Calculations executed for the 3-bladed rotor of the VIRYA-3.3S windmill (λ_d = 4.5, steel blades) using a 34-pole PM-generator for coupling to a pump motor

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1 Introduction

The VIRYA-3.3S windmill is developed for manufacture western countries and in developing countries. The VIRYA-3.3S has a 3-bladed rotor with galvanised steel blades and a design tip speed ratio $\lambda_d = 4.5$. The blades are the same as the blades of the VIRYA-3S rotor described in report KD 559 (ref. 1) but the spokes are longer and wider. The VIRYA-3.3S rotor generates a higher power at the rated rotational speed of 176.5 rpm than the VIRYA-3S rotor.

The generator of the VIRYA-3.3S is made from the housing of an Indian 6-pole asynchronous motor frame size 112F-6P. It has an armature with 34 poles and a stator with 36 slots. The generator is described in report KD 560 (ref. 2). The generator makes use of the original motor shaft which has a shaft end with a diameter of 28 mm, a length of 60 mm, an 8 mm key and a central threaded hole M10.

The VIRYA-3.3S is meant to be coupled to the asynchronous motor of a centrifugal pump if the generator has a high voltage winding. The frequency f = 50 Hz at a rotational speed n = 176.5 rpm. The generator can also be used for 24 V battery charging if the generator has a low voltage winding rectified in star. A sketch of the rotor is given in appendix 1.

The head is about the same as the head of the VIRYA-3S but the eccentricity and the vane blade are enlarged. The alternative tubular tower of the VIRYA-3S will be used.

2 Description of the rotor of the VIRYA-3.3S windmill

The 3-bladed rotor of the VIRYA-3.3S windmill has a diameter D = 3.3 m and a design tip speed ratio $\lambda_d = 4.5$. Advantages of a 3-bladed rotor are that the gyroscopic moment in the rotor shaft isn’t fluctuating and that a 3-bladed rotor looks nicer than a 2-bladed rotor.

The rotor has blades with a constant chord and no twist and is provided with a 7.14 % cambered airfoil. A blade is made of a galvanised steel strip with dimensions of 208 * 1250 * 2.5 mm and twelve blades for four rotors can be made from a standard sheet of 1.25 * 2.5 m with almost no material waste. Because the blade is cambered, the chord c is a little less than the blade width, resulting in c = 205 mm = 0.205 m. The rotor is drawn of A1 format and the drawing number is 1404-01. The blade press to camber the blades is also drawn on A1 format and the drawing number is 1405- 01.

The blades are connected to each other by a spoke assembly which is made of three spokes welded together at the centre. Each spoke has a length of 650 mm, a width of 60 mm and a thickness of 10 mm. As the spokes are rather thin, the rotor is rather flexible and this flexibility neutralises vibrations caused by wind turbulence.

The overlap in between spoke and blade is 250 mm resulting in a rotor diameter of 3.3 m and in a free blade length of 1 m. This free blade length, in combination with a sheet thickness of 2.5 mm, is expected to be short enough to prevent flutter at high wind speeds. However, the connection in between the blade and the end of the central strip must be very tight. This is realised by bevelling the left and right backside of the spokes under an angle of 3° and by an extra cambered sheet size 3 * 50 * 50 mm which is placed under the head of the outer connecting bolt. A sketch of the rotor is given in appendix 1.

The spokes are twisted in between the hub and the blade root to realise the correct blade setting angle. Three M10 * 40 mm bolts are used for connection of blade and strip. These bolts can also be used for connection of the balancing weights.

The stainless steel hub has a diameter of 60 mm and a length of 60 mm and is provided with an internal key groove. The spoke assembly is clamped in between the hub and a clamping disk to prevent that the welds are loaded by a bending moment. Four hexagon socket head cap screws M10 * 40 mm are used for clamping. The mass of the whole rotor including the hub is about 25.5 kg which seems acceptable for a 3-bladed steel rotor with a diameter of 3.3 m.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 576) has its own formula numbering. Substitution of λ_d = 4.5 and R = 1.65 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 2.7273 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 3 and c = 0.205 m in formula (5.4) of KD 35 gives:

$C_l = 40.866\, r\, (1 - \cos\phi)$ (-) (4)

Substitution of V = 6 m/s and c = 0.205 m in formula (5.5) of KD 35 gives:

$R_{e\,r} = 0.820 * 10^5 * \sqrt{(\lambda_{r\,d}^2 + 4/9)}$ (-) (5)

