Ideas about a self starting, 3-bladed H-Darrieus rotor for water pumping

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1 Introduction

The Darrieus rotor is a vertical axis wind turbine (VAWT) provided with two or more blades having an aerodynamic airfoil. The blades of the original Darrieus rotor are bent into a chain line and are connected to the hub at the upper and lower side. However, also Darrieus rotors with straight blades (H-Darrieus) have been developed which therefore have large hubs provided with spokes. The energy is taken from the wind by a component of the lift force L working in the direction of rotation. The same principle is used for a horizontal axis wind turbine (HAWT).

A Darrieus rotor has some advantages and many disadvantages. An overview of advantages and disadvantages is given in my report KD 215 (ref. 1). The main advantage is that a Darrieus rotor accepts wind from any direction so it needs no device which turns the rotor in the wind. One of the main disadvantages is that the rotor has a negative torque coefficient for low tip speed ratios and that therefore the generator must also work as a motor to start the rotor and reach the region of the C_q-λ curve where the torque coefficient is positive.

The rotor can be made self starting if the blades are oscillating in such a way that the front blades have a positive blade angle and that the back blades have a negative blade angle. This was realised in the Gyromill which has been tested long ago on the test field of Energie Anders in Hoek van Holland. A similar principle is also used in the Voith Schneider ship propulsion. It was already patented in 1931. A very nice description of the functioning of the Voith Schneider system is given on: www.voithturbo.de/545950.htm. For the Gyromill, the oscillation is realised by an eccentric. The centre of the eccentric has a small eccentricity e with respect to the rotor axis. Each blade is connected to the eccentric by a long lever. The position of the eccentric must be such that e is in line with the wind direction. So now a vane is needed to realise this and the main advantage that the rotor accepts wind from any direction, is lost.

I believe that for electricity generation one can better use a HAWT. However, for water pumping, using a slow running centrifugal pump, a Darrieus rotor might be an acceptable option because no gearing in between the windmill rotor and the centrifugal pump is needed. But it would be stupid to need an electric motor to start the Darrieus rotor and therefore I have investigated if there is a way to make the rotor self starting without loosing the main advantage that the rotor accepts wind from any direction.

2 Making a H-Darrieus rotor self starting

The first choice made is to use a H-Darrieus rotor because for a H-Darrieus rotor a blade is straight and has the same radius R for the whole blade. Therefore it is possible to oscillate the blade. The original Darrieus rotor has blades bent in a chain line. The original H-Darrieus rotor has straight blades with a fixed blade angle $\beta = 0°$. If the rotor is turning at the design tip speed ratio each blade is streamed at a positive angle of attack if it is at the front side and at a negative angle of attack if it is at the back side. Therefore one has to use a symmetrical airfoil. The angle of attack α is created because of the blade speed U and the wind speed in the rotor plane which is about 2/3 V if maximum power is extracted. This was derived by Betz already in 1926. Betz also found a value for the theoretical maximum C_p of 16/27 which can be realised. The derivation of the Betz coefficient is given in chapter 4.2 of my report KD 35 (ref. 2). This report deals with HAWT's but the derivation of the Betz coefficient is also valid for VAWT's.

As explained in chapter 1, a way to make the rotor self starting is to make the blade oscillate such that the blade angle β is positive if the blade is at the front side of the rotor and negative if the blade is at the back side. It might be possible to realise this oscillation without using an eccentric. However, using no eccentric, introduces certain new problems.

If a blade oscillates, it must have a turning point. The best choice for the tuning point seems the aerodynamic centre which lies about at the quart-chord point which lies at 0.25 c from the nose of the airfoil and at the zero line of the airfoil. The aerodynamic moment is zero around the aerodynamic centre if the angle of attack α is smaller than about 12° (depending on the airfoil and the Reynolds number). If the airfoil is made out of massive wood it will have a centre of gravity which lies at about 0.4 c from the airfoil nose. The centrifugal force at the airfoil exerts in the centre of gravity and will therefore have a tendency to turn a blade to a smaller blade angle. This is the good direction if the blade is at the back side of the rotor but the wrong direction if the blade is at the front side. So the centrifugal force can't be used to activate the oscillation and it is even necessary to cancel the effect of the centrifugal force.

