distance of 0.24 m of one to another. Station F corresponds to the end of the spoke. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 2). The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for areas with a good wind regime. Those airfoil Reynolds numbers are used which are lying closest to the calculated values.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	1.7	4.5	8.4	0.205	0.74	0.76	3.11	3.4	-0.2	-0.1	8.6	8.5	0.038
B	1.46	3.865	9.7	0.205	0.85	0.87	2.68	2.5	1	1.2	8.7	8.5	0.033
C	1.22	3.229	11.5	0.205	1.00	1.02	2.26	2.5	2.8	3.0	8.7	8.5	0.038
D	0.98	2.594	14.1	0.205	1.20	1.19	1.83	1.7	5.8	5.6	8.3	8.5	0.050
E	0.74	1.959	18.0	0.205	1.48	1.43	1.42	1.2	-	9.5	-	8.5	0.1
F	0.5	1.324	24.7	0.205	1.87	1.28	1.01	1.2	-	16.2	-	8.5	0.26

table 1 Calculation of the blade geometry of the VIRYA-3.4 rotor

No value for α_{th} and therefore for β_{th} is found for stations E and F because the required C_l values can't be generated. The theoretical blade angle β_{th} for stations A to D varies in between 8.3° and 8.7°. If a constant blade angle of 8.5° is chosen, the linearised angle of attack α_{lin} differs only a little from the theoretical value α_{th} for the most important outer side of the blade. Each spoke is twisted 8.5° right hand to get the correct blade angles for a rotor with a right hand direction of rotation.

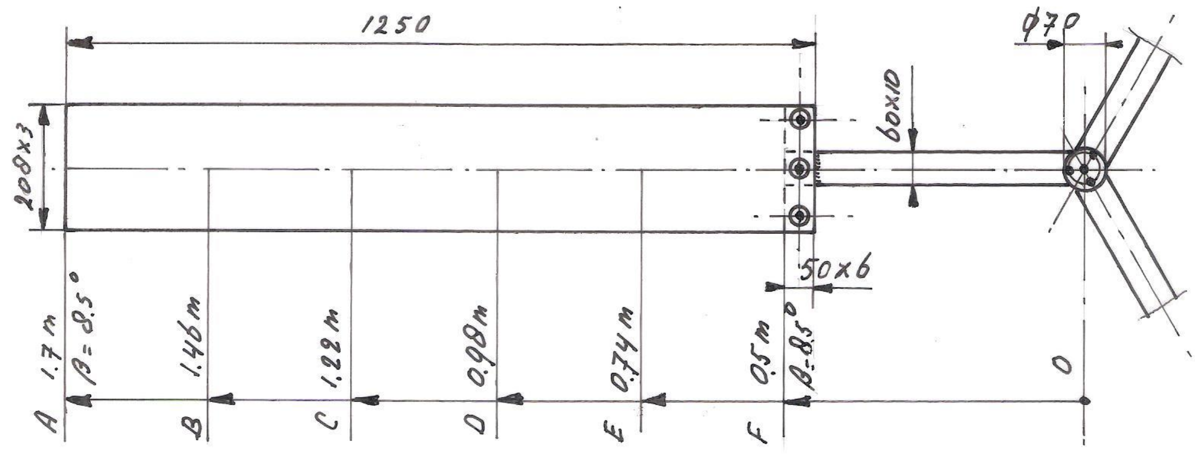

fig. 1 Sketch VIRYA-3.4 rotor

At this moment no detailed drawing of the rotor is made but the following remarks may help to make it.

If the rotor is used to drive a centrifugal pump or a generator, a shaft diameter of 30 mm might be enough. The shaft must have a halve cone angle of 5° over 48 mm. The hub has a tapered inside hole and the half cone angle is also 5°. Both cone angles must be made very accurately to guarantee a good fitting. The hub diameter must be 70 mm and the hub length must be 50 mm. The hub is pulled to the shaft by one central bolt M16 * 50 quality at least 8.8, which has to be turned very tightly. The three bolts at a pitch circle of 50 mm are M10 quality 8.8. One should not forget to place a disk of ϕ 70 * 8 mm on the front side of the spoke assembly to prevent too high bending stresses at the welds and the holes.

The three spokes are welded together at the centre (at the front and at the back side) and the angle in between the strips must be accurately 120° which can be realised by using three 822.7 mm long auxiliary strips bolted to the central outside holes. The welds must be ground flat. The three spokes are twisted 8.5° right hand after welding. The 210 mm longs strips size 50 * 6 mm must be cambered with the same camber as the blade at the contact area. Each strip should be welded exactly square to a spoke at the front side and at the back side.

The pitch in between the holes in the blades is 79 mm in a flat plane. The pitch in between the holes in the strip size 210 * 50 * 6 mm is 80 mm in a flat plane. This value is larger because the heart line of the strip is lying at a larger camber radius. The three bolts with which a blade is connected to the strip must be M10 quality 8.8. These bolts must have a cylindrical part where the blade touches the bolt to guarantee the fitting and to prevent high surface stresses. The holes in the blade and the strip must be made as small as possible (try 10 mm holes and make the hole pattern very accurately) to realise a rigid geometry for which the rotor balance is not changing during mounting because of clearance in between bolts and holes. The bolt heads should be at the back side of the blade.