The blade is calculated for six stations A till F which have a distance of 0.25 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 4). The Reynolds values for the stations are calculated for a wind speed of 6 m/s because this is a reasonable wind speed for a windmill with V_{rated} = 11 m/s (see chapter 5). Those airfoil Reynolds numbers are used which are lying closest to the calculated values.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 6 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	1.65	4.5	8.4	0.205	0.72	0.67	3.73	3.4	-0.3	-0.6	8.7	9.0	0.047
B	1.4	3.818	9.8	0.205	0.83	0.86	3.18	3.4	0.3	0.8	9.5	9.0	0.037
C	1.15	3.136	11.8	0.205	0.99	1.00	2.63	2.5	2.7	2.8	9.1	9.0	0.035
D	0.9	2.455	14.8	0.205	1.22	1.28	2.09	2.5	5.1	5.8	9.7	9.0	0.053
E	0.65	1.773	20.0	0.205	1.60	1.43	1.55	1.7	-	11.0	-	9.0	0.15
F	0.4	1.091	28.3	0.205	1.96	1.27	1.05	1.2	-	19.3	-	9.0	0.31

table 1 Calculation of the blade geometry of the VIRYA-3.3S rotor

No value for α_{th} and therefore for β_{th} is found for stations E and F because the required C_l values can't be generated. The theoretical blade angle β_{th} varies only in between 8.7° and 9.7°. If the blade angle is taken 9° for the whole blade, the linearised blade angles are lying close to the theoretical values. The spokes are twisted 9° to get the correct blade angle at the blade root.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.7 of KD 35 (for B = 3) en λ_{opt} = 4.5 and C_d/C_l = 0.04 gives $C_{p\ th}$ = 0.43.
The blade is stalling in between station E and F and the airfoil is disturbed because of the blade connection. Therefore not the whole blade length k = 1.25 m but only the part up to 0.1 m outside station F is used for the calculation of the C_p. This gives a reduced blade length k' = 1.15 m. Substitution of $C_{p\ th}$ = 0.43, R = 1.65 m and blade length k = k' = 1.15 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.39. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.39 / 4.5 = 0.0867.
Substitution of $\lambda_{opt} = \lambda_d$ = 4.5 in formula 6.4 of KD 35 gives λ_{unl} = 7.2.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

The blade angle is 9° for the whole blade. For a non rotating rotor, the angle of attack α is therefore 90° - 9° = 81°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 81° it can be read that C_l = 0.3. The whole blade is stalling during starting and therefore now the whole blade length k = 1.25 m is taken.
Substitution of B = 3, R = 1.65 m, k = 1.25 m, C_l = 0.3 and c = 0.205 m in formula 6 gives that $C_{q\ start}$ = 0.013. For the ratio in between the starting torque and the optimum torque we find that it is 0.013 / 0.0867 = 0.15. This is acceptable for a rotor with λ_d = 4.5.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \qquad (m/s) \qquad (7)$$

For the VIRYA-3D generator with 5RN90L04V housing, a sticking torque Q_s of 0.4 Nm has been measured if the rotor is not rotating. It is assumed that the sticking torque of the 32-pole generator with 112F-6P housing is somewhat higher. Assume Q_s = 0.6 Nm. Substitution of Q_s = 0.6 Nm, $C_{q\ start}$ = 0.013, ρ = 1.2 kg/m^3 and R = 1.65 m in formula 7 gives that V_{start} = 2.3 m/s. This is very low for a rotor with a design tip speed ratio of 4.5 and a rated wind speed of 11 m/s. Because the generator is not rectified for coupling to an asynchronous motor and rectified in star for 24 V battery charging, the unloaded Q-n curve is rising only a little at increasing rotational speed. The C_q-λ curve of a rotor equipped with cambered steel blades is rising rather fast for V = 2.3 m/s. Therefore, the real starting wind speed will be about the same as the calculated value.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing a S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 5). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-3.3S rotor are given in figure 1 and 2.

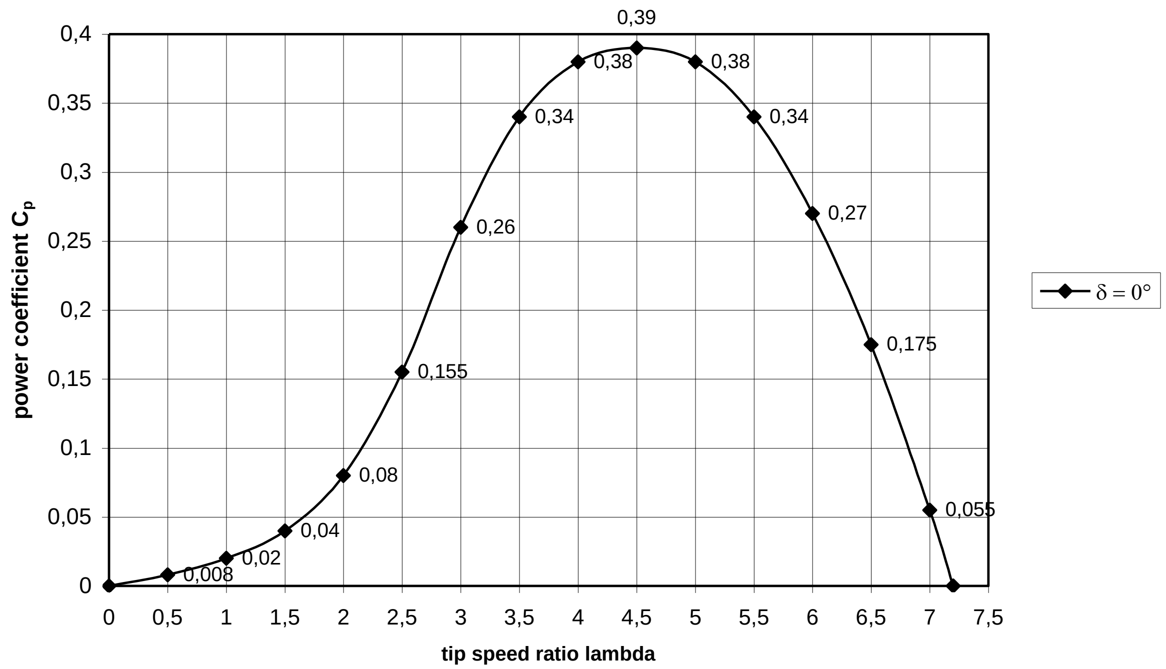

fig. 1 Estimated C_p-λ curve for the VIRYA-3.3S rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