This can be done by creating a mass distribution in the airfoil such that the centre of gravity coincides with the turning point but this requires an airfoil with a hollow tail section. Another option is to use a rotor with two or four blades and to connect two opposite blades by a long strip such that the centrifugal force in one blade is counterbalanced by the centrifugal force in the opposite blade. However, this creates new turning points for the hinges and the long strips will give inertia effects. As I prefer a 3-bladed rotor, this option is cancelled. So it is decided to use a wooden blade with a hollow tail section. The airfoil is made out of two halves which are glued together. An advantage of this construction is that now it is also possible to glue a steel shaft in the blade for the turning point at the quart chord point.

The bending moment caused by the centrifugal force in a straight blade is rather large and therefore a rather thick airfoil is chosen. Provisionally the airfoil NACA 0015 is chosen. This airfoil has a maximum thickness of 15 % of the chord at 0.3 c. The aerodynamic characteristics of the NACA 0015 are given at page 3-94 of report R-443-D (ref. 3) for a large range of Reynolds values. As this report is no longer available, a scan has been made of the characteristics and this scan is given as figure 1. The measuring points aren't given in R-443-D so it isn't possible to make more accurate graphs using the measuring points.

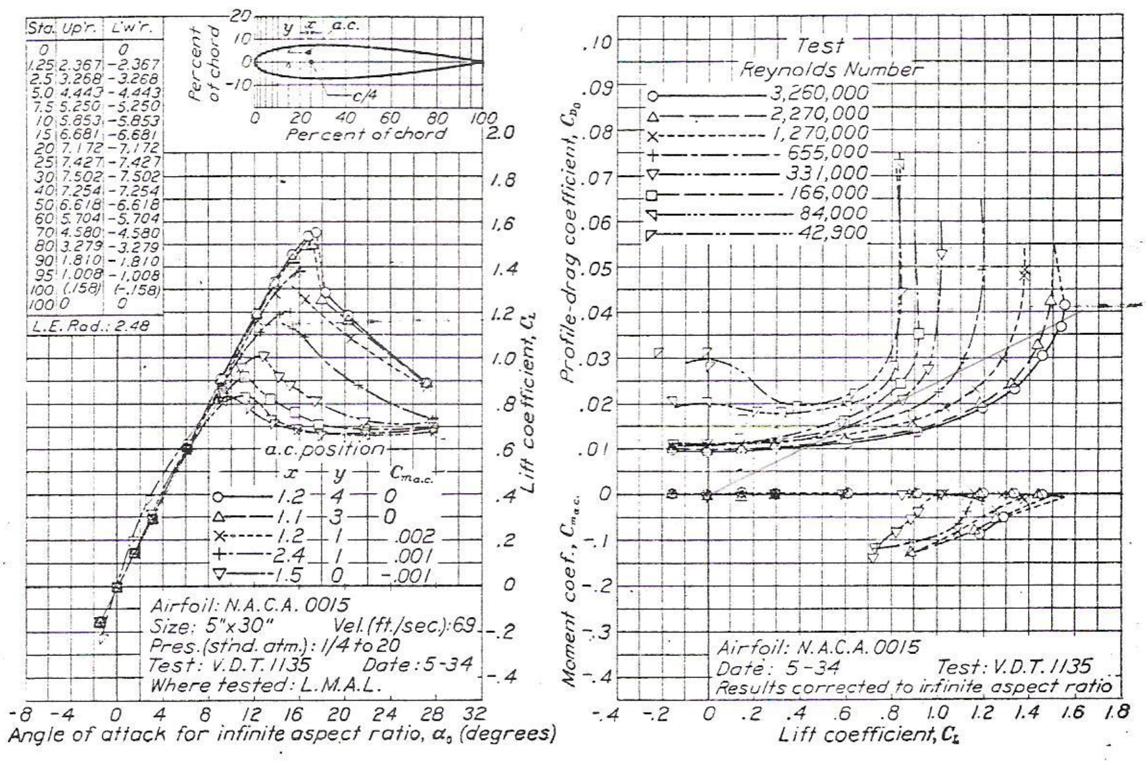

Fig. 1 Aerodynamic characteristics of the NACA 0015 airfoil.

