Basic knowledge about electrical, chemical, mechanical, potential and kinetic energy to understand literature about the generation of energy by small wind turbines

ing. A. Kragten

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Engineering office Kragten Design
Populierenlaan 51
5492 SG Sint-Oedenrode
The Netherlands
telephone: +31 413 475770
e-mail: info@kdwindturbines.nl
website: www.kdwindturbines.nl

Contains page

1 Introduction

Up to now, Kragten Design (KD) has developed 14 small battery charging windmills for which licences are available. More than 400 KD-reports and KD-notes about technical aspects of small wind turbines have been written. The reports which are public are given on a list of public KD-reports. Information about the reports, the windmills and KD-consultancy can be found on: www.kdwindturbines.nl. For all the reports it is assumed that the reader has some basic knowledge of electrical, chemical and mechanical engineering.

However, not all people interested in wind energy have this basic knowledge and will therefore meet problems to understand wind energy literature even if it is written on a low level. Especially for people living in developing countries, access to this knowledge might be difficult. In this report KD 378 some basic information about electrical, chemical and mechanical engineering will be given. But it will be clear that much more can be said about each item and that this report can't replace a good technical education.

Energy is available in many different forms. For understanding functioning of small wind turbines one must know about electrical, chemical, mechanical, potential and kinetic energy. Electrical energy is generated by the generator. Chemical energy is used when the energy is stored in a battery. Mechanical energy is generated by the windmill rotor. Potential energy is generated when the windmill is used to pump water from a well into a reservoir. Kinetic energy is available in the wind. One has to know the difference in between energy and power. One has to be able to make some basic calculations using the correct dimensions.

The main formulas for each kind of energy will be given in five separate chapters. At the end of each chapter some examples will be given to demonstrate how the formulas have to be used to make basic calculations. Only the modern magnitudes will be used. The five fundamental magnitudes and its dimensions are: length (m), time (s), mass (kg), temperature (°C) and current (A). All other magnitudes can be derived from these five magnitudes. The advantage of using the modern magnitudes is that the formulas which give the relation in between the different kinds of energy, are very simple. So if you live in a country where one uses inches, pounds and °F, you have to transfer these magnitudes to the modern magnitudes.

Almost all VIRYA windmills designed by Kragten Design are small battery chargers. Batteries supply direct current (DC) and have to be charged by direct current. So the information given in this report about electrical energy will be limited to direct current. 3 phase alternating current can be transformed into direct current by a 3-phase rectifier with only a small fluctuation of the voltage. How this works is described in report KD 340 (ref. 1). Almost all VIRYA windmills make use of permanent magnet (PM) 3-phase generators which are made of standard asynchronous motors. These generators are described in report KD 341 (ref. 2). The aerodynamic theory which was used to design the windmill rotors of all VIRYA windmills is given in report KD 35 (ref. 3). This report has been used in many courses. Questions and answers about subjects which are discussed in this report are given in report KD 196 (ref. 4).

2 Electrical DC energy

2.1 General

If a constant voltage is placed over a resistor, a certain DC current will flow through the resistor. The relation in between the voltage U, the current I and the resistance R is given by the law of Ohm:

$$U = I * R \quad (V) \qquad (1)$$

The dimension of the voltage U is Volt (V). The dimension of the current I is Ampere (A). The dimension of the resistance R is Ohm (Ω). The dimension is placed in between brackets.

Formula 1 can also be written as:

$I = U / R$ (A) (2)

or as:

$R = U / I$ (Ω) (3)

The DC current I which is flowing through a load with a certain resistance, is generating an electrical power P_{el} in that load. The load can be anything through which a DC current can flow. This can be an electric device, a DC motor, a lamp or a pure resistor. The electrical power P_{el} is given by:

$P_{el} = U * I$ (W) (4)

The dimension of the electrical power P_{el} is given in Watt (W). Formula 4 can be written as:

$I = P_{el} / U$ (A) (5)

or as:

$U = P_{el} / I$ (V) (6)

(1) + (4) gives:

$P_{el} = I^2 * R$ (W) (7)

(2) + (4) gives:

$P_{el} = U^2 / R$ (W) (8)

The electrical energy E_{el}, is the product of the electrical power P_{el} and the time t, or in formula:

$E_{el} = P_{el} * t$ (Ws) (9)

The dimension of the time t is given in seconds (s) so the electrical energy E_{el} is given in Watt-seconds (Ws) or Joule.

