Design report of the VIRYA-2.02 rotor (λ_d = 6, B = 2, tapered stainless steel blades)

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KD 617

It is allowed to copy this report for private use. The VIRYA-2.02 has not been built and tested. The rotor should not be used for a wind turbine without a proper safety system which limits the maximum rotational speed and thrust.

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1 Introduction

In report KD 616 (ref. 1), wind tunnel measurements are given for a 2-bladed stainless steel rotor with a diameter of 1.8 m. The full title of report KD 616 is: “Translation of parts of report R 343 D of June 1978 from Dutch into English. R 343 D gives wind tunnel measurements for a rotor with tapered blades made out of a cylinder”. Rotors with different sheet thicknesses and made from different materials are described in R 343 D. Only the measurements for 2 mm stainless steel are given in KD 616 as the measurements for 3 mm aluminium and especially for 2 mm aluminium showed flutter problems at high wind speeds.

So the wind tunnel measurements have been performed long ago but I believe that they are still relevant because the tested rotor had a high maximum C_p and it was very silent in between $6 < \lambda < 7.5$. However, the tested rotor was difficult to manufacture and a lot of material was wasted. In chapter 5 of KD 616, ideas are given about a somewhat larger stainless steel rotor which makes a more efficient use of material and which is easier to manufacture. A sketch of this VIRYA-2.02 rotor is given in figure 5 of KD 616. This figure is copied as figure 1. The measures are given for a non-cambered blade and a non-twisted connecting strip.

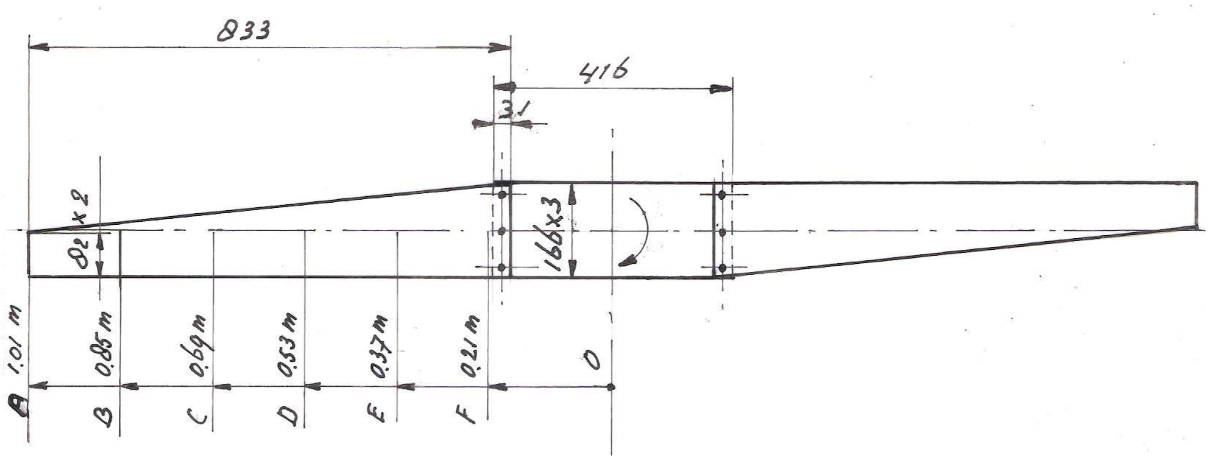

fig. 1 Sketch VIRYA-2.02 rotor, 2 mm stainless steel blades, B = 2, λ_d = 6

Two blades are made out of a 2 mm thick stainless steel sheet size 250 * 833 mm and 15 of these sheets can be made out of a standard sheet size 1.25 * 2.5 m. The sheet is cambered first and then ground in two trapezium shaped blades with a 2 mm thick rectangular grinder. The bending radius is chosen 165 mm. The connecting sheet is made out of 3 mm thick stainless steel sheet size 416 * 166 mm and 45 of these sheets can be made out of a standard sheet size 1.25 * 2.5 m. The connecting sheet is connected to the hub at the centre but the hub can only be designed if the generator is chosen. The connecting strip is twisted in between the hub and the blade root to realise the correct blade angle at the blade root. The ends are cambered with about the same radius as used for the blade. However, the required twisting angle isn’t given in KD 616. This angel will be determined in chapter 2 of this report KD 617.