The left figure gives the C_l-α curves for different Reynolds values and a table with the airfoil geometry. The right figure gives the C_l/C_d curve and the C_l/C_{mac} curve. In the C_l-α curve it can be seen that the curves for different Reynolds value almost coincide for -2° < α < 8°. Stalling starts about at α = 10° for low Reynolds values but stalling is extended to much higher angles for high Reynolds values. If the Reynolds value is 3.31 * 10^5, the airfoil starts stalling at about α = 12° and the maximum C_l value for this Reynolds value is about 1.0.

In the C_l/C_d graph it can be seen that the drag coefficient for a certain lift coefficient is larger as the Reynolds number is lower. The rotor will have the maximum C_p if the blades are designed such that the airfoil is used at the optimum lift coefficient. The optimum lift coefficient is the lift coefficient for which the C_l/C_d ratio is minimal. The minimal C_l/Cd ratio can be found easily if a straight line is drawn through the origin and if this line is touching the C_l/C_d curve for a certain Reynolds value. Assume the Reynolds value is 3.31 * 10^5. A straight line through the origin touches the C_l/C_d curve for Reynolds = 3.31 * 10^5 about at a C_l value C_l = 0.7. So C_l = 0.7 is the optimum lift coefficient for Reynolds = 3.31 * 10^5. In the C_l-α curve it can be seen that α = 7° for C_l = 0.7 and Reynolds = 3.31 * 10^5. So α = 7° is the optimum angle of attack for this Reynolds value. The C_d/C_l ratio is about 0.025 for α = 7°.

The C_l/C_{mac} curves are only given for Reynolds = 3.31 * 10^5 or higher. In the C_l/C_{mac} curve for Reynolds = 3.31 * 10^5 it can be seen that C_{mac} is zero for C_l values smaller than about 1 corresponding to α < 12° but that for larger angles α, C_{mac} becomes negative. As C_{mac} is defined positive if it is working right hand, a negative C_{mac} means that the aerodynamic moment has a tendency to decrease the angle of attack α. This can also be seen as if the point of exertion of the lift force acting on the airfoil is lying behind the aerodynamic centre for large angles α. So the negative aerodynamic moment at large angles α can be used to activate the oscillating movement of the blades if the rotor is standing still or rotating only slowly. Unfortunately the aerodynamic characteristics of the NACA 0015 are only given up to angles α of about 28°. So it isn't possible to calculate the aerodynamic moment for larger angles α. But if the rotor starts rotating, at a certain tip speed ratio the angle of attack α will become smaller than 12°. From that tip speed ratio, the aerodynamic moment becomes zero and therefore the blade oscillation stops and the rotor works as a normal H-Darrieus rotor with fixed blades at β = 0°.

The blade must have stops for the oscillation of the blade around the quart chord point in both directions. Without stops, a blade of a non rotating rotor will always take a position such that the angel of attack α is almost zero. It is estimated that the starting torque of the rotor is large enough if the stops are positioned such that the blade angle β can vary in between -10° and +10°. So the angle of attack α is 80° for the front and the back position of a blade if the rotor isn't rotating. A centrifugal pump has no starting torque and the only required torque is for some friction in the bearings of the vertical shaft in between rotor and pump. The stops must have some elasticity to prevent shocks if the blade is oscillating at low rotational speeds of the rotor.

3 Determination of the rotor geometry of a H-Darrieus

As far as I know there is no simple aerodynamic theory to design a H-Darrieus rotor. For a HAWT, the design theory is given in my report KD 35 (ref. 2) and may be with some modification, it is possible to use the formulas of this theory also for a H-Darrieus rotor.

For designing of a HAWT, six parameters have to be determined. These six parameters are given in the beginning of chapter 5.2 of KD 35 and are: Rotor tip radius R, number of blades B, design tip speed ratio λ_d, blade aerofoil as a function of the local radius r, blade chord c as a function of the local radius and blade setting angle β as a function of the local radius r. For a HAWT, the first four parameters are chosen and the last two parameter are calculated as a function of the local radius r using the formulas given in chapter 5.3 of KD 35.