The blade is cambered 7.14 % over the whole length. The blade isn't twisted. The VIRYA-3.4 rotor can also be seen as an alternative for the VIRYA-3.3S rotor. It has as advantage that the spoke assembly is a lot smaller. A similar hydraulic blade press can be used as designed for the VIRYA-3.3S but it has to be adjusted for a blade thickness of 3 mm. The camber radius r_c at the heart line of the sheet for a sheet width of 208 mm can be calculated using chapter 5.1 of report KD 398 (ref. 2). It can be calculated that r_c = 366.1 mm for the blade and that r_c = 370.6 mm for the strip.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.7 of KD 35 (for B = 3) and λ_{opt} = 4.5 and C_d/C_l = 0.04 gives $C_{p\ th}$ = 0.43 (interpolation in between the lines for C_d/C_l = 0.03 and C_d/C_l = 0.05). The blade is stalling in between station E and F. Therefore not the whole blade length k = 1.25 m, but only the part up to 0.13 m outside station F is used for the calculation of the C_p. This gives an effective blade length k' = 1.12 m.

Substitution of $C_{p\ th}$ = 0.43, R = 1.7 m and effective blade length k = k' = 1.12 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.38. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.38 / 4.5 = 0.0844.
Substitution of $\lambda_{opt} = \lambda_d$ = 4.5 in formula 6.4 of KD 35 gives λ_{unl} = 7.2.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R – ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

The blade angle is 8.5° for the whole blade. For a non rotating rotor the average angle of attack α is therefore 90° - 8.5° = 81.5°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 81.5° it can be read that C_l = 0.29. The whole blade is stalling during starting and therefore now the whole blade length k = 1.25 m is taken.

Substitution of B = 3, R = 1.7 m, k = 1.25 m, C_l = 0.29 and c = 0.205 m in formula 6 gives that $C_{q\ start}$ = 0.012. For the ratio in between the starting torque and the optimum torque we find that it is 0.012 / 0.0844 = 0.142. This is good for a rotor with a design tip speed ratio of 4.5. The ratio is expected to be high enough for combination of the windmill with a centrifugal pump because a centrifugal pump without a foot valve looses its water in the pressure pipe if the pump isn't running and the water level in the pressure pipe increases with the square of the rotational speed until the static water height is reached.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3}\right)} \qquad (m/s) \qquad (7)$$

The average starting torque Q_s is only caused by friction of the bearings on the rotor shaft and the vertical shaft and by some friction of the Polycord string. Assume Q_s = 1 Nm measured on the rotor shaft. Substitution of Q_s = 1 Nm, $C_{q\ start}$ = 0.012, ρ = 1.2 kg/m^3 and R = 1.7 m in formula 7 gives that V_{start} = 3 m/s. This is acceptable low for a 3-bladed rotor with a design tip speed ratio of 4.5. The Q-n curve of the rotor for V = 3 m/s is rising rather fast and therefore it is allowed that the pump torque is also rising at increasing rotational speed. The pump torque is rising because the water level in the rising main is rising as the vertical shaft has a higher rotational speed.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 4). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-3.4 rotor are given in figure 2 and 3.

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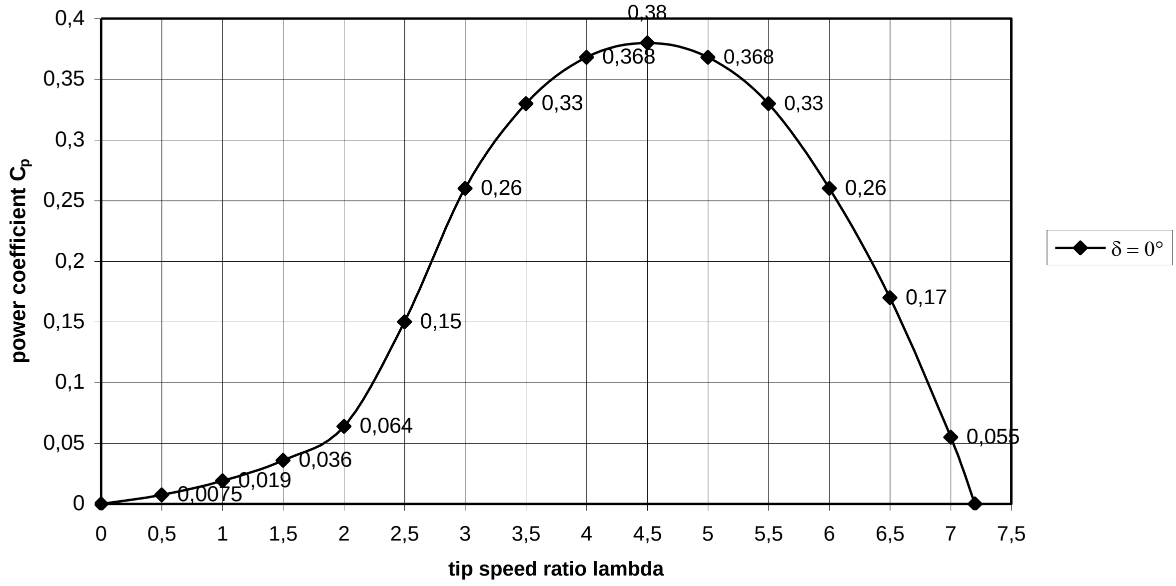

fig. 2 Estimated C_p-λ curve for the VIRYA-3.4 rotor for the wind direction perpendicular to the rotor (δ = 0°)