The original 1.8 m rotor was calculated in 1978 using the aerodynamic theory which was available at that time but the design calculations were not given in R 343 D. But the results of the calculations are given in a table in top of the drawing of the rotor which has drawing number 7701-2 and of which a reduced copy is given in figure 1 of KD 616. The drawing is given for a rotor with left hand rotation but a mirror image of the blades have been manufactured which made the prototype rotating right hand. The VIRYA-2.02 rotor is also rotating right hand. In chapter 3 it will be checked if the rotor geometry matches with the aerodynamic theory as given in report KD 35 (ref. 2). The P-n curves of the rotor are derived in chapter 5.

2 Calculation of the chord c, the camber C and the blade angle β for different stations

The outside part of the blades of the VIRYA-2.02 rotor are about the same as the outside part of the blades of the original 1.8 m rotor but the blades are not exactly congruent. The angle in between the leading edge and the tailing edge is 4.74° for the original 1.8 m rotor and 5.76° for the VIRYA-2.02. If the original 1.8 m rotor would be scaled up to a diameter of 2.02 m, it would have a non-cambered width at the tip of 84 mm but this value is 82 mm for the VIRYA-2.02 rotor. At cross section B, the sheet width is about the same but at smaller radii, the sheet width of the VIRYA-2.02 is somewhat larger. The blade angle β at the tip is 2.1° for the original 1.8 m rotor. Assume that the twisting angle of the connecting sheet is chosen such that the same blade angle at the tip is realised for the VIRYA-2.02 rotor.

A blade is not twisted but is made out of a cylinder and the cylinder axis is in parallel to the tailing edge. Different blade angles for different stations are realised because the blade width and so the chord, is increasing at decreasing radius. The increase of the blade width is shown in figure 1. However, the corresponding chord for a certain sheet width is a little smaller than the sheet width because of the camber and the reduction is stronger as the camber is larger. The formulas which give the relation in between the sheet width b, the airfoil thickness a, the chord c, the bending radius r_c, the half bending angle α_c and the camber C are given in chapter 5 of report KD 398 (ref. 3).

In figure 1 it can be seen that the sheet width at the tip (station A) is 82 mm. The sheet width at the blade root is 166 mm and the blade has a length of 833 mm. As the blade shape is a trapezium, the sheet width at the stations B, C, D, E and F can be calculated easily. The result of the calculations is given in table 1.

station	r (mm)	width b (mm)	chord c (mm)	Camber C (%)	blade angle β (°)
A	1010	82	81.16	6.24	2.1
B	850	98.13	96.69	7.49	5.4
C	690	114.27	112.00	8.74	8.0
D	530	130.40	130.40	10.16	10.9
E	370	146.54	141.77	11.29	13.7
F	210	162.67	156.16	12.58	16.5

table 1 Variation of b, c and b as a function of r

The bending radius r_c at the heart line of the airfoil is chosen 165 mm for all six stations.

The half bending angle α_c is given by formula 1 of KD 398 (α_c in radian). The chord c is given by formula 5 of KD 398. Substitution of formula 1 in formula 5 of KD 398 gives:

$$c = 2\, r_c * \sin(b / 2\, r_c) \qquad (1)$$

To perform the calculations of c, the pocket calculator has to be set in the mode “radian”.

Substitution of r_c = 165 mm and b = 82 mm in formula 1 gives that c = 81.16 mm.
Substitution of r_c = 165 mm and b = 98.13 mm in formula 1 gives that c = 96.69 mm.
Substitution of r_c = 165 mm and b = 114.27 mm in formula 1 gives that c = 112.00 mm.
Substitution of r_c = 165 mm and b = 130.40 mm in formula 1 gives that c = 127.03 mm.
Substitution of r_c = 165 mm and b = 146.54 mm in formula 1 gives that c = 141.77 mm.
Substitution of r_c = 165 mm and b = 162.67 mm in formula 1 gives that c = 156.16 mm.