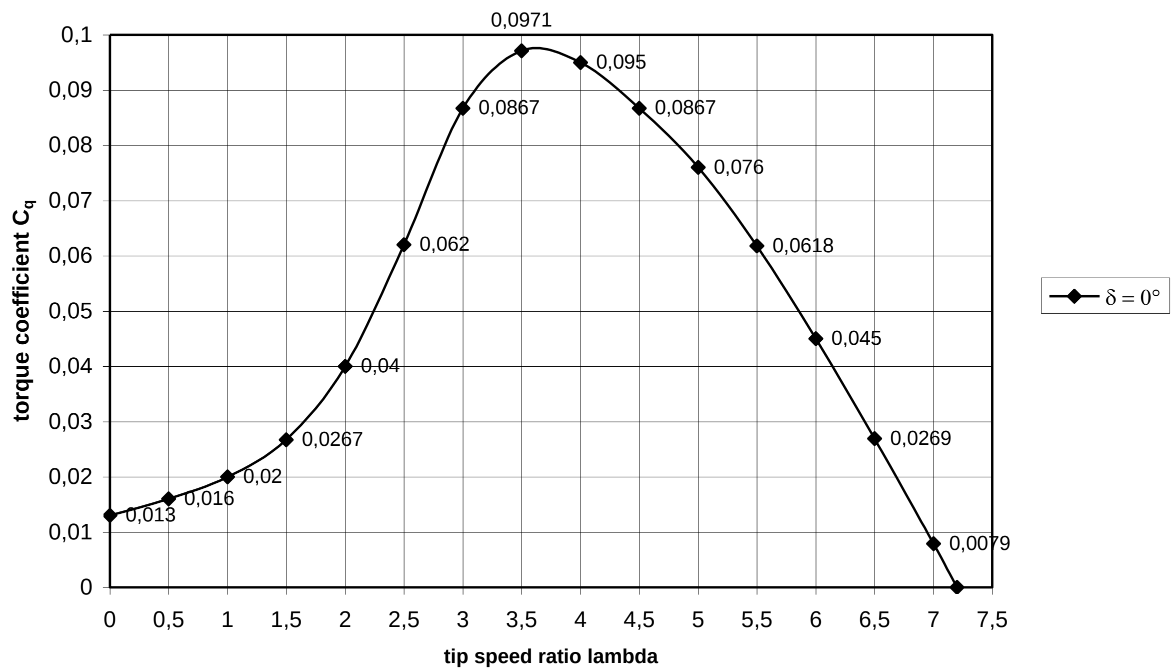

fig. 2 Estimated C_q-λ curve for the VIRYA-3.3S rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

5 Determination of the P-n curves and the optimum cubic line

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The C_p-λ curve is given in figure 1. The δ-V curve of the safety system depends on the vane blade mass per area. The vane blade is made of 9 mm water proof plywood with a density of about $0.6 * 10^3$ kg/m^3. In report KD 223 (ref. 6) a method is given to check the estimated δ-V curve and the estimated δ-V curve of the VIRYA-3.3D windmill is checked as an example. This windmill has also a vane blade made of 9 mm meranti plywood, so the δ-V curves of both windmills will be about the same. The estimated and calculated curves appear to lie very close to each other so it is allowed to use the estimated curve. The VIRYA-3.3D windmill has a rated wind speed of about 11 m/s. The estimated δ-V curve is given in figure 3.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 11 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 11 m/s will therefore also be valid for wind speeds higher than 11 m/s.