(4) + (9) gives:

$E_{el} = U * I * t$ (Ws) (10)

The dimensions W, s and Ws are rather small.

For larger powers one often uses kilowatt (kW) or megawatt (MW).
1 kW = 1000 W. 1 MW = 1000000 W.

For larger times one often uses hour (h), day (day) or year (year).
1 h = 3600 s. 1 day = 3600 * 24 = 86400 s. 1 year has 365.24 days.
So 1 year = 3600 * 24 * 365.24 = 31556736 s.

For larger energies one often uses kilowatt-hour (kWh).
1 kWh = 1000 * 3600 = 3600000 Ws.
1 Ws = 1 / 3600000 = 0.0000002778 kWh

Sometimes one is interested how many kWh are used in one year. So in this case one is using a special dimension of power (kWh/year) because now the a certain amount of energy, kWh, is divided by a certain time, a year.
1 kWh/year = 3600000 / 31556736 = 0.1141 W.
1 W = 31556736 / 3600000 = 8.766 kWh/year.

2.2 Example

Suppose one uses a 12 V water heater with a power of 130 W.
a) What is the current flowing through the heater?

Substitution of P_{el} = 130 W and U = 12 V in formula 5 gives I = 10.833 A.

b) What is the resistance of this heater?

Substitution of U = 12 V and I = 10.833 A in formula 3 gives R = 1.108 Ω.

c) What is the energy consumption in kWh/year if this heater is used continually for one year?

1 W = 8.766 kWh/year.
So 130 W = 130 * 8.766 = 1139.6 kWh/year.

d) What are the yearly costs if 1 kWh costs 0.2 Euro?

1139.6 kWh/year will cost 1139.6 * 0.2 = 227.9 Euro/year.

3 Chemical energy

3.1 General

Electrical DC energy can be stored directly in a capacitor. However, the disadvantage of a capacitor is that even in a large capacitor, only a small amount of energy can be stored. Another disadvantage is that the voltage of the capacitor will be higher as it contains more energy. Therefore capacitors are normally only used to store small amounts of energy or to flatten fluctuations in a DC current. The efficiency of a capacitor is very high.

To store large amounts of energy one needs a galvanic element. For use in windmills, lead sulphuric acid batteries are the most common option. A battery is build up from several cells which are connected in series. One lead sulphuric acid cell has a nominal voltage of 2 V so six cells have to be connected in series to obtain a 12 V battery. One can use six separate 2 V cells but six cells mounted in one casing is most general. For a 24 V battery, two 12 V batteries are connected in series.

Description of the chemical process which is active in the battery during charging and discharging is out of the scope of this report. The efficiency of a battery is much lower than of a capacitor because there is a transformation of electrical energy into chemical energy during charging and a transformation of chemical energy into electrical energy during discharging. Another reason is that a battery has a certain internal resistance resulting in internal heat losses when a current is flowing. But even if no external current is flowing, some energy will be lost because of internal leakage. The maximum efficiency of a battery is about 70 % or 0.7 (-) if given as a factor of 1. (-) means that it is dimensionless.

A normal lead sulphuric acid battery will loose about 1 % of its capacity each day so a full battery will be almost empty after about three months even if it is not used at all.

What is important to know if a battery is used in combination with a windmill, is how the voltage varies during charging and discharging and how the energy content can be derived from the battery capacity. One can distinguish three voltages, the charging voltage, the discharging voltage and the open voltage when nothing is connected to the battery poles.

The open voltage is not very stable directly after charging or discharging. If a 12 V battery is charged with a voltage of for instance 13.6 V and if the charger is removed one can measure the open voltage depending on the time which has passed. One will see that the open voltage starts at 13.6 V but that it drops slowly if time goes by. Finally after at least 20 minutes it will end somewhere in between 12 V and 13 V depending on the charging state. So when I speak about the open voltage I mean the stable open voltage which is realised after at least 20 minutes of non charging or non discharging.