The calculated values of c are also given in table 1. The camber C is given by formula 7 of KD 398. Substitution of formula 1 in formula 7 of KD 398 gives:

$$C = \frac{1 - \cos(b / 2\, r_c)}{2 * \sin(b / 2\, r_c)} * 100 \quad (\%) \qquad (2)$$

Substitution of r_c = 165 mm and b = 82 mm in formula 2 gives that C = 6.24 %.
Substitution of r_c = 165 mm and b = 98.13 mm in formula 2 gives that C = 7.49 %.
Substitution of r_c = 165 mm and b = 114.27 mm in formula 2 gives that C = 8.74 %.
Substitution of r_c = 165 mm and b = 130.40 mm in formula 2 gives that C = 10.16 %.
Substitution of r_c = 165 mm and b = 146.54 mm in formula 2 gives that C = 11.29 %.
Substitution of r_c = 165 mm and b = 162.67 mm in formula 2 gives that C = 12.58 %.

The calculated values of C are also given in table 1. One should not forget to put the pocket calculator back in the mode “degree” if the next calculations are made.

The cylinder with a bending radius r_c = 165 mm must be positioned such that the blade angle β at station A is 2.1°. It is very difficult to find the blade angles β for the other stations theoretically but they can be found easily if a cross sectional drawing is made scale 1 : 1 on a drawing board and if the calculated chords are drawn in this drawing such that all chords start at the same point. A reduced version of this drawing is given as figure 2.

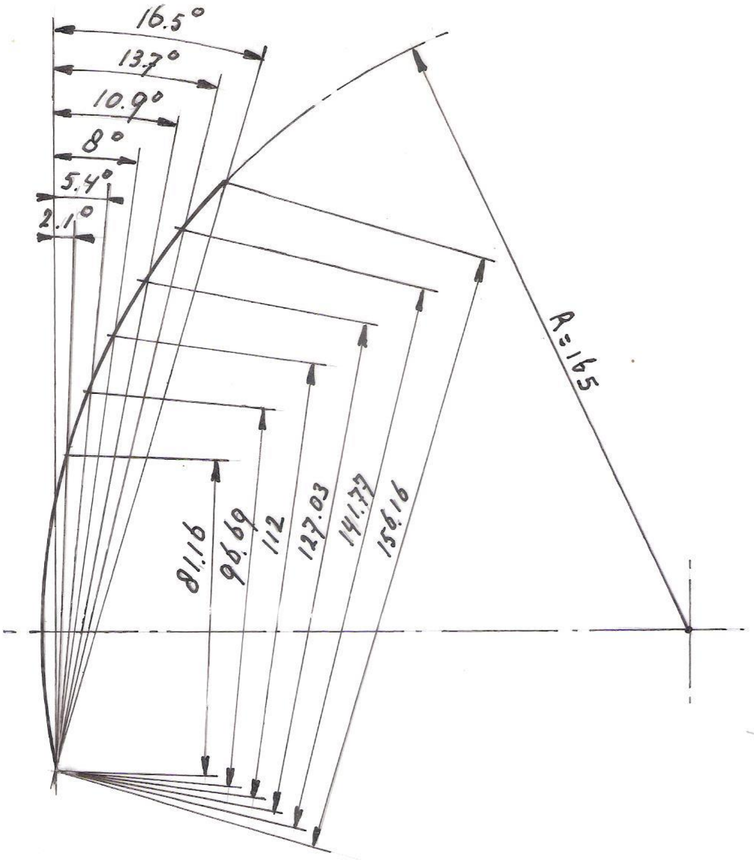

fig. 2 Cross section over the six blade sections

The blade angle β was read from the 1 : 1 drawing. The results are also given in table 1. It is found that the blade angle for station F is 16.5°. Station F is lying only 2 mm from the end of the connecting strip and this difference is neglected. So the connecting strip hast to be twisted 16.5° left hand in between the hub and the blade root.