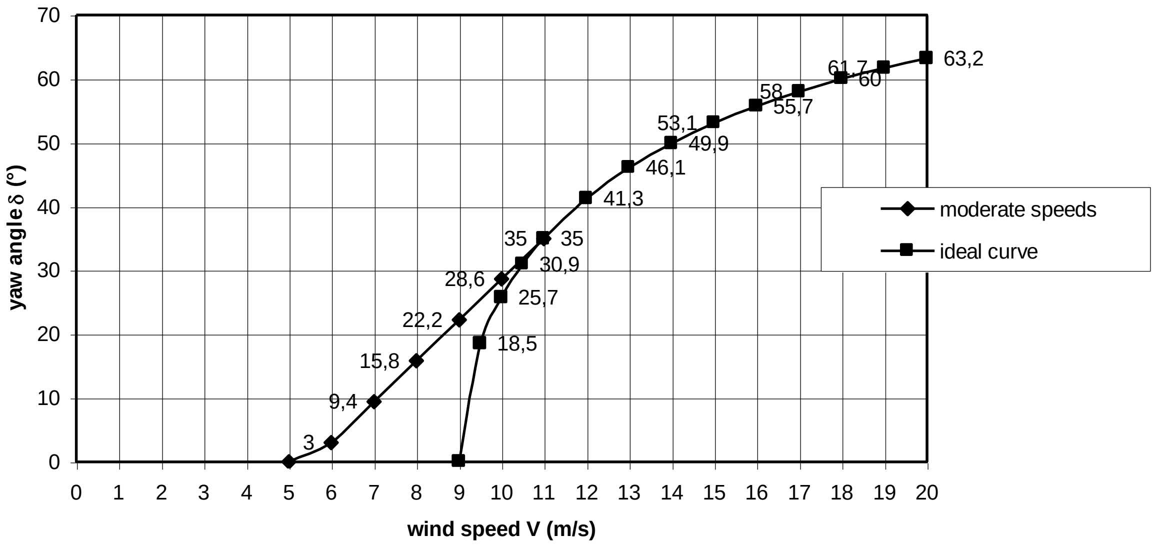

fig. 3 δ-V curve VIRYA-3.3S safety system with V_{rated} = 11 m/s

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator for a certain gear ratio i (the VIRYA-3.3S has no gearing so i = 1). Because we are especially interested in the domain around the optimal cubic line and because the P-n curve for low values of λ appears to lie very close to each other, the P-n curves are not determined for low values of λ. The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9, 10 and 11 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of R = 1.65 m in formula 7.1 of KD 35 gives:

$n_\delta = 5.7875 * \lambda * \cos\delta * V$ (rpm) (8)

Substitution of $\rho = 1.2$ kg / m^3 en R = 1.65 m in formula 7.10 of KD 35 gives:

$P_\delta = 5.1318 * C_p * \cos^3\delta * V^3$ (W) (9)

The P-n curves are determined for C_p values belonging to λ = 2.5, 3.5, 4.5, 5.5, 6.5 and 7.2. (see figure 1). For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 5, is taken into account. The result of the calculations is given in table 2.

		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 3°		V = 7 m/s δ = 9.4°		V = 8 m/s δ = 15.8°		V = 9 m/s δ = 22.2°		V = 10 m/s δ = 28.6°		V = 11 m/s δ = 35°	
λ (-)	C_p (-)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)
2.5	0.155	43.4	21.5	57.9	50.9	72.3	99.4	86.7	171.1	99.9	262.0	111.4	362.8	120.6	460.2	127.0	538.3	130.4	581.9
3.5	0.34	60.8	47.1	81.0	111.7	101.3	218.1	121.4	375.3	139.9	574.7	155.9	795.9	168.8	1010	177.8	1181	182.5	1276
4.5	0.39	78.1	54.0	104.2	128.1	130.2	250.2	156.0	430.5	179.9	659.2	200.5	912.9	217.0	1158	228.7	1355	234.7	1464
5.5	0.34	95.5	47.1	127.3	111.7	159.2	218.1	190.7	375.3	219.8	574.7	245.0	795.9	265.2	1010	279.5	1181	286.8	1276
6.5	0.175	112.9	24.2	150.5	57.5	188.1	112.3	225.4	193.2	259.8	295.8	289.6	409.6	313.5	519.6	330.3	607.8	339.0	657.0
7.2	0	125.0	0	166.7	0	208.4	0	249.7	0	287.8	0	320.8	0	347.2	0	365.9	0	375.5	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-3.3S rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which is going through the points of maximum power, is also drawn in figure 4.