The open voltage of a 12 V battery depends on the charging state. A battery is normally not used below a charging state of 10 %. Complete discharge of a battery will certainly result in damage of the battery. The open voltage of a 12 V lead sulphuric acid battery is about 12 V for a charging state of 10 % and about 12.6 V for a charging state of 90 %. As the difference is only small and as the value depends on the time after disconnection of the poles, measuring of the open voltage is not an accurate way to determine the charging state of a battery. The charging state is normally determined by measuring the density of the electrolyte.

The open voltage for a charging state of 100 % is about 13 V so the open voltage is rising rather fast during the last part of the charging process. At a charging state of more than 90 %, the battery starts gassing which means that the water of the electrolyte starts to separate into oxygen and hydrogen. The gassing at a charging state of 100 % is very large at high currents and this results in loss of water and in decrease of the lifetime. The current is normally limited by limiting the charging voltage to a certain value.

The maximum charging voltage is normally limited to 2.3 V / cell which means to 13.8 V for a 12 V battery and 27.6 V for a 24 V battery. For lower charging rates, a higher charging voltage is allowed but it is difficult to design a voltage controller which varies the maximum charging voltage as a function of the charging state of the battery. All VIRYA windmills are executed with a voltage controller and a dump load which limits the charging voltage up to about 2.3 V / cell. The magnetic field of a permanent magnet generator (PM-generator) can’t be regulated. So the charging voltage can only be limited if a voltage controller allows just that much energy to flow to the dump load, that the voltage is maintained on the required level.

The current depends on the difference in between the charging voltage and the open voltage and on the battery capacity. The voltage difference is used to overcome the internal resistance of the battery and this internal resistance is smaller as the battery capacity is larger. As the open voltage increases with the charging state, the current will decrease if a battery is charged with the maximum charging voltage. A small battery will reach the maximum charging voltage already at a rather low current even if it is not almost full. This means that a large part of the energy generated by the windmill will be dissipated in the dump load which is not favourable.

So one of the criteria for selection of the battery capacity is that it must be able to absorb the maximum generator current at a voltage which is lying below the maximum charging voltage for a charging state below about 90 %. The other criteria is that it must be large enough to store enough energy for overcoming some wind still days. But to make the required calculations, the battery capacity has to be transformed into the energy content.

The capacity of a battery is normally not given in Ws or kWh but in Ah. The capacity of a lead sulphuric acid battery is normally given for a discharge time of 10 hours. During the discharge period the voltage is decreasing and therefore the power is decreasing too. The discharge current I is taken so large that, if the battery is discharged with this current, the minimum discharge voltage is reached after ten hours.

The minimum discharge voltage for a 12 V battery is about 10.6 V. As soon as the discharge of a full battery starts, the open voltage will drop to a loaded voltage of about 12 V. This is because of the internal resistance of the battery. For the average loaded voltage U_{av} during the complete discharge period it is assumed that $U_{av} = 11.3$ V.

The capacity decreases significantly if the discharge time is taken shorter than 10 hour. This is because high currents result in large heat losses and therefore in decrease of the efficiency. The capacity increases a little if the discharge time is taken longer than 10 hour.

The available energy E_a for a period of 10 hour or 36000 s is given by:

$$E_a = I * U_{av} * 36000 \qquad (Ws) \qquad (11)$$

For the current I, one has to take a current in A equal to 1/10 of the capacity in Ah. The battery has a certain efficiency η (-). So the required charging energy E_{ch} is given by:

$$E_{ch} = E_a / \eta \qquad (Ws) \qquad (12)$$

The required charging energy E_{ch} for a certain charging current I_{ch} and an average charging voltage U_{chav} is given by:

$$E_{ch} = I_{ch} * U_{chav} * t \qquad (Ws) \qquad (13)$$

3.2 Example

Suppose one has a 12 V battery of 200 Ah.
a) What is the nominal energy content of a full battery in Ws and in kWh?

This battery can be discharged with a current of 1/10 * 200 = 20 A during 10 hours until the minimum discharging voltage of 10.6 V is reached. Substitution of I = 20 A and $U_{av} = 11.3$ V in formula 11 gives $E_a = 8136000$Ws. 1 Ws = 0.0000002778 kWh so 8136000 Ws = 2.25 kWh.

b) How much energy is required to charge this battery if it is empty and if the battery efficiency is 0.7?