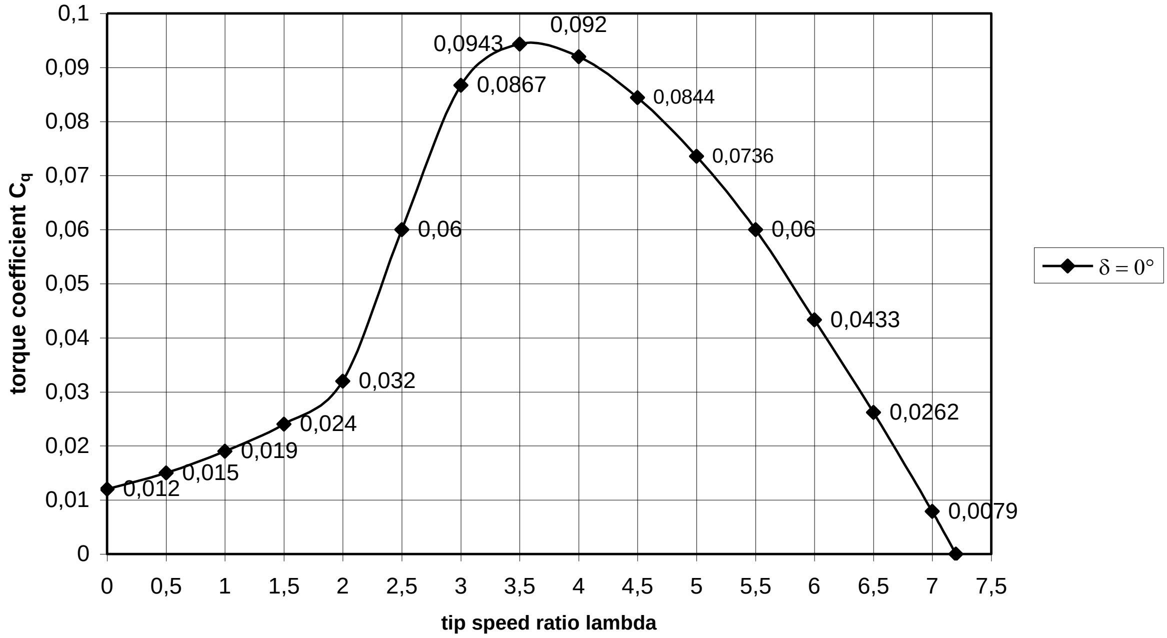

fig. 3 Estimated C_q-λ curve for the VIRYA-3.4 rotor for the wind direction perpendicular to the rotor (δ = 0°)

5 Determination of the Q-n curves

The determination of the Q-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_q-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the torque Q and the rotational speed n. The C_q-λ curve is given in figure 3. The δ-V curve of the safety system depends on the safety system. As this safety system is not yet designed, the δ-V curve of it is estimated. The estimated δ-V curve is given in figure 4.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 8 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The Q-n curve for 8 m/s will therefore also be valid for wind speeds higher than 8 m/s.

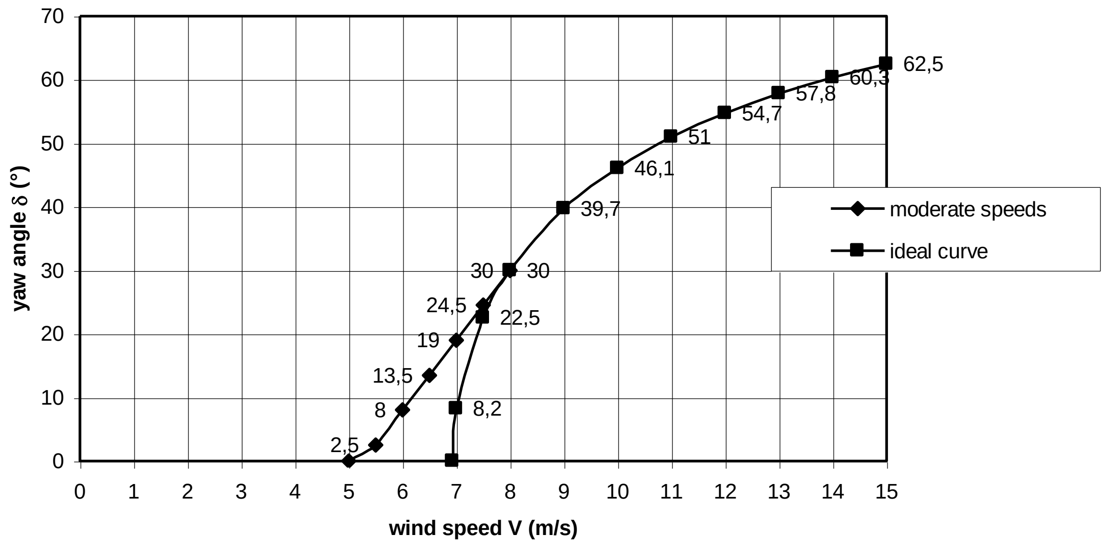

fig. 4 Estimated δ-V curve VIRYA-3.4 safety system with V_{rated} = 8 m/s

The Q-n curves are used to check the matching with the Q-n curve of the pump. The Q-n curves are determined for wind the speeds 2, 3, 4, 5, 6, 7 and 8 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for Q and n are used which are given in chapter 7 of KD 35.

Substitution of R = 1.7 m in formula 7.1 of KD 35 gives:

$$n_\delta = 5.6172 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of ρ = 1.2 kg / m^3 and R = 1.7 m in formula 7.7 of KD 35 gives:

$$Q_\delta = 9.2608 * C_q * \cos^2\delta * V^2 \qquad \text{(Nm)} \qquad (9)$$

The Q-n curves are determined for C_q values belonging to λ is 0, 1, 2, 2.5, 3, 3.5, 4.5, 5.5, 6.5 and 7.2 (see figure 3). For a certain wind speed, for instance V = 2 m/s, related values of C_q and λ are substituted in formula 8 and 9 and this gives the Q-n curve for that wind speed. For the higher wind speeds, the yaw angle as given by figure 5, is taken into account. The result of the calculations is given in table 2.