For a HAWT, the swept area A is only determined by the rotor radius R. For a normal H-Darrieus rotor the swept area is determined by the rotor radius R and by the height H of the turbine. A is given by:

$A = 2\ R * H$ (m^2) (1)

For a normal H-Darrieus rotor, the blade angle β is zero so it can't be calculated. In stead of the calculation of β one can calculate the design tip speed ratio λ_d which matches best with $\beta = 0°$. Assume the following parameters are chosen:
R = 1 m, H = 1.5 m (so A = 3 m^2), number of blades B = 3, airfoil NACA 0015, $\beta = 0°$. Next the optimum tip speed ratio λ_d and the optimum chord c have to be calculated.

For a HAWT, the angle ϕ in between the direction of the relative wind W and the rotor plane is given by formula 5.3 of KD 35. The derivation of this formula is very complex but if the wake rotation is neglected, a simplified formula can be derived using figure 5.1 of KD 35. The difference in between the correct formula 5.3 and the simplified formula 5.9 is that the factor 2/3 is at a different position. The effect of the different position of the factor 2/3 is illustrated in figure 5.2 of KD 35. In this figure it can be seen that there is almost no difference in between both formulas if λ_{rd} is larger than 2. For a H-Darrieus rotor there is only one value for R so in stead of λ_{rd} we can use λ_d. The optimum value for λ_d will certainly be much larger than 2, so it is allowed to use formula 5.9. Formula 5.9 of KD 35 is now copied as formula 2 but in stead of λ_{rd} we use λ_d.

$\phi = \arctan 2/3 / \lambda_d$ (°) (2)

A difference in between a HAWT and a H-Darrieus is that all blades of a HAWT turn in the same plane. For a H-Darrieus a blade can be in the front position or in the back position and in all positions in between front and back. It is assumed that the wind speed is 2/3 V at the axis of the H-Darrieus but because of the expansion of the wake (see KD 35 figure 4.1) it will be slightly higher than 2/3 V at the front position and slightly lower than 2/3 V at the back position. This effect is neglected and so it is assumed that the absolute wind speed in the rotor plane is 2/3 V for any position of the blade. So figure 4.4 of KD 35 is valid if the blade is at the front position and if it is at the back position. For other positions of the blade one gets a different figure because the blade speed U isn't perpendicular to the absolute wind speed in the rotor plane. This effect is investigated in chapter 4. At this moment only the front and back position of the blade are taken into account.

The relation in between α, β and ϕ is given by formula 5.2 of KD 35 and illustrated by figure 3.2. This formula is copied as formula 3.

$\beta = \phi - \alpha$ (°) (3)

For a H-Darrieus rotor $\beta = 0°$ so this means that $\phi = \alpha$. In chapter 2 it was determined that the optimum α for the NACA 0015 airfoil is 7° for Reynolds = $3.31 * 10^5$. The real Reynolds value can only be calculated if the chord and the relative wind speed W are known but at this moment it is assumed that Reynolds = $3.31 * 10^5$ is about correct. In chapter 2 we found that the optimum α for NACA 0015 for Reynolds = $3.31 * 10^5$ is 7°. So as $\phi = \alpha$, the optimum ϕ is also 7°. The real angle α varies in between the maximum value at the front and the back position and zero at the left and the right position. So α is chosen somewhat larger than the optimum value at the front and back position to get an acceptable average over a whole revolution. Assume it is chosen that $\alpha = \phi = 9°$ for the front position and that $\alpha = \phi = -9°$ for the back position. In figure 1 it can be seen that $C_l = 0.85$ for Reynolds = $3.31 * 10^5$ and $\alpha = 9°$. Next a certain value of λ_d is estimated and the corresponding value of ϕ is calculated.

Assume λ_d = 4. Substitution of this value in formula 2 gives ϕ = 9.46°. This is too large.
Assume λ_d = 5. Substitution of this value in formula 2 gives ϕ = 7.59°. This is too small.
Assume λ_d = 4.2. Substitution of this value in formula 2 gives ϕ = 9.02°. This is about correct. So the optimum design tip speed ratio of a H-Darrieus rotor using blades with a NACA 0015 airfoil and a fixed value β = 0° is about 4.2.