In table 1 it can be seen that the camber varies in between 6.24 % at the blade tip and 12.58 % at station F. Aerodynamic characteristics are available for 7.14 %, 10 % and 12.5 % camber, so a bending radius of 165 mm seems a good choice for this rotor.

3 Checking if the geometry matches with the aerodynamic theory

The 2-bladed rotor of the VIRYA-2.02 windmill has a diameter D = 2.02 m and a design tip speed ratio λ_d = 6. Advantages of a 2-bladed rotor are that manufacture and balancing of the rotor is easy and that it is possible to transport a completely mounted rotor. The gyroscopic moment in the rotor shaft of a 2-bladed rotor is fluctuating but this is almost completely neutralised because the blades are connected to the hub by a flexible connecting sheet.

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 2). This report (KD 617) has its own formula numbering. Substitution of $\lambda_d = 6$ and R = 1.01 m in formula (5.1) of KD 35 gives:

$$\lambda_{r\,d} = 5.9406 * r \quad (-) \qquad (3)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \quad (°) \qquad (4)$$

$$\phi = 2/3 \text{ arc tan } 1 / \lambda_{r\,d} \quad (°) \qquad (5)$$

The rotor has tapered blades and the lift coefficient will therefore vary much less than for a constant chord blade (see examples in chapter 5.4 of KD 35). The Reynolds values are calculated for a wind speed of 5.5 m/s as the original 1.8 m rotor with 2 mm aluminium blades has also been measured for this wind speed and had good characteristics.

Substitution of B = 2 and in formula (5.4) of KD 35 gives:

$$C_l = 12.566 \text{ r } (1 - \cos\phi) / c \quad (-) \qquad (6)$$

The chord in table 1 is given in mm but for formula 6, one should take the chord in m. Substitution of V = 5.5 m/s in formula (5.5) of KD 35 gives:

$$R_{e\,r} = 3.669 * 10^5 * c * \sqrt{}\,(\lambda_{r\,d}^2 + 4/9) \quad (-) \qquad (7)$$

The blade is calculated for six stations A till F which have a distance of 0.16 m of one to another. The calculated camber for each station is given in table 1. Aerodynamic characteristics are given in report KD 398 (ref. 3) and are only available for three different cambers. The airfoil camber is chosen which is lying closest to the calculated camber. Those airfoil Reynolds numbers are used which are lying closest to the calculated Reynolds values.

First the theoretical values are determined for $C_{l\,th}$, α_{th} and β_{th}. Next β_{th} is compared to the real angle β as given in table 1. The result of the calculations is given in table 2.

Station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$R_{e\,r} * 10^{-5}$ V = 5.5 m/s	Calculated camber (%)	Airfoil camber (%)	$R_e * 10^{-5}$	α_{th} (°)	β_{th} (°)	β (°)
A	1.01	6	6.3	0.0812	0.95	1.80	6.24	7.14	1.7	3.0	3.3	2.1
B	0.85	5.050	7.5	0.0967	0.94	1.81	7.49	7.14	1.7	2.9	4.6	5.4
C	0.69	4.099	9.1	0.1120	0.98	1.71	8.74	10	1.7	1.6	7.5	8.0
D	0.53	3.149	11.8	0.1270	1.10	1.50	10.16	10	1.7	2.3	9.5	10.9
E	0.37	2.198	16.3	0.1418	1.36	1.19	11.29	12.5	1.2	7.0	9.3	13.7
F	0.21	1.248	25.8	0.1562	1.68	0.81	12.58	12.5	1.2	10.0	15.8	16.5

table 2 Checking of the blade geometry of the VIRYA-2.02 rotor

The theoretical blade angle β_{th} varies in between 3.3° and 15.8°. The real blade angle varies in between 2.1° and 16.5°. The calculation isn't very accurate because the aerodynamic characteristics are only available for three different cambers and not for the calculated camber of every section. However, the differences in between the theoretical blade angles β_{th} and the real blade angles β aren't very large, especially at the most important outer part of the blade. So it is decided that the chosen blade geometry is OK for a design tip speed ratio $\lambda_d = 6$.