		V = 2 m/s δ = 0°		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 8°		V = 7 m/s δ = 19°		V = 8 m/s δ = 30°	
λ (-)	C_q (-)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	$n_δ$ (rpm)	$Q_δ$ (Nm)	$n_δ$ (rpm)	$Q_δ$ (Nm)	$n_δ$ (rpm)	$Q_δ$ (Nm)
0	0.012	0	0.44	0	1.00	0	1.78	0	2.78	0	3.92	0	4.87	0	5.33
1	0.019	11.2	0.70	16.9	1.58	22.5	2.82	28.1	4.40	33.4	6.21	37.2	7.71	38.9	8.45
2	0.032	22.5	1.19	33.7	2.67	44.9	4.74	56.2	7.41	66.8	10.46	74.4	12.98	77.8	14.22
2.5	0.06	28.1	2.22	42.1	5.00	56.2	8.89	70.2	13.89	83.4	19.62	92.9	24.34	97.3	26.67
3	0.0867	33.7	3.21	50.6	7.23	67.4	12.85	84.3	20.07	100.1	28.34	111.5	35.17	116.8	38.54
3.5	0.0943	39.3	3.49	59.0	7.86	78.6	13.97	98.3	21.83	116.8	30.83	130.1	38.26	136.2	41.92
4.5	0.0844	50.6	3.13	75.8	7.03	101.1	12.51	126.4	19.54	150.2	27.59	167.3	34.24	175.1	37.52
5.5	0.06	61.8	2.22	92.7	5.00	123.6	8.89	154.5	13.89	183.6	19.62	204.5	24.34	214.0	26.67
6.5	0.0262	73.0	0.97	109.5	2.18	146.0	3.88	182.6	6.07	216.9	8.57	241.7	10.63	253.0	11.65
7.2	0	80.9	0	121.3	0	161.8	0	202.2	0	240.3	0	267.7	0	280.2	0

table 2 Calculated values of n and Q as a function of λ and V for the VIRYA-3.4 rotor

The calculated values for n and Q are plotted in figure 5.

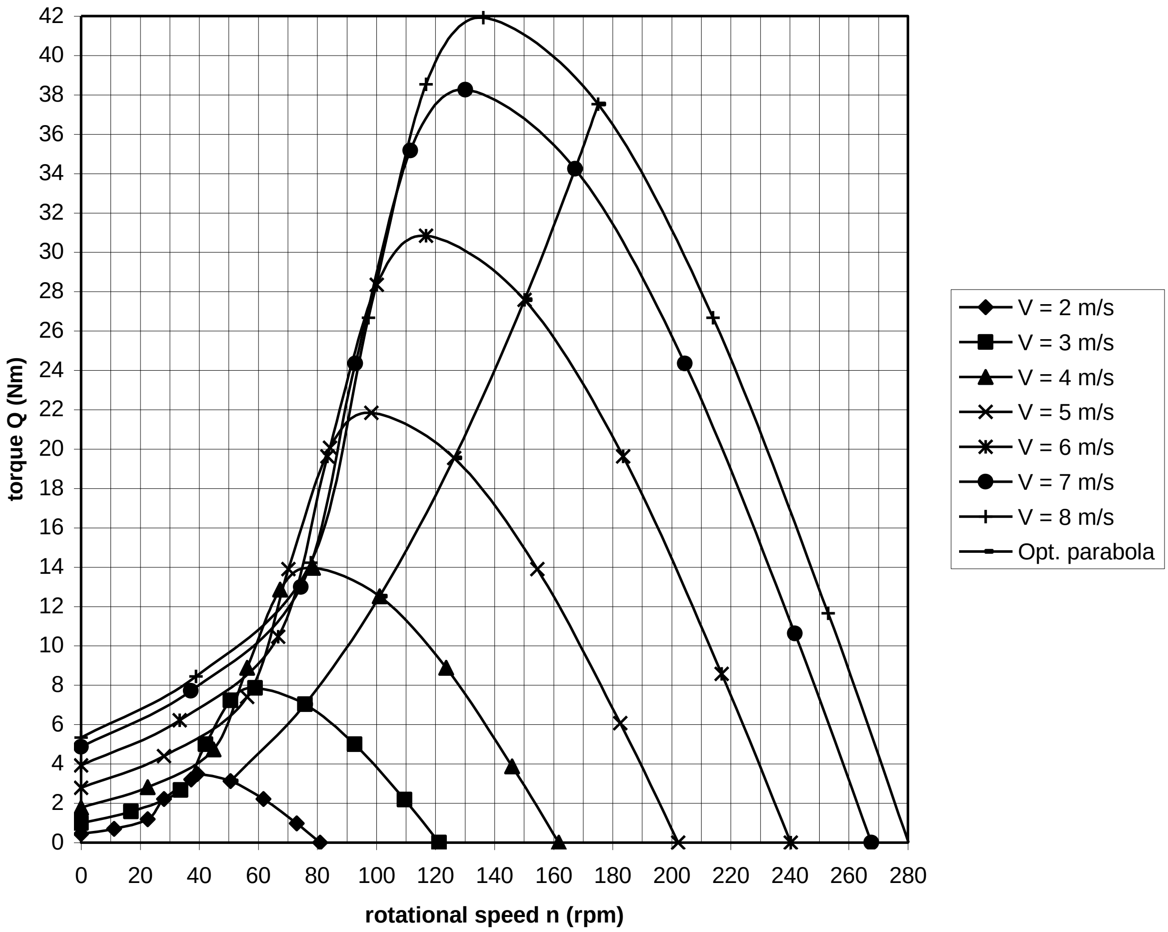

fig. 5 Q-n curves of the VIRYA-3.4 rotor

The optimum parabola which is going through the points with λ = 4.5, where C_p is maximum, is also drawn in figure 5. Figure 5 shows the Q-n curves on the rotor shaft. The Q-n_v curves on the vertical shaft can be derived from figure 5 if the gear ratio i and the transmission efficiency η_{tr} are taken into account.