The last parameter to calculate is the blade chord c. For a HAWT, the blade chord c is given by formula 5.4 of KD 35. In this formula it can be seen that a certain product of C_l * B has to be realised. For a HAWT all blades are active during a full revolution of the rotor and at a certain radius r, the lift coefficient C_l is constant for every position of the blade. For a H-Darrieus, C_l is maximal for the front and the back position but zero for the left and the right position. However, the average C_l value during one revolution is higher than half the peak value. It is assumed that the average C_l value is a factor 0.6 of the peak value. In chapter 4 it will be checked if this assumption is correct. Formula 5.4 of KD 35 is now copied as formula 4 but R is used in stead of r and 0.6 C_l is used in stead of C_l.

$$c = 8 \pi R (1 - \cos\phi) / (0.6 * B * C_l) \quad \text{(m)} \qquad (4)$$

Substitution of R = 1 m, ϕ = 9.02°, B = 3 and C_l = 0.85 in formula 4 gives that c = 0.203 m rounded to 0.2 m = 200 mm. This seems acceptable for a blade with a length H = 1.5 m which is supported at the upper and the lower side. At this moment it is expected that a wooden blade with a steel pin inside is strong and stiff enough for a chord of 200 mm, at least, if the maximum rotational speed is limited at high wind speeds. But strength and stiffness calculations have to be executed before a prototype can be designed and built.

Formula 5.5 out of KD 35 for the calculation of the Reynolds value can also be used for a H-Darrieus. This formula is copied as formula 5 but R_e is taken in stead of $R_{e\,r}$ and λ_d is taken in stead of $\lambda_{r\,d}$.

$$R_e = 0.667 * 10^5 * V * c * \sqrt{(\lambda_d^2 + 4/9)} \quad \text{(-)} \qquad (5)$$

It is assumed that the minimum wind speed for which the rotor must work optimally is 5 m/s. Substitution of V = 5 m/s, c = 0.2 m and λ_d = 4.2 in formula 5 gives that $R_e = 2.84 * 10^5$. This is somewhat lower than the Reynolds value of $3.31 * 10^5$ which was used for the determination of the optimum value of λ_d but I think that the calculated values for λ_d and c are good enough for making a prototype of the rotor and test if oscillating of the blade angle in between -10° and +10° gives a sufficiently large starting torque coefficient. The calculation also shows that the chord must be rather large otherwise the Reynolds value becomes too low to get an acceptable low C_d/C_l ratio at low wind speeds.

If this idea of an oscillating blade works, one of the disadvantages of a H-Darrieus rotor is neutralised. However, many other disadvantages remain. One of the most serious disadvantages is that it isn’t possible to limit the rotational speed and thrust at high wind speeds aerodynamically. The only option is a brake which slows down the rotor up to the region of the C_q-λ curve where the torque is low or to stop the rotor completely. For a water pumping windmill this means that an extra disk brake is required on the vertical shaft. This brake should be spring loaded and released by a mechanism activated by the centrifugal force. Technically this seems possible but it is an extra investment and the brake has to be released manually once the high wind speeds for which the brake was activated, are over. So I doubt if this water pumping windmill with a special self starting H-Darrieus rotor is a realistic option. I will certainly not develop it myself.

4 Following a blade during one revolution

Up to now, the angle of attack α has only been determined for the front and the back position of a blade and for these positions α is maximal. For formula 4 it was assumed that the average C_l value is a factor 0.6 of the peak value. To be sure that this is correct, a blade is followed during one revolution and a speed diagram is made every 30°. The positions are numbered 1-12. The front position is no. 1. The rotor is turning right hand seen from above. The wake is somewhat expanding at the left and right positions but it is assumed that the absolute wind speed in the rotor plane 2/3 V, is in line with the direction of the undisturbed wind speed V for every position. The blade speed is called U. The relative wind speed is called W. It is assumed that λ = 4.2 so U = 4.2 * V or U = 6.3 * 2/3 V and that the length of 2/3 V = 10 mm on the scale of the drawing. So U = 63 mm on the scale of the drawing. The airfoil is drawn at positions 1, 5 and 9. The speed diagrams for the twelve positions are given in figure 2.