Substitution of $\eta = 0.7$ and $E_a = 8136000$ Ws in formula 12 gives that $E_{ch} = 11622857$ Ws or 3.22 kWh.

c) How long will it take to charge this battery if the charging current is 5 A and if the average charging voltage is 13 V?

Formula 13 can be written as:

$$t = E_{ch} / (I_{ch} * U_{chav}) \qquad (s) \qquad (14)$$

Substitution of $E_{ch} = 11622857$ Ws, $I_{ch} = 5$ A and $U_{chav} = 13$ V in formula 14 gives that t = 178813 s or t = 178813 / 3600 = 49.67 hour so somewhat more than two days.

4 Mechanical energy

4.1 General

Mechanical energy E is required if a subject is displaced with certain force F over a certain distance s. The relation is given by:

$$E = F * s \quad (Nm) \qquad (15)$$

The power P which is supplied is the energy per second so:

$$P = E / t \quad (Nm/s) \qquad (16)$$

(15) + (16) gives:

$$P = F * s / t \quad (Nm/s) \qquad (17)$$

The velocity V is the displaced distance s per second so:

$$V = s / t \quad (m/s) \qquad (18)$$

(17) + (18) gives:

$$P = F * V \quad (Nm/s) \qquad (19)$$

So the power is proportional to the speed V. There are three possibilities for the speed.

a) V = 0. This is the case when we are pushing against the wall of a building. We can become very tired by doing this but mechanically we are supplying no power.

b) V = constant. This is the case when we are lifting a bucket of water out of a well with no acceleration. On the bucket there is an upwards force of the rope and there is a downwards force of the weight of the water and the bucket. The upwards force and the downwards force are equal when water is lifted with a constant speed.

c) V = varying. V can increase or decrease. The variation of the speed can be with a constant acceleration or with a varying acceleration. Only a constant acceleration will be discussed in this report. A body will accelerate if there is only one force working on the body. This normally happens only in outer space because there is no friction. On the earth there is always friction which causes a force opposite the external force F. Suppose we let a stone fall down from an helicopter. In the beginning, the speed will increase with a constant acceleration. But the air molecules will cause a friction and this friction increases proportional to V^2. So at a certain speed, the upwards force caused by the friction will become the same as the downwards force because of the weight and from this point the speed will be constant. The maximum speed depends on the mass and the shape of the stone.

So a falling body accelerates only as long as the driving force is larger than the friction force. The same counts for all horizontal movements. If you drive a car with a constant speed, the driving force is the same as the counteracting force caused by the friction in between the car body and the air and the friction in between the car wheels and the road. It accelerates only if the driving force is larger than the friction forces.

If there is only one force working on a body, there is a very simple relation in between the force F, the mass of the body m and the acceleration a.

The force F is given by the law of Newton which is:

$F = m * a$ (N or kgm/s^2) (20)

m is the mass in kg and a is the acceleration in m/s^2. So the dimension of the force F is kgm/s^2 if only fundamental magnitudes are used.

(19) + (20) gives:

$P = m * a * V$ (kgm^2/s^3 or W) (21)

It can be proven that the dimension kgm^2/s^3 is the same as the Watt or W which is used more generally as dimension for power.

Formula 20 is normally used when a certain subject is really accelerated by an external force. However it can also be used to calculate the weight G if the force is caused by the earth gravity and if the subject is supported, and so not moving. In this case one has to use the acceleration of gravity g in stead of a. g = 9.81 m/s^2. So formula 20 changes into:

$G = m * g$ (N) (22)

So a mass m = 1 kg has a weight G = 9.81 N. In old literature one says that the weight is 1 kgf (kilogram force) if the mass is 1 kg but this dimension of force is no longer used in modern mechanical engineering.