6 Calculation of the strength of the spokes which connects the blades

The three blades are connected to each other by the spoke assembly. Every spoke has a length of 500 mm, a width b = 60 mm and a height h = 10 mm. A spoke is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. A spoke is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because the spoke is thin and long it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1^e The load which appears for a rotating rotor at V_{rated} = 8 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 30° for V_{rated} = 8 m/s.

2^e The load which appears for a locked rotor. The rotor is locked by connecting a spoke to the head (never connect a spoke to the tower) or by locking the vertical shaft in the tower centre.

6.1 Bending stress in the strip for a rotating rotor and V = 8 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 / B \qquad (N) \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The blade length k of the VIRYA-3.4 rotor is 1.25 m but the rotor radius R = 1.7 m. Therefore there is a disk in the centre with an area of about 0.07 of the rotor area on which no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.93 = 0.827. Because of the tip losses the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75.

Substitution of C_t = 0.75, δ = 30°, ρ = 1.2 kg/m^3, V = 8 m/s, R = 1.7 m and B = 3 in formula 10 gives $F_{t\,\delta\,bl}$ = 65.4 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 1.133 m. Because the blade length is only k, there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about r_1 = 1.2 m.

The maximum bending stress is not caused at the heart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at r_2 = 0.035 m. At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_1 - r_2) \qquad \text{(Nm)} \qquad (11)$$

Substitution of $F_{t\,\delta\,bl}$ = 65.4 N, r_1 = 1.2 m and r_2 = 0.035 m gives $M_{b\,t}$ = 76.2 Nm = 76200 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$$\sigma_b = M / W \qquad (\text{N/mm}^2) \qquad (12)$$

The moment of resistance W of a strip is given by:

$$W = 1/6\ bh^2 \qquad (\text{mm}^3) \qquad (13)$$

(12) + (13) gives:

$$\sigma_b = 6\ M / bh^2 \qquad (\text{N/mm}^2) \qquad (\text{M in Nmm}) \qquad (14)$$

Substitution of M = 76200 Nmm, b = 60 mm and h = 10 mm in formula 14 gives σ_b = 76 N/mm^2. For this stress the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the connecting strip is bending only in between the hub and the blade root. So it is assumed that the blade itself is not bending. The blade root is lying at r_3 = 0.45 m = 450 mm. So the length of the strip l which is loaded by bending is given by:

$$l = r_3 - r_2 \qquad \text{(mm)} \qquad (15)$$

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$$M = F * (r_1 - r_3) \qquad \text{(Nmm)} \qquad (16)$$

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$$\phi = l * (M + ½\ Fl) / EI \qquad \text{(rad)} \qquad (17)$$

The bending moment of inertia I of a strip is given by:

$$I = 1/12\ bh^3 \qquad (\text{mm}^4) \qquad (18)$$

(15) + (16) + (17) + (18) gives:

$$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½\ (r_3 - r_2)\} / (E * bh^3) \qquad \text{(rad)} \qquad (19)$$

Substitution of F = 65.4 N, r_3 = 450 mm, r_2 = 35 mm, r_1 = 1200 mm, E = 2.1 * 10^5 N/mm^2, b = 60 mm and h = 10 mm in formula 19 gives: ϕ = 0.02475 rad = 1.42°. This is an angle which can not be neglected. In report R409D (ref. 5) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-3.4 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin \left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2} \right) \quad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 208 mm and a plate thickness of 3 mm it is found that A_{pr} = 0.000624 m^2. The blade is made of steel sheet with a density ρ_{pr} of about ρ_{pr} = 7.8 * 10^3 kg/m^3. If the rotor is coupled to a centrifugal pump it will run about with the design tip speed ratio if the matching is well. So it is supposed that the tip speed ratio is 4.5. Substitution of C_t = 0.75, ρ = 1.2 kg/m^3, R = 1.7 m, B = 3, A_{pr} = 0.000624 m^2, ρ_{pr} = 7.8 * 10^3 kg/m^3 and λ = 4.5 in formula 20 gives: ε = 1.58°. This angle is larger than the calculated angle of 1.42° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.42°.

The real bending angle ε is determined a follows. A thrust moment M_t = 76.2 Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. M_b = 76.2 Nm for ε = 1.42°. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. M_c = 76.2 Nm for ε = 1.58°. The path of these three moments is given in figure 6. The sum total of M_b + M_c is determined and the line M_b + M_c is also given in figure 6.