The angle α at point 1 is the same as at point 7. The relative wind speeds W for the left positions 2, 3, 4, 5 and 6 are somewhat smaller than for right the positions 8, 9, 10, 11 and 12. The angles α for a certain left position is somewhat larger than for the opposite right position.

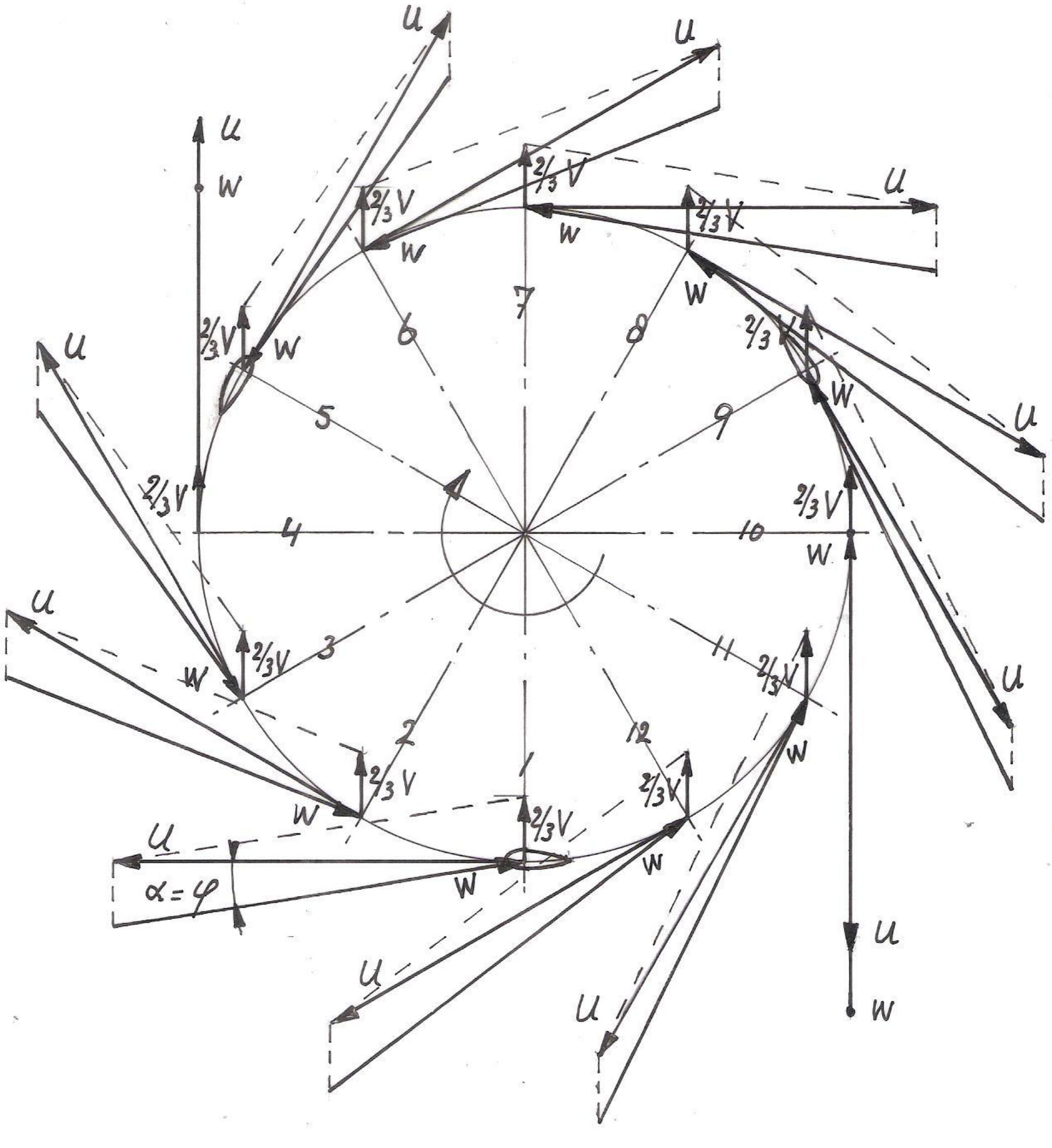

Fig. 2 Speed diagrams of a H-Darrieus rotor for twelve positions

The angle α has been measured in figure 2 for every position and the result is given in table 1. The angle α is zero for positions 4 and 10. The angle is positive for positions 11, 12, 1, 2 and 3. The angle α is negative for the positions 5, 6, 7, 8 and 9. The angle α is 9° for position 1 and -9° for position 7 but that was the design condition. The lift coefficient C_l for α = 9° is 0.85. For a symmetrical airfoil the C_l-α curve is rota symmetric for negative angles α. So this means that C_l = - 0.85 for α = -9°. However, a negative C_l has a positive influence on the rotor torque for positions 5, 6, 7, 8 and 9 and therefore the C_l value is taken positive for negative angles α. The C_l-α curve is about a straight line for 0° < α < 9°. So the C_l value for smaller angles can easily be calculated if the C_l value at α = 9° is known. The calculated C_l values are also given in table 1.

Position nr.	α (°)	C_l (-)
1	9	0.85
2	8.5	0.80
3	5.5	0.52
4	0	0
5	-5.2	0.49
6	-8.2	0.77
7	-9	0.85
8	-7.2	0.68
9	-3.2	0.30
10	0	0
11	3.7	0.35
12	7.2	0.68

Table 1 Measured angles α and calculated C_l values for 12 positions