The speed V at a certain time t for a constant acceleration a can be calculated with:

$V = V_0 + a * t$ (m/s) (23)

V_0 is the beginning speed for t = 0. a is the acceleration in m/s^2. The displaced distance s at a certain time t for a constant acceleration a can be calculated with:

$s = V_o * t + ½ a * t^2$ (m) (24)

Up to now the formulas are valid if the force is acting along a straight line. But for a windmill rotor, the mechanical energy is generated in a rotating shaft. The formula for the power for a rotating shaft is about similar to formula 19 but one has to use the torque Q in stead of the force F and the angular velocity ω in stead of the velocity V. This results in:

$P = Q * \omega$ (W) (25)

A torque Q is formed by two identical forces F which are working in opposite direction at a distance r. The torque Q is given by:

$Q = F * r$ (Nm) (26)

The angular velocity ω is given in radian per second. A radian is an angle for which the length of the arc of a circle is the same as the radius. So 360° = 2 π radian and so:

1 radian = 360 / 2 π (°) (27)

So 1 radian is about 57.296°. The radian itself has no dimension, so the dimension of ω is 1/s but one can also write radian/s.

The relation in between the angular velocity ω and the rotational speed n (rpm) is given by:

$\omega = \pi * n / 30$ (1/s) (28)

(25) + (28) gives:

$P = Q * \pi * n / 30$ (W) (29)

The rotor torque Q is caused by aerodynamic forces on the rotor blades. The relation in between the torque, the shape of the rotor and the wind speed is explained in report KD 35 (ref. 3).

Similar to formula 20 for a linear movement, a formula can be derived for the torque for a rotating shaft. This formula is:

$Q = I_m * d\omega/dt$ (Nm) (30)

I_m is the mass moment of inertia (kgm^2). If m is a point mass at a radius r from the axis around which it rotates, I_m is given by:

$I_m = m * r^2$ (kgm^2) (31)

The symbol for the angular acceleration is an ω with a dot on it but I can't find this symbol on my computer. But it can also be called $d\omega/dt$ because in fact it is the derivative of the angular velocity to the time. Similarly the linear acceleration a can also be called dv/dt as it is the derivative of the speed to the time. If the mass is formed by a rotating body like a disk or a shaft, I_m can be found by integration of I_m of all point masses but there are also formulas available in handbooks for different shapes of the rotating body.

Similar to formula 23 for a linear movement, a formula can be derived for the angular velocity ω if the rotational speed of a shaft is increasing with a constant angular acceleration. The formula for ω is:

$\omega = \omega_0 + d\omega/dt * t$ (radian/s) (32)

Similar to formula 24 for a linear movement, a formula can be derived for the angle α (in radians) if the rotational speed of a shaft is increasing with a constant angular acceleration $d\omega/dt$. The formula for α is:

$\alpha = \omega_0 * t + ½\, d\omega/dt * t^2$ (radian) (33)

4.2 Examples

A bucket of water with a total mass of 10 kg is lifted with a constant speed of 0.8 m/s.

a) How much power is generated?

Substitution of m = 10 kg and g = 9.81 m/s^2 in formula 22 gives G = 98.1 N.
Substitution of F = G = 98.1 N and V = 0.8 m/s in formula 19 gives P = 78.48 W.

Suddenly the cord breaks so the bucket falls down. The friction in between the bucket and the air can be neglected.

b) What is the speed of the bucket after 2 seconds?

The downward speed is taken as the positive direction. So the upward speed with which the bucket was lifted when the chord broke has to be seen as a negative starting speed V_0. Substitution of V_0 = -0.8 m/s, a = g = 9.81 m/s^2 and t = 2 s in formula 23 gives V = 18.82 m/s.

c) How far has the bucket fallen after two seconds?

Substitution of V_0 = -0.8 m/s, a = g = 9.81 m/s^2 and t = 2 s in formula 24 gives s = 18.02 m.

A windmill rotor is rotating with a constant rotational speed of 120 rpm and it is loaded such by the generator that it is producing a mechanical power of 2 kW on the rotor shaft.

d) What is the torque in the rotor shaft?

Substitution of n = 120 rpm in formula 28 gives that ω = 12.566 radian/s.

Formula 25 can be written as:

$Q = P / \omega$ (Nm) (34)

2 kW = 2000 W. Substitution of P = 2000 W and ω = 12.566 radian/s in formula 34 gives Q = 159.16 Nm.