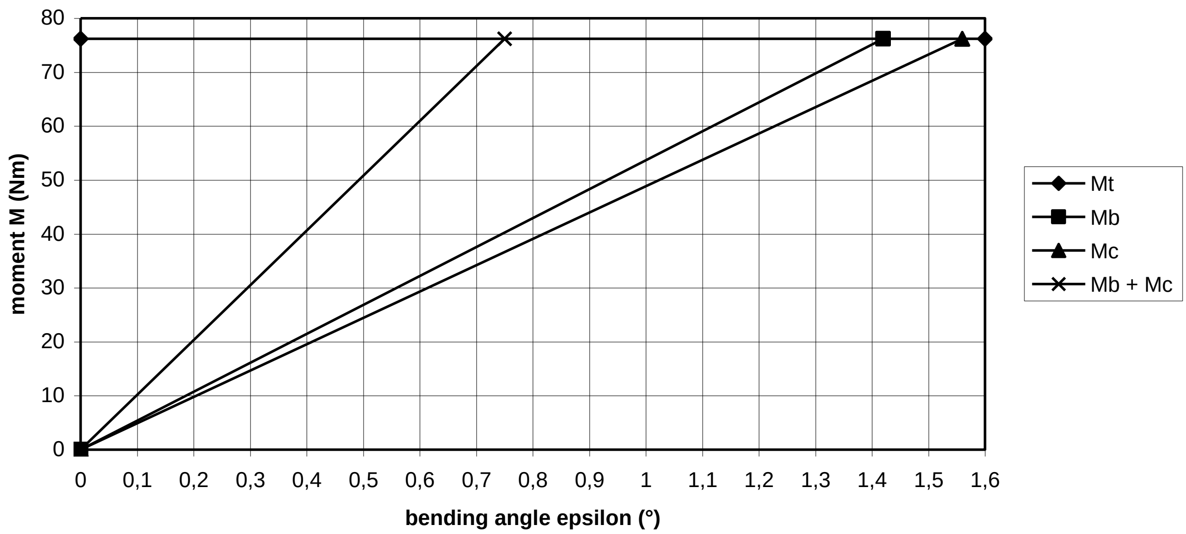

fig. 6 Path of M_t, M_b, M_c, and M_b + M_c as a function of ε

The point of intersection of the line of M_t with the line of M_b + M_c gives the final angle ε. In figure 6 it can be seen that ε = 0.75°. This is a factor 0.528 of the calculated angle of 1.42°. Because the bending stress is proportional to the bending angle it will also be a factor 0.528 of the calculated stress of 76 N/mm^2 resulting in a stress of about 40 N/mm^2. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with $\sin\alpha$ with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment $dF * r$ which is exerted by this force with respect to the blade is therefore varying sinusoidal too.

However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \qquad \text{(Nm)} \qquad (21)$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three $\sin^2\alpha$ functions which make an angle of 120° which each other, appear to result in a constant value. The three functions are given in figure 7. It can be proven for a three bladed rotor that the sum value of the three blades is equal to 3/2 of the peak value of one blade.

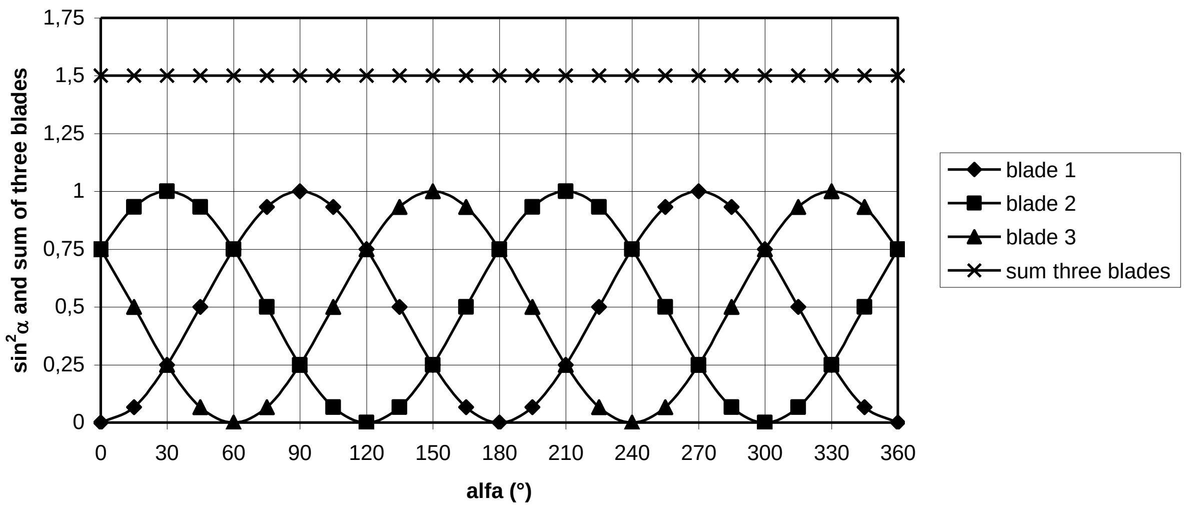

fig. 7 Path of $\sin^2\alpha$ and the sum of three blades

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then 2/3 of the sum moment as given by formula 21. The variation of the moment in the blade with respect to a rotating axis frame is therefore given by:

$M_{gyr\ bl} = 2/3 \sin\alpha * I_{rot} * \Omega_{rot} * \Omega_{head}$ (Nm) (22)

For a three bladed rotor, the moment of inertia of the whole rotor I_{rot} is three times the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$M_{gyr\ bl} = 2 \sin\alpha * I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (23)

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 22 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $\sin^2\alpha$ function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened peak value is given by:

$M_{gyr\ bl\ max} = 1.2 * I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (24)

For the chosen blade geometry it is calculated that I_{bl} = 8 kgm^2. The maximum loaded rotational speed of the rotor for the optimum cubic line can be read in figure 5 if the optimum parabola is followed and it is found that n_{max} = 175 rpm. This gives $\Omega_{rot\ max}$ = 18.3 rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-3.4 is provided with the hinged side vane safety system which has a light van blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be 0.2 rad/s at very high wind speeds.