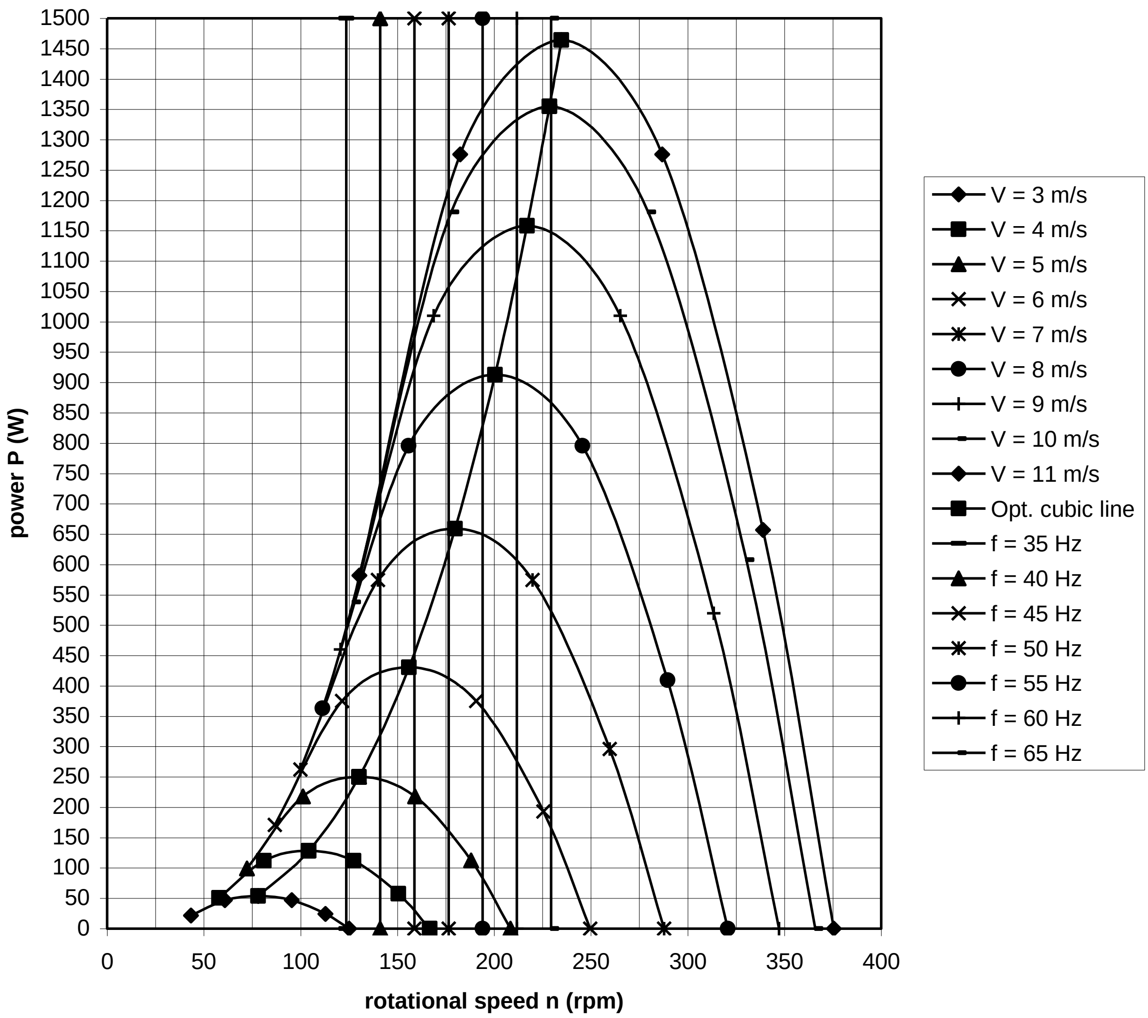

fig. 4 P-n curves of the VIRYA-3.3S rotor for V_{rated} = 11 m/s, optimum cubic line and lines for 35 Hz, 40 Hz, 45 Hz, 50 Hz, 55 Hz, 60 Hz and 65 Hz.

The 34-pole generator has not yet been built and tested. So measured P_{mech}-n and P_{el}-n curves for a low voltage winding for 26 V star are not available. Measurements of the generator for a high voltage winding in combination with an asynchronous motor of a centrifugal pump are also not available.

A 2-pole generator has a frequency of 50 Hz at a rotational speed of 3000 rpm. So a 34-pole generator has a frequency of 50 Hz at a rotational speed of 3000 * 2 / 34 = 176.47 rpm. The rotational speeds for frequencies of 35, 40, 45, 50, 55, 60 and 65 Hz are respectively 123.53, 141.18, 158.82, 176.47, 194.12, 211.76 and 229.41 rpm. The lines for frequencies of 35, 40, 45, 50, 55, 60 and 65 Hz are also given in figure 4.

In figure 4 it can be seen that the line for a frequency of 50 Hz is intersecting with the optimum cubic line at a mechanical power of about 630 W belonging to a design wind speed of about 6.9 m/s. The available electrical power will be lower because of the generator efficiency. Assume the generator efficiency is 0.8, so the electrical power is about 500 W. The mechanical power which can be generated at f = 50 Hz and V = 8 m/s is about 880 W and the maximum electrical power will be about 700 W.

A centrifugal pump with a 0.55 kW pump motor used at a factor 0.8 of its nominal power and with a motor efficiency of 0.73 will absorb an electrical power of about 600 W, so a 0.55 kW pump motor seems an acceptable choice. The working point will lie a bit left from the optimum cubic line for a pump with a 0.55 kW motor.

In figure 4 it can be seen that the maximum power at a wind speed of 11 m/s is 1464 W if the optimum cubic line is followed. The frequency is about 66 Hz which is rather high. The load characteristic of a centrifugal pump is about a cubic line which means that the optimum cubic line of the windmill will be followed upwards from the design point if the design point is lying on the optimum cubic line. However, the design point is lying left from the optimum cubic line for a pump with a 0.55 kW pump motor and this means that a cubic line will be followed which is also lying left from the optimum cubic line. This means that the maximum frequency will be about 63 Hz at V = 11 m/s. I expect that this is allowed for the pump and for the pump motor but this must be verified in practice.

Below a frequency of about 35 Hz, belonging to a rotational speed of 123.53 rpm, the pump is no longer able to produce the static water height so no water will be pumped. Probably it is necessary to disconnect the generator and the pump motor by a 3-phase switch below a frequency of about 35 Hz. This makes that the rotor will always start unloaded at low wind speeds. If the connection is broken at f = 35 Hz for a running rotor, this results in acceleration of the rotor. The connection can be made at a frequency of 52 Hz belonging to a rotational speed of 183.5 rpm. This frequency will be reached for an unloaded rotor for a wind speed of about 4.5 m/s. So the pump will start pumping at this wind speed but it will stop only if the frequency becomes lower than 35 Hz. This means that even at low wind speeds there will be some intermittent output.