4 Determination of the C_p-λ and the C_q-λ curves

As the geometry of the VIRYA-2.02 rotor is almost the same as for the measured 1.8 m rotor, it seems logic to use the measured C_p-λ and C_q-λ curves as given in figure 2 and 3 of KD 616. However, these measurements were performed for a tunnel wind speed of 11 m/s and this high wind speed resulted in high Reynolds numbers. In figure 4 of KD 616 it can be seen that the C_l/C_d curve for $Re = 2 * 10^5$ is lying much more to the left than for $Re = 1 * 10^5$ and that therefore much lower C_d/C_l ratios are realised for a high tunnel wind speed than for a low tunnel wind speed. The rather high Reynolds values for a tunnel wind speed of 11 m/s must be responsible for the peak in the C_p-λ curve for $6.4 < \lambda < 7.5$.

The rotor with 2 mm stainless steel blades has only be measured for a tunnel speed of 11 m/s but the rotor with 2 mm aluminium blades has also been measured for a tunnel speed of 5.5 m/s. The C_p-λ curve for this wind speed doesn't show a peak for $6.4 < \lambda < 7.5$ and has a maximum C_p value of about 0.39 for $\lambda = 6$. The unloaded tip speed ratio is about 9. The 2 mm aluminium rotor was rounded at the leading edge and sharpened at the tailing edge. Sharpening is a tricky procedure and it is assumed that the stainless steel rotor is rounded at both sides with a radius equal to half the sheet thickness, just as it is also done in figure 4 of KD 616 for the measurements of the 10 % cambered plate airfoil. The VIRYA-2.02 rotor has chords which are more than 10 % larger than the chords of the measured 1.8 m rotor and this has a favourable influence on the Reynolds numbers. So it is decided to use C_p-λ and C_q-λ curves which are about the same as the measured curves for the 2 mm aluminium rotor. The estimated C_p-λ and C_q-λ curves are given in figure 3 and 4.

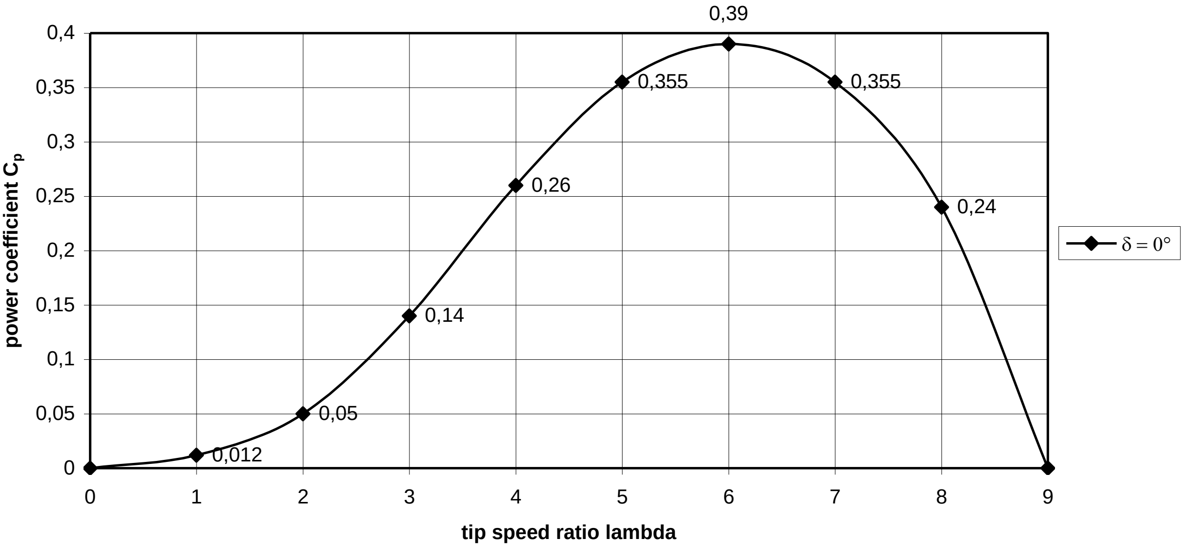

fig. 3 Estimated C_p-λ curve for the VIRYA-2.02 rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

