

**Calculations executed for the 4-bladed rotor of the VIRYA-3.5 windmill
($\lambda_d = 4$, stainless steel blades) meant to drive a centrifugal pump through a Polycord
transmission with an accelerating gear ratio 2.5 : 1 and a vertical shaft in the tower.**

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It is allowed to copy this report for private use. It is allowed to use the idea of the 4-bladed rotor and the Polycord transmission. The VIRYA-3.5 has not yet been built and tested and no responsibility is accepted for the correct functioning of it.

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1 Introduction

The VIRYA-3.5 windmill has a 4-bladed rotor with stainless steel blades and a rotor diameter of 3.5 m. The VIRYA-3.5 windmill is designed to drive a centrifugal pump using a Polycord transmission with an accelerating gearing with a gear ratio of 2.5 : 1 in the windmill head and a vertical shaft in the tower. The hinged side vane safety system which turns the rotor out of the wind at high wind speeds and the Polycord transmission are described in chapter 6. The VIRYA-3.5 can be used for drainage or for low head irrigation.

The vane has a light 2 mm thick aluminium vane blade and the rated wind speed is about 8 m/s. The tower is derived from the tower of the VIRYA-3D windmill. It has a 6 m long lowest part made of angle iron and strip and a 2 m long upper part made of pipe. The overlap is 0.2 m, so the total tower height is about 7.8 m. This tower looks about the same as the tower which is used for the VIRYA-2.2S windmill but the pipe diameter is 2" instead of 1½" and the clamping of the pipe is different because the overlap is shorter. This shorter overlap makes that the blade tip of the rotor can't touch the lower tower part.

2 Description of the rotor of the VIRYA-3.5 windmill

The 4-bladed rotor of the VIRYA-3.5 windmill has a diameter $D = 3.5$ m and a design tip speed ratio $\lambda_d = 4$. Two opposite blades are connected to each other by means of a twisted strip. Advantages of this construction are that no welded spoke assembly is required, that the rotor can be balanced easily and that the assembly of two blades can be transported completely mounted.

The rotor has blades with a constant chord and is provided with a 7.14 % cambered airfoil. A blade is made of a stainless steel strip with dimensions of 208 * 1250 * 2.5 mm and 12 blades can be made from a standard sheet of 1.25 * 2.5 m. Because the blade is cambered, the chord c is a little less than the blade width, resulting in $c = 205$ mm = 0.205 m.

Two opposite blades are connected to each other by a 1.5 m long twisted stainless steel strip size 60 * 10 mm. The overlap in between blade and strip is 0.25 m which results in a free blade length of 1 m. This blade length in combination with a design tip speed ratio of 4 and a blade thickness of 2.5 mm, is expected to be enough to prevent flutter of the blade at high wind speeds. The blade is connected to the strip using three bolts M10. The 250 mm long outer parts of the strip are slightly bevelled to prevent strong deformation of the