If the pump is a centrifugal pump, the system will probably also work if there is no 3-phase switch which disconnects the generator and the pump motor but in this case water is not pumped intermittently if the wind speed is just above 4.5 m/s. A switch will certainly be needed for a positive displacement pump as such pump demands a torque directly from stand still position.

The generator winding must be chosen such that the loaded voltage is 230 V at a frequency of 50 Hz. This means that the unloaded voltage at 50 Hz must be a lot higher. I expect about 280 V but this must be tested for a prototype of the generator.

6 Calculation of the strength of the spoke assembly

The blades are connected to each other by the spoke assembly. Each spoke has a length l = 650 mm, a width b = 60 mm and a height h = 10 mm. The blade has a thickness of 2.5 mm but the blade is cambered and the moment of resistance therefore increases substantially (see formula 12 report KD 398, ref. 4). The bending moment in a spoke at the hub is also much higher than in the blade at the end of the spoke. The spoke is therefore the weakest component. The spoke is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The spoke is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because a spoke is long and rather thin, it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1^e^ The load which appears for a rotating rotor at V_{rated} = 11 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 35° for V_{rated} = 11 m/s.

2^e^ The load which appears for a slowed down rotor. The rotor is slowed down by making short-circuit in the generator winding. The rotor will turn at a very low rotational speed and for this very low rotational speed the effect of compensation by the centrifugal moment is negligible. Therefore it is assumed that the rotor stands still.

6.1 Bending stress in the spoke for a rotating rotor and V = 11 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 / B \qquad (N) \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation, the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The effective blade length k' of the VIRYA-3.3S rotor is only 1.15 m but the rotor radius R = 1.65 m. Therefore there is a disk in the centre with an area of 0.09 of the rotor area on which no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.91 = 0.81. Because of the tip losses, the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75. It is assumed that the thrust coefficient is constant for values of λ in between λ_d and $\lambda_{unloaded}$.

Substitution of C_t = 0.75, δ = 35°, ρ = 1.2 kg/m^3, V = 11 m/s, R = 1.65 m and B = 3 in formula 10 gives $F_{t\,\delta\,bl}$ = 104 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 1.1 m.

Because the effective blade length is only k', there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k' falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about r_1 = 1.18 m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at r_2 = 0.03 m for a hub with a diameter of 60 mm. At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$M_{b\,t} = F_{t\,\delta\,bl} * (r_1 - r_2)$ (Nm) (11)

Substitution of $F_{t\,\delta\,bl}$ = 104 N, r_1 = 1.18 m and r_2 = 0.03 m gives $M_{b\,t}$ = 120 Nm = 120000 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$\sigma_b = M / W$ (N/mm^2) (12)

The moment of resistance W of a strip is given by:

$W = 1/6\ bh^2$ (mm^3) (13)

(12) + (13) gives:

$\sigma_b = 6\ M / bh^2$ (N/mm^2) (M in Nmm) (14)

Substitution of M = 120000 Nmm, b = 60 mm and h = 10 mm in formula 14 gives σ_b = 120 N/mm^2. For this stress, the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the spoke is bending in between the hub and the inner connection bolt of blade and spoke. This bolt is lying at r_3 = 0.425 m = 425 mm. So the length of the spoke l which is loaded by bending is given by:

$l = r_3 - r_2$ (mm) (15)

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$M = F * (r_1 - r_3)$ (Nmm) (16)

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$\phi = l * (M + \frac{1}{2} Fl) / EI$ (rad) (17)

The bending moment of inertia I of a strip is given by:

$I = 1/12\ bh^3$ (mm^4) (18)

(15) + (16) + (17) + (18) gives:

$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½ (r_3 - r_2)\} / (E * bh^3)$ (rad) (19)

Substitution of F = 104 N, r_3 = 425 mm, r_2 = 30 mm, r_1 = 1180 mm, E = 2.1 * 10^5 N/mm^2, b = 60 mm and h = 10 mm in formula 19 gives: ϕ = 0.037265 rad = 2.14°. This is an angle which can not be neglected. In report R409D (ref. 7) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-3.3S this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin \left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2} \right) \quad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 208 mm and a plate thickness of 2.5 mm it is found that A_{pr} = 0.00052 m^2. The blade is made of steel sheet with a density ρ_{pr} of about ρ_{pr} = 7.8 * 10^3 kg/m^3. It is assumed that the optimum cubic line is followed so the rotor runs at the design tip speed ratio λ = 4.5. Substitution of C_t = 0.75, ρ = 1.2 kg/m^3, R = 1.65 m, B = 3, A_{pr} = 0.00052 m^2, ρ_{pr} = 7.8 * 10^3 kg/m^3 and λ = 4.5 in formula 20 gives: ε = 1.79°. This angle is smaller than the calculated angle of 2.14° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.79°.

The real bending angle ε is determined a follows. A thrust moment M_t = 120 Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. M_b = 120 Nm for ε = 2.14°. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. M_c = 120 Nm for ε = 1.79°. The path of these three moments is given in figure 5. The sum total of M_b + M_c is determined and the line M_b + M_c is also given in figure 5.