The average C_l value $C_{l\ av}$ for all twelve sections is the sum of all lift coefficients divided by 12. This gives $C_{l\ av}$ = 6.29 / 12 = 0.52. So the average lift coefficient is a factor 0.52 / 0.85 = 0.61 times the peak value at position 1. So formula 4 is about right.

5 Estimation of the C_p-λ and C_q-λ curves

For checking of the matching with a certain load, C_p-λ and C_q-λ curves are needed. I could find no reliable measurements of a H-Darrieus rotor on the Internet (I found only measurements for a rotor for which an asymmetrical airfoil was used but this is absolutely wrong as it gives a lot of drag when the blade is at the backside of the rotor). So the C_p-λ and C_q-λ curves are estimated. The C_p-λ and C_q-λ curves will differ for different Reynolds numbers as the drag coefficient of a symmetrical airfoil increases strongly at low Reynolds numbers. So the estimated C_p-λ and C_q-λ curves are only valid if the Reynolds number is larger than about $3 * 10^5$.

First the characteristics are estimated for a H-Darrieus rotor with fixed blades. It is assumed that $C_{p\ max}$ = 0.35 for λ_d = 4.2 and that the C_q-λ curve is negative for 0 < λ < 1.4. Next it is assumed that the pitch mechanism increases the C_q value for 0 < λ < 2.8. The estimated C_p-λ and C_q-λ curves are given in figure 3 and 4.

The P-n and Q-n curves for different wind speeds can be derived from the C_p-λ and C_q-λ curves but this derivation is out the scope of this report. The method is given in chapter 8 of KD 35. One needs the C_p-λ and C_q-λ curves and the formulas for P, Q and n.