Substitution of I_{bl} = 8 kgm^2, $\Omega_{rot\ max}$ = 18.3 rad/s and $\Omega_{head\ max}$ = 0.2 rad/s in formula 24 gives: $M_{gyr\ bl\ max}$ = 35.1 Nm = 35100 Nmm.

Substitution of M = 35100 Nmm, b = 60 mm and h = 10 mm in formula 14 gives $\sigma_{b\ max}$ = 35 N/mm^2. This value has to be added to the bending stress of 40 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max}$ = 40 + 35 = 75 N/mm^2. The minimum stress is 40 - 35 = 5 N/mm^2. So the stress is not becoming negative and therefore it is probably not necessary to take the load as a fatigue load.

For the strip material hot rolled strip Fe 360 is chosen. For hot rolled strip the allowable stress for a load in between zero and maximum is about 190 N/mm^2 and for a fatigue load it is about 140 N/mm^2. However, these are the stresses for a tensile stress. The allowable bending stresses are higher. It is assumed that the allowable bending stress for a load in between zero and maximum is 230 N/mm^2 and for a fatigue load is 170 N/mm^2.

The calculated stress is even much lower than the allowable fatigue stress, so the strip is strong enough. In reality the blade is not extremely stiff and will also bend somewhat. This reduces the bending of the strip and therefore the stress is the strip will be somewhat lower.

6.2 Bending stress in the strip for a locked rotor

The rotational speed for a rotor which is stopped by locking of the rotor is zero. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 65.4 N for $V = V_{rated} = 8$ m/s and $\delta = 30°$. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 30° if the force on one blade is 65.4 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at $r_4 = 1.075$ m because the relative wind speed is constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 11 changes into:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_4 - r_2) \qquad \text{(Nm)} \qquad (25)$$

Substitution of $F_{t\,\delta\,bl} = 65.4$ N, $r_4 = 1.075$ m and $r_2 = 0.035$ m in formula 27 gives $M_{b\,t} = 68$ Nm = 68000 Nmm. Substitution of M = 68000 Nmm, b = 60 mm and h = 10 mm in formula 14 gives $\sigma_b = 68$ N/mm^2. This is lower than the calculated stress for a rotating rotor. The load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable stress is 190 N/mm^2 for hot rolled strip, so the strip is strong enough.

7 Determination of the Polycord transmission

The Polycord transmission in the head of the VIRYA-2.8B4 windmill is described in chapter 3 of KD 340 (ref. 1). Figure 3 out of KD 340 gives the string path and wheels for a transmission with an accelerating gear ratio i = 2.5. This figure is copied as figure 8. The determination of the string dimensions is described in chapter 4 of KD 340. A copy of the Polycord data sheet (in German) is given in appendix 1 of KD 340.

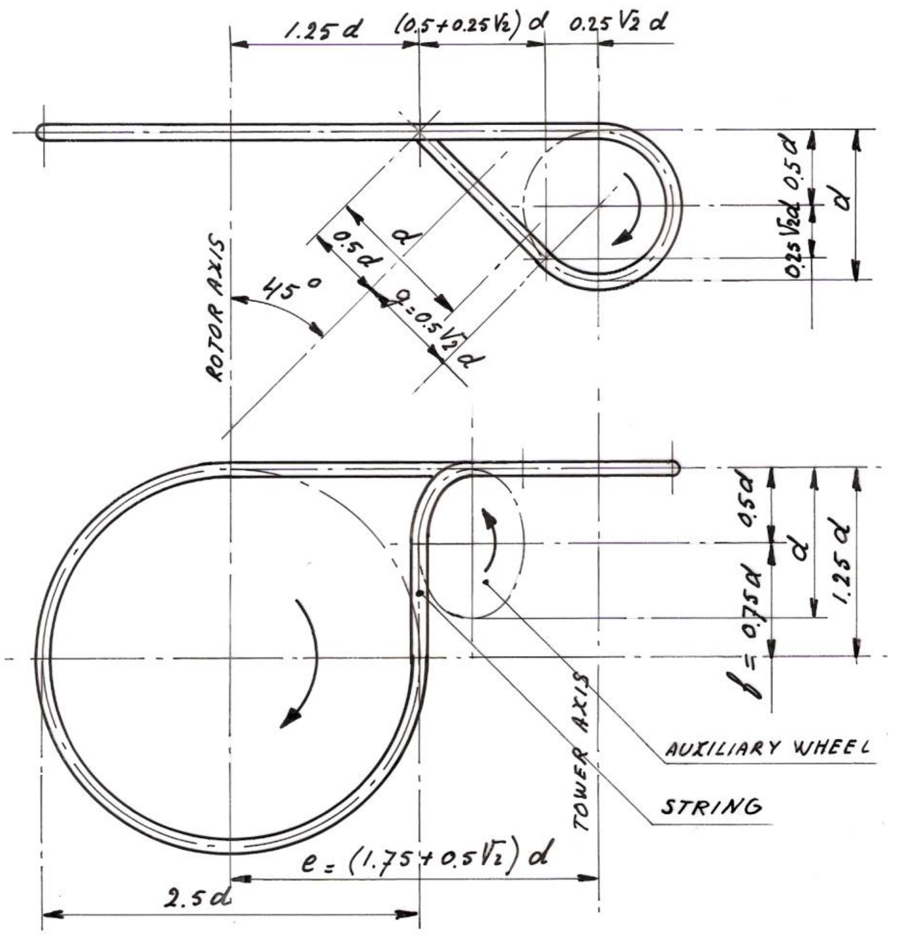

fig. 8 String path and wheels for VIRYA-3.4 transmission with i = 2.5

The rotor shaft is rotating right hand if seen from the front side of the rotor. This has as advantage that the central bolt which clamps the hub to the tapered shaft end will not unscrew if the hub may shift over the shaft. The vertical shaft is rotating right hand if seen from above. This has as advantage that one may use couplings in the shaft made from long nuts and that these couplings will not unscrew because of the torque.