Something is going wrong with the transmission in between the rotor and the generator and suddenly the generator is disconnected from the rotor. The mass moment of inertia of the rotor I_m = 80 kgm^2. The rotor starts to accelerate and it is assumed that the moment which is supplied by the rotor blades is constant (in reality it decreases with increasing speed but this makes the example too complicated).

e) What is the rotational speed in rpm of the rotor after five seconds?

Formula 30 can be written as:

$d\omega/dt = Q / I_m$ (radian/s^2) (35)

Substitution of Q = 159.16 Nm and I_m = 80 kgm^2 in formula 35 gives $d\omega/dt$ = 1.99 radian/s^2. Substitution of ω_0 = 12.566 radian/s, $d\omega/dt$ = 1.99 radian/s^2 and t = 5 s in formula 32 gives ω = 22.516 radian/s.

Formula 28 can be written as:

$n = 30 * \omega / \pi$ (rpm) (36)

Substitution of ω = 22.516 radian/s in formula 36 gives n = 215 rpm.

f) How many revolutions has the rotor made after 5 seconds?

Substitution of ω_0 = 12.566 radian/s, t = 5 s and $d\omega/dt$ = 1.99 radian/s^2 in formula 33 gives α = 87.705 radian. One revolution = 360° = 2 π radian, so 1 radian = 1 / 2 π revolution. So 87.705 radian = 87.705 / 2 π = 13.959 revolution.

5 Potential energy

5.1 General

The potential energy of a body is normally defined with respect to the surface of the earth. But if a certain body can fall into a deep hole, the potential energy can also be defined with respect to the bottom of the hole. Bringing it back to the surface of the earth requires energy.

The same counts if water is pumped from a deep well up to a reservoir situated above the earth surface. In this case it is the difference H in between the water level in the well and the water level in the reservoir which determines the required potential energy to lift the water column.

First we look at a solid mass m which is situated at a height H above the surface of the earth. This mass has a certain weight G. The potential energy E_p of this mass is given by:

$$E_p = G * H \quad \text{(Nm)} \tag{37}$$

(22) + (37) gives:

$$E_p = m * g * H \quad \text{(Nm)} \tag{38}$$

All formulas given in chapter 4 for mechanical energy are also valid for potential energy because there is no fundamental difference if the force F is an external force or if it is supplied by the weight G of an object. However, there is a special form of potential energy to which some more attention will be paid and that is when the potential energy is available in water.

Water is supplied or needed as a constant stream and so it is not a point mass. For a stream of water one speaks about the flow q (m^3/s) which is the volume of water passing per second. Water has a density $\rho_w = 1000$ kg/m^3. The power which can be generated by falling water or which is required to lift water is called the hydraulic power P_{hyd}. P_{hyd} is given by:

$$P_{hyd} = \rho_w * g * H * q \tag{39}$$

In this formula the efficiency of the device to extract power from falling water or to lift water is not taken into account. More information about this subject concerning the lift of water with a piston pump is given in report KD 294 (ref. 5). Information about water pumping with an electricity generating windmill and a pump with an electric motor is given in report KD 341 (ref. 6).

5.2 Example

Someone has the idea to store the energy generated by a 2 MW windmill by lifting an heavy weight in an old straight mine shaft. The mine shaft has a depth of 200 m and a diameter of 8 m. The idea is to make a drum with a diameter of 7 m and a height of 5 m and to fill this drum with water. The total mass of drum and water is about 250000 kg and the drum is hanging on a strong steel cable which is connected to a winch situated above the mine shaft.

a) How much energy in kWh can be stored in this device?

Substitution of m = 250000 kg, g = 9.81 m/s^2 and H = 200 m in formula 38 gives E_p = 490500000 Ws. 1 Ws = 0.0000002778 kWh so 490500000 Ws = 136 kWh. If the windmill would have an average power of 550 kW, the energy production of only about a quarter of an hour can be stored (if the efficiency of the winch is taken 1). So it can be doubted is this way of storing energy is a good option for a big windmill.

6 Kinetic energy

6.1 General

The kinetic energy E_k of a point mass m moving with a speed V is given by:

$E_k = ½ m * V^2$ (kgm²/s² or Ws) (40)

With air flow, it is conventional to consider the energy of a mass flow per second ϕ_m, passing through a certain area A. With $\phi_m = m / t$, formula 40 can be written as:

$E_k = ½ \phi_m * t * V^2$ (Ws) (41)

The mass flow per second through the area A is given by:

$\phi_m = V * A * \rho$ (kg/s) (42)

The density ρ of air is about 1.2 kg/m³ at a temperature of 20° C at sea level.