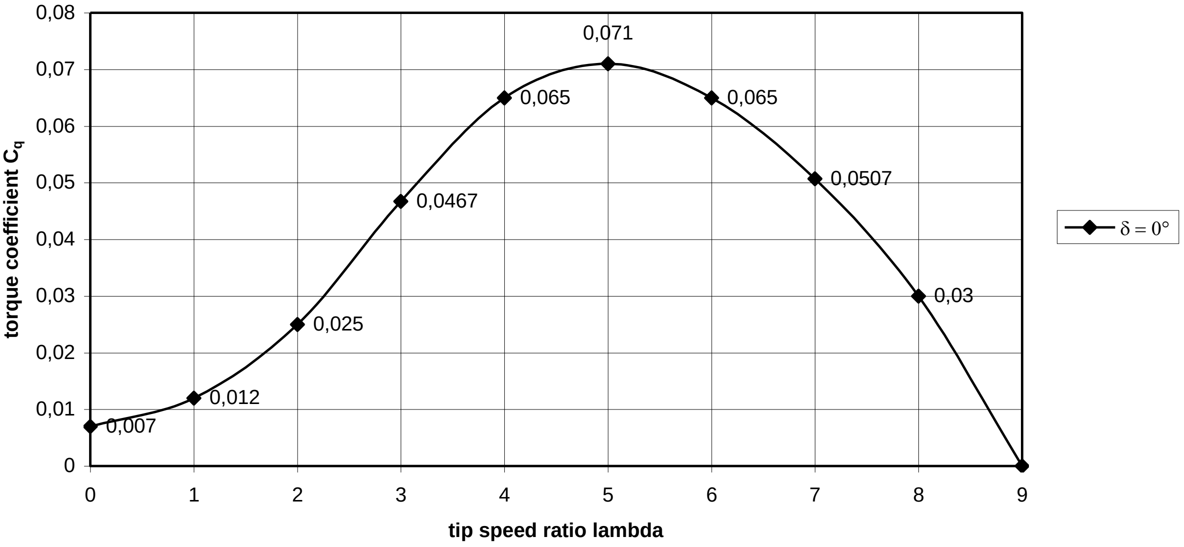

fig. 4 Estimated C_q-λ curve for the VIRYA-2.02 rotor for the wind direction perpendicular to the rotor (δ = 0°)

5 Determination of the P-n curves and the optimum cubic line

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The head of the VIRYA-2.02 will be derived from the head of the VIRYA-2.2S. The VIRYA-2.2S has a vane blade made out of 1 mm stainless steel sheet. The rated wind speed for this vane blade is about 11 m/s. The estimated δ-V curve is given in figure 5.

The head starts to turn away at a wind speed of about 7 m/s. For wind speeds above 11 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 11 m/s will therefore also be valid for wind speeds higher than 11 m/s.