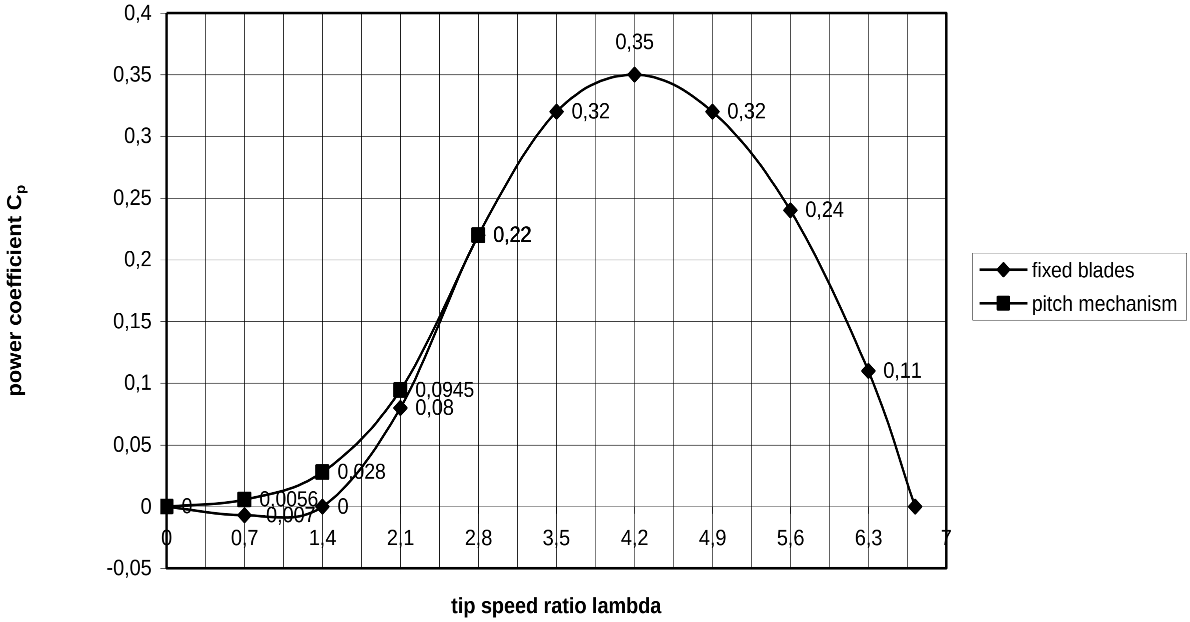

Fig. 3 Estimated C_p-λ curve H-Darrieus rotor with fixed blades and with a pitch mechanism

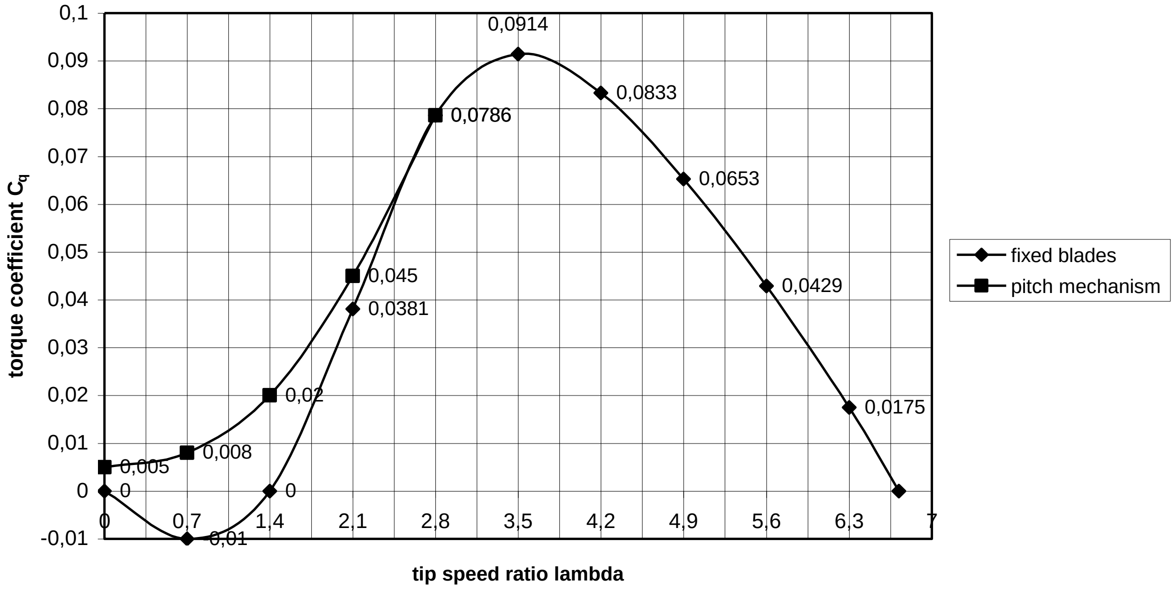

Fig. 4 Estimated C_q-λ curve H-Darrieus rotor with fixed blades and with a pitch mechanism

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