(41) + (42) gives:

$E_k = ½\rho V^3 * A * t$ (Ws) (43)

Power is the energy per second and is found by dividing E_k by the time t so:

$P_k = ½\rho V^3 * A$ (W) (44)

Not all the kinetic power available in the wind can be extracted by a windmill. More information about this subject is given in report KD 35 (ref. 3).

Kinetic energy can also be stored in a rotating body. Similar to formula 40 for a linear movement, a formula can be derived for the kinetic energy of a rotating body. This formula is:

$E_k = ½ I_m * \omega^2$ (kgm²/s² or Ws) (45)

The mass moment of inertia I_m and the angular velocity ω are already defined in chapter 5.

6.2 Examples

A mass of 20 kg falls down from a tower from stand still position.

a) What is the kinetic energy after three seconds (the friction in between the mass and the air can be neglected).

Substitution of $V_0 = 0$ m/s, a = g = 9.81 m/s² and t = 3 s in formula 23 gives V = 29.43 m/s.
Substitution of m = 20 kg and V = 29.43 m/s in formula 40 gives $E_k = 8661$ Ws.

b) How much kinetic power in kW is available in wind flowing through a disk with a diameter of 10 m for a wind speed of 12 m/s?

A disk with a diameter of 10 m has an area A = 78.54 m². Substitution of ρ = 1.2 kg/m³, V = 12 m/s and A = 78.54 m² in formula 44 gives $P_k = 81430$ W = 81.43 kW.

A massive steel disk with a diameter of 2 m and a thickness of 0.25 m is rotating at a speed of 5000 rpm. The disk is very well balanced and rotates in a vacuum to minimise friction losses.

c) How much kinetic energy is available in the disk?

Substitution of n = 5000 rpm in formula 28 gives $\omega = 523.6$ radian/s.

The mass moment of inertia I_m for a disk with a diameter d is found in an handbook. It is given by:

$$I_m = 1/8\ m * d^2 \qquad (kgm^2) \qquad (46)$$

The mass m of a steel cylinder with a diameter d, a height h and a density ρ_{st} is given by:

$$m = \rho_{st} * \pi/4 * d^2 * h \qquad (kg) \qquad (47)$$

(46) + (47) gives:

$$I_m = 1/32\ \rho_{st} * \pi * d^4 * h \qquad (kgm^2) \qquad (48)$$

(45) + (48) gives:

$$E_k = 1/64\ \rho_{st} * \pi * d^4 * h * \omega^2 \qquad (Ws) \qquad (49)$$

Steel has a density of 7800 kg/m^3. Substitution of ρ_{st} = 7800 kg/m^3, d = 2 m, h = 0.25 m and ω = 523.6 radian/s in formula 49 gives E_k = 41987827 Ws = 116.31 kWh.

Suppose one is extracting a constant power of 500 W from this flywheel.

d) How long will it take until the flywheel comes to a stand still?

$$P_k = E_k / t \qquad (W) \qquad (50)$$

Formula 50 can be written as:

$$t = E_k / P_k \qquad (s) \qquad (51)$$

Substitution of E_k = 419878827 Ws an P_k = 500 W in formula 51 gives t = 839758 s or t = 233.27 hour, so almost ten days.

This example demonstrates that a lot of energy can be stored in a big flywheel especially if it is rotating at a high rotational speed. A problem is to prevent vibrations due to imbalance. If the disk would brake due to high centrifugal or imbalance forces, all stored energy comes free suddenly and this is very dangerous. In practice one is not using a massive steel disk for modern flywheels but the disk is build up from a light carbon fibre centre and a heavy ring to minimise the total weight but to create a large moment of inertia.

A problem is the friction of the flywheel with the surrounding air. This can be solved if the flywheel is running in a vacuum. The bearings must be of very good quality with only very low internal friction. Another problem is to find a generator which can add or extract energy for a large speed range of the flywheel. So building a good flywheel requires a high level of technology but it is a realistic option to store energy in the future.

6 References

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