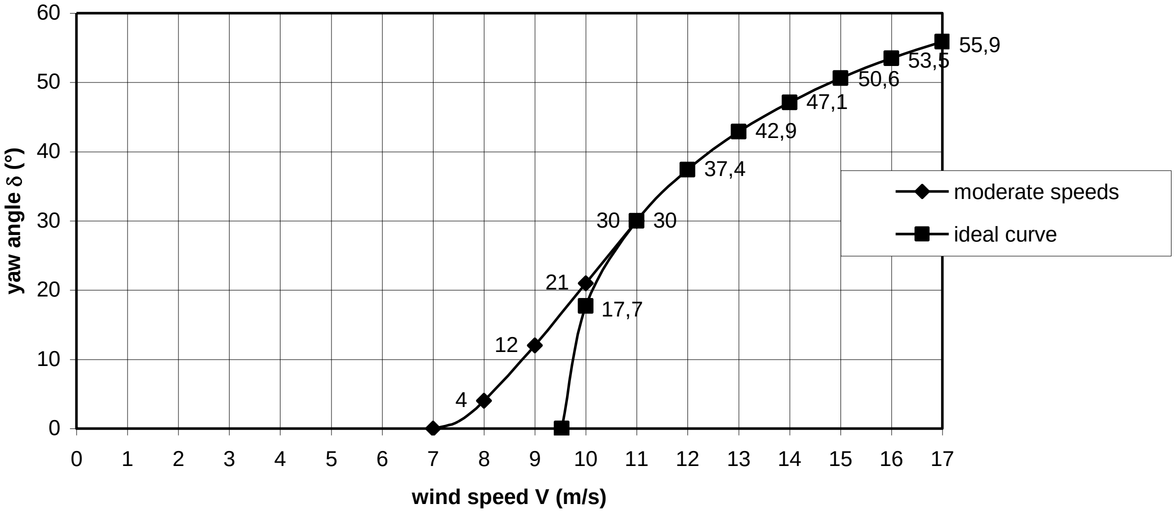

fig. 5 Estimated δ-V curve VIRYA-2.02 for a 1 mm stainless steel vane blade

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator for a certain gear ratio i (the VIRYA-2.02 has no gearing so i = 1). Because we are especially interested in the domain around the optimal cubic line and because the P-n curves for low values of λ appear to lie very close to each other, the P-n curves are not determined for low values of λ. The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9, 10 and 11 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of R = 1.01 m in formula 7.1 of KD 35 gives:

$$n_\delta = 9.455 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of $\rho = 1.2$ kg / m^3 and R = 1.01 m in formula 7.10 of KD 35 gives:

$$P_\delta = 1.923 * C_p * \cos^3\delta * V^3 \qquad \text{(W)} \qquad (9)$$

The P-n curves are determined for C_p values belonging to λ = 3, 4, 5, 6, 7, 8 and 9. (see figure 3). For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 5, is taken into account. The result of the calculations is given in table 3.

		$V = 3$ m/s $\delta = 0^\circ$		$V = 4$ m/s $\delta = 0^\circ$		$V = 5$ m/s $\delta = 0^\circ$		$V = 6$ m/s $\delta = 0^\circ$		$V = 7$ m/s $\delta = 0^\circ$		$V = 8$ m/s $\delta = 4^\circ$		$V = 9$ m/s $\delta = 12^\circ$		$V = 10$ m/s $\delta = 21^\circ$		$V = 11$ m/s $\delta = 30^\circ$	
λ (-)	C_p (-)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)
3	0.14	85.1	7.3	113.5	17.2	141.8	33.7	170.2	58.2	198.6	92.3	226.4	136.8	249.7	183.7	264.8	219.1	270.2	232.7
4	0.26	113.5	13.5	151.3	32.0	189.1	62.5	226.9	108.0	264.7	171.5	301.8	254.1	332.9	341.1	353.1	406.8	360.3	432.2
5	0.355	141.8	18.4	189.1	43.7	236.4	85.3	283.7	147.5	330.9	234.2	377.3	347.0	416.2	465.7	441.4	555.5	450.4	590.2
6	0.39	170.2	20.2	226.9	48.0	283.7	93.7	340.4	162.0	397.1	257.2	452.7	381.2	499.4	511.7	529.6	610.2	540.4	648.4
7	0.355	198.6	18.4	264.7	43.7	330.9	85.3	397.1	147.5	463.3	234.2	528.2	347.0	582.6	465.7	617.9	555.5	630.5	590.2
8	0.24	226.9	12.5	302.6	29.5	378.2	57.7	453.8	99.7	529.5	158.3	603.6	234.6	665.9	314.9	706.2	375.5	720.6	399.0
9	0	255.3	0	340.4	0	425.5	0	510.6	0	595.7	0	679.1	0	749.1	0	794.4	0	810.6	0

table 3 Calculated values of n and P as a function of λ and V for the VIRYA-2.02 rotor

The calculated values for n and P are plotted in figure 6. The optimum cubic line which can be drawn through the maximum of all P-n curves is also given in figure 6.