If the vertical shaft is rotating right hand it means that the reaction torque of the vertical shaft on the head is left hand. The eccentricity e is chosen at the left side of the tower axis. This has as advantage that the reaction torque of the vertical shaft on the head is working in the opposite direction as the torque which is product of the rotor thrust times the eccentricity. This means that the rotor will turn out of the wind more if the pump load disappears (for instance if the well is emptied).

The string diameter is 12 mm for the VIRYA-2.8B4 and 15 mm for the VIRYA-3.4. All dimensions of the transmission are scaled up by a factor 15/12 = 5/4.

The pitch diameter d of the small wheel on the vertical shaft and of the auxiliary wheel is 10 times the string diameter. The string diameter of the VIRYA-3.4 is 15 mm, so the pitch diameter d is 10 * 15 = 150 mm. For the gear ratio it is chosen that i = 2.5 because for this value the auxiliary wheel isn’t touching the big wheel on the rotor shaft. The pitch diameter D of the big wheel on the rotor shaft is 2.5 * 150 = 375 mm.

The eccentricity e is given by formula 16 out of KD 320. Substitution of d = 150 mm in formula 16 of KD 320 gives that e = 368.6 mm = 0.3686 m. As the rotor diameter D is 3.4 m, the ratio e / D = 0.3686 / 3.4 = 0.1084. This rather large ratio and the fact that the optimum torque coefficient is only 0.0844 for a rotor with λ_d = 4.5 (see figure 3) makes that the influence of the reaction torque of the vertical shaft on the moment which turns the head out of the wind is therefore rather small.

The theoretical string length l_{th} is calculated with formula 19 of KD 320. The real string length l for a pre-tension of 8 % is calculated with formula 20 of KD 320. For the VIRYA-2.8B4 it was found that l = 1396 mm. For the VIRYA-3.4, all transmission dimensions are increased by a factor 5/4 = 1.25 so l = 1.25 * 1396 = 1745 mm.

The maximum moment which can be passed through for a certain string diameter and a certain wheel diameter depends on the angle of the spanned bow. The smallest angle for a loaded wheel is 225° for the small wheel on the vertical shaft.

The allowable net pulling force F_{net} is given in the first table (Tafel 1) of appendix 1 of KD 320 (in German) as Nennumfangskraft F_{un}). F_{net} for a 15 mm diameter string is 225 N. This value is valid for a spanned bow (Umschlingungswinkel) β = 180°. In figure 1 of appendix 1 of KD 320, the bow factor (Winkelfaktor) c_1 is given as a function of the spanned bow β. For a spanned bow of 225° we can read that c_1 is 0.87 (by interpolation just below β = 220°). The given value of F_{net} for 180° has to be divided by c_1 to find the corrected value $F_{net\ cor}$ or in formula:

$$F_{net\ cor} = F_{net} / c_1 \qquad \text{(N)} \qquad (26)$$

Substitution of F_{net} = 225 N and c_1 = 0.87 in formula 26 gives $F_{net\ cor}$ = 258 N.

The force for 8 % pre-tension = 315 N (see upper table appendix 1 KD 320). This means that the total pulling force for the loaded part of the string is 315 + ½ * 258 = 444 N and that the pulling force for the unloaded part of the string is 315 – ½ * 258 = 186 N. So the unloaded part has still a considerable large pulling force and will therefore not run out of the wheels. This is an indication that even for a higher torque than the allowable torque, there is a large reserve. May be the string will slip over the smallest wheel if the torque is too large. The string will never break because the breaking force is 7000 N.

The maximum rotor torque Q_{max} for $F_{net\ cor}$ is given by:

$$Q_{max} = F_{net\ cor} * D / 2 \qquad \text{(Nm)} \qquad (27)$$

Substitution of $F_{net\ cor}$ = 258 N and D = 375 mm = 0.375 m in formula 27 gives Q_{max} = 48.4 Nm.

The VIRYA-2.8B4 is driving a rope pump and the VIRYA-3.4 is driving a centrifugal pump. A rope pump is a positive displacement pump and the required torque is about constant (see KD 320 figure 1). A centrifugal pump has about a parabolic Q-n curve. It is assumed that the pump geometry and the water height are chosen such that the optimum parabola of the windmill rotor is followed. Design of the centrifugal pump is out of the scope of this report. In figure 5 it can be seen that the maximum torque at a wind speed of 8 m/s (and higher) is 37.5 Nm if the optimum parabola is followed. The maximum peak torque is about 42 Nm for V = 8 m/s (and higher). As the maximum allowable torque is 48.4 Nm, the Polycord string is certainly strong enough for the pumping conditions at high wind speeds. It is even strong enough to stop the rotor by using of a brake on the vertical shaft if this brake isn't used too abruptly. If the rotor is braked too abruptly, a large extra moment is needed to decelerate the rotor.

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