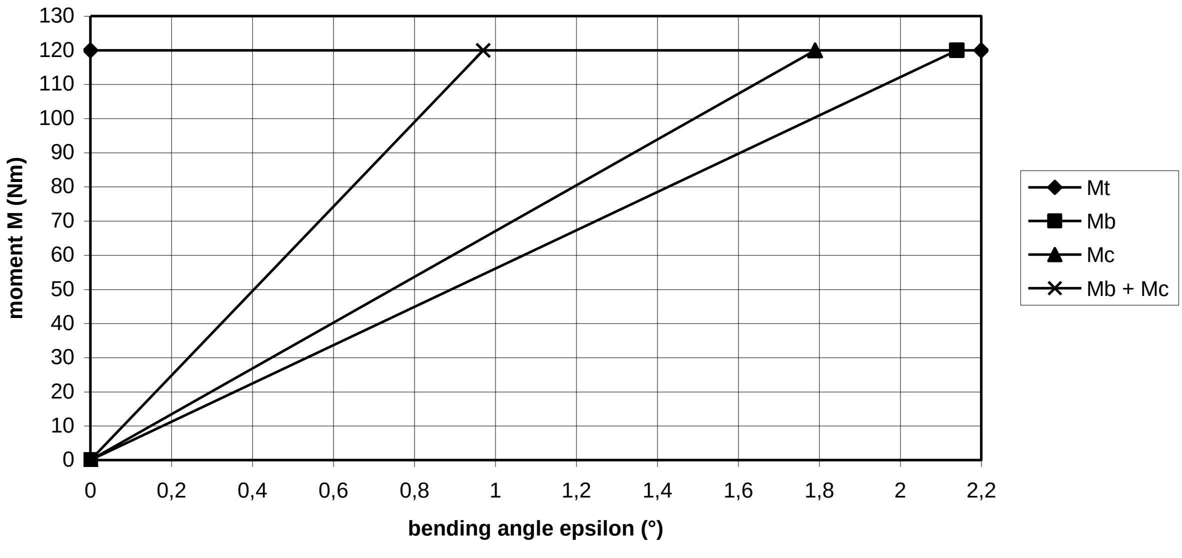

fig. 5 Path of M_t, M_b, M_c, and M_b + M_c as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 5 it can be seen that ε = 0.97°. This is a factor 0.453 of the calculated angle of 2.14°. Because the bending stress is proportional to the bending angle it will also be a factor 0.453 of the calculated stress of 120 N/mm^2 resulting in a stress of about 54 N/mm^2. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft, and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with sinα with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment dF * r which is exerted by this force with respect to the blade is therefore varying sinusoidal too. However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \tag{21}$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three $\sin^2\alpha$ functions which make an angle of 120° which each other, appear to result in a constant value. The three functions are given in figure 6. It can be proven for a three bladed rotor that the sum value of the three blades is equal to 3/2 of the peak value of one blade.

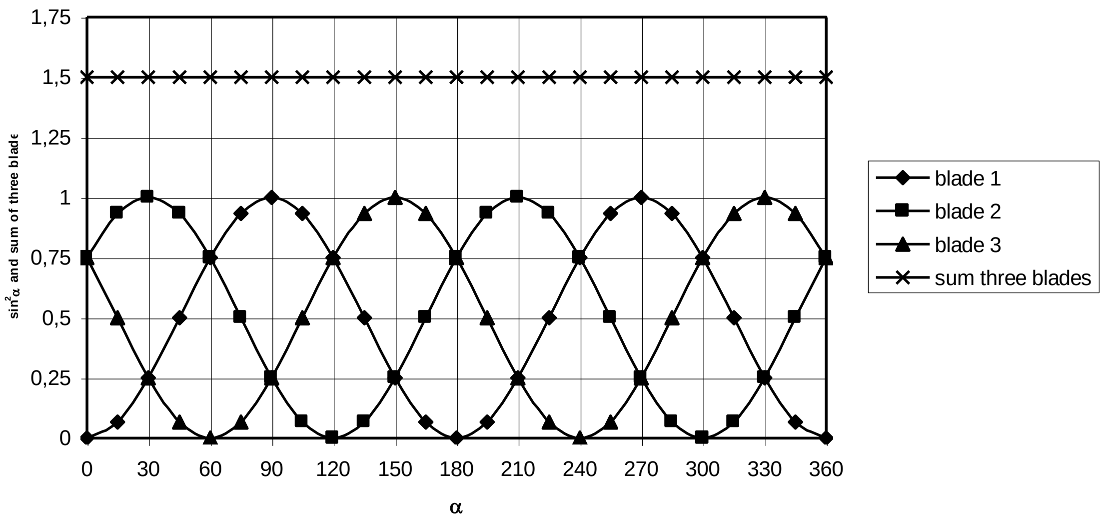

fig. 6 Path of $\sin^2\alpha$ and the sum of three blades

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then 2/3 of the sum moment as given by formula 21. The variation of the moment in the blade with respect to a rotating axis frame is therefore given by:

$M_{gyr\ bl} = 2/3 \sin\alpha * I_{rot} * \Omega_{rot} * \Omega_{head}$ (Nm) (22)

For a three bladed rotor, the moment of inertia of the whole rotor I_{rot} is three times the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$M_{gyr\ bl} = 2 \sin\alpha * I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (23)

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 22 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $\sin^2\alpha$ function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened peak value is given by:

$M_{gyr\ bl\ max} = 1.2 * I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (24)

For the chosen blade geometry it is calculated that I_{bl} = 6.42 kgm^2. The maximum loaded rotational speed of the rotor for the optimum cubic line can be read in figure 4 and it is found that n_{max} = 235 rpm. This gives $\Omega_{rot\ max}$ = 24.6 rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-3.3S is provided with the hinged side vane safety system which has a light van blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be 0.2 rad/s at very high wind speeds.