The generator of the VIRYA-2.02 is not yet chosen, so measured so P_{mech}-n and P_{el}-n curves are not yet available. They therefore can’t be drawn in the P-n graph of the rotor to determine the P_{el}-V curve.

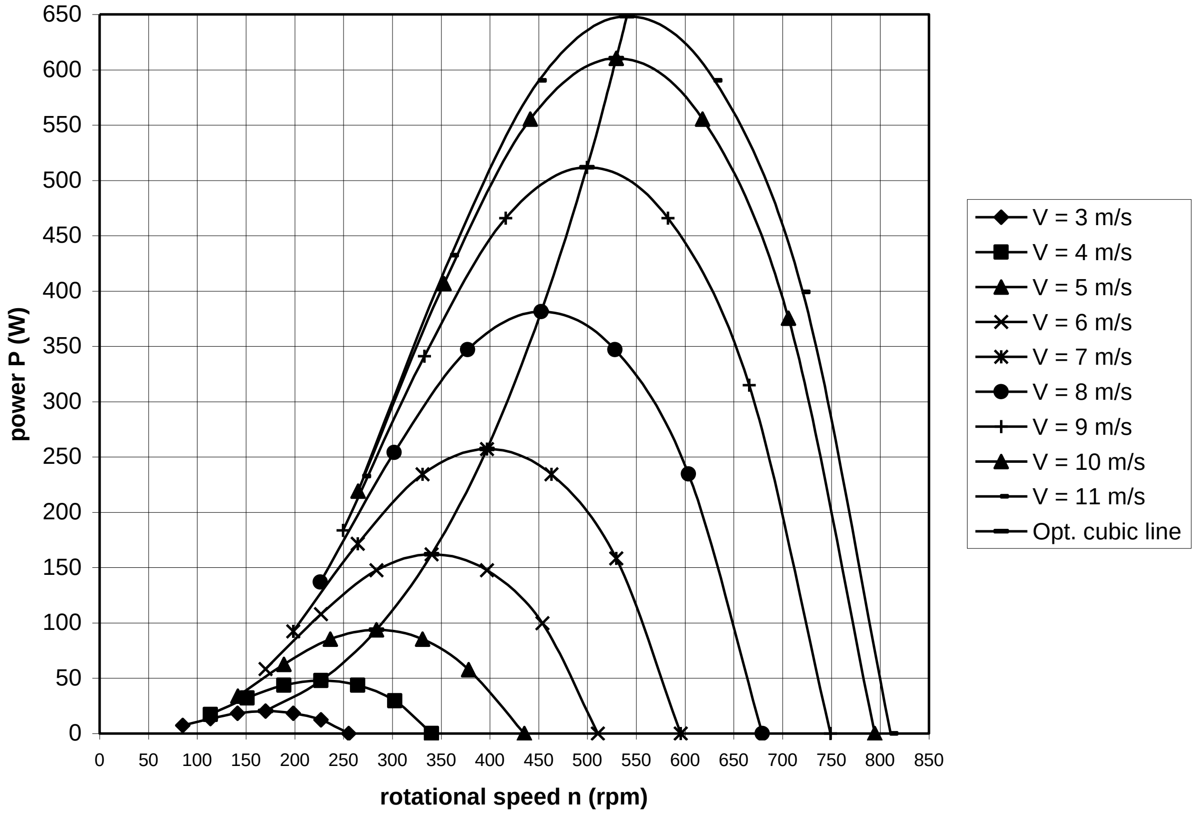

fig. 6 P-n curves and the optimum cubic line of the VIRYA-2.02 rotor

6 References

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