Substitution of I_{bl} = 6.42 kgm^2, $\Omega_{rot\ max}$ = 24.6 rad/s en $\Omega_{head\ max}$ = 0.2 rad/s in formula 24 gives: $M_{gyr\ bl\ max}$ = 38 Nm = 38000 Nmm.

Substitution of M = 38000 Nmm, b = 60 mm and h = 10 mm in formula 14 gives $\sigma_{b\ max}$ = 38 N/mm^2. This value has to be added to the bending stress of 54 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max}$ = 92 N/mm^2. The minimum stress is 54 - 38 = 16 N/mm^2. So the stress is not becoming negative and therefore it is probably not necessary to take the load as a fatigue load.

For the strip material hot rolled strip Fe 360 is chosen. For hot rolled strip the allowable stress for a load in between zero and maximum is about 190 N/mm^2 and for a fatigue load it is about 140 N/mm^2. However, these are the stresses for a tensile stress. The allowable bending stresses are higher. It is assumed that the allowable bending stress for a load in between zero and maximum is 230 N/mm^2 and for a fatigue load is 170 N/mm^2.

The calculated stress is even much lower than the allowable fatigue stress, so the strip is strong enough. In reality the blade is not extremely stiff and will also bend somewhat. This reduces the bending of the strip and therefore the stress is the strip will be somewhat lower.

6.2 Bending stress in the strip for a slowed down rotor

The rotational speed for a rotor which is slowed down by making short-circuit of the generator is very low. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 104 N for $V = V_{rated} = 11$ m/s and $\delta = 35°$. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 35° if the force on one blade is 104 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at $r_4 = 1.025$ m because the relative wind speed is almost constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 16 changes into:

$$M_{b\,t} = F_{t\,\delta\,bl} * (r_4 - r_2) \quad \text{(Nm)} \qquad (25)$$

Substitution of $F_{t\,\delta\,bl} = 104$ N, $r_4 = 1.025$ m en $r_2 = 0.03$ m in formula 25 gives $M_{b\,t} = 103$ Nm = 103000 Nmm. Substitution of M = 103000 Nmm, b = 60 mm and h = 10 mm in formula 14 gives $\sigma_b = 103$ N/mm^2. This is higher than the calculated stress for a rotating rotor. However, this load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable bending stress is about 230 N/mm^2 for hot rolled strip Fe 360, so the strip is strong enough.

Because the strip and the blade are rather flexible it has to be checked if a slowed down rotor can’t hit the tower. In chapter 6.1 it has been calculated, for no compensation of the gyroscopic moment, that the bending angle is 2.14° for a stress of 120 N/mm^2. So for a stress of 103 N/mm^2 the bending angle will be 2.14 * 103 / 120 = 1.84°. For a rotor radius of R = 1.65 m this results in a movement at the tip of about 0.053 m. Because the blade itself will bend too, the movement will be larger and it is expected that it will be about 0.08 m. The minimum distance in between the blade tip and the tower pipe is much larger if the blade is not bending. So there is no chance that the blade hits the tower for a slowed down rotor.

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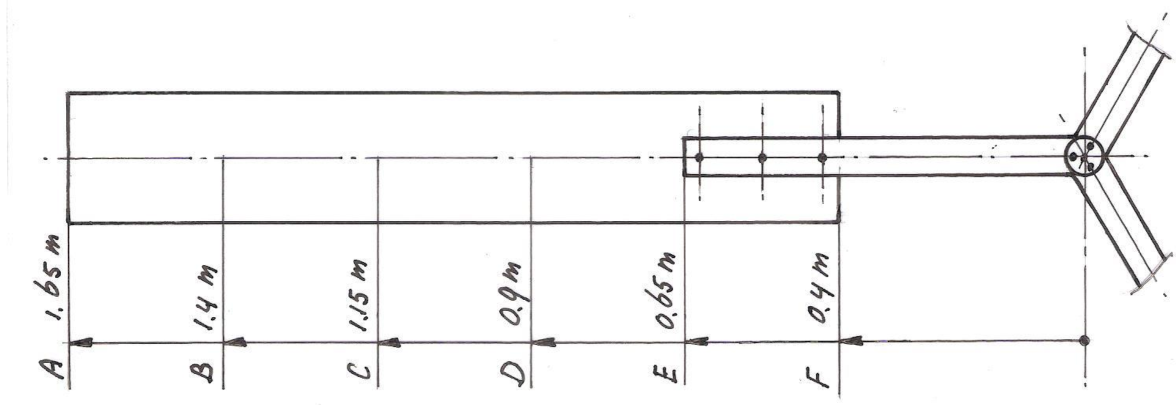
Appendix 1



Sketch VIRYA-3.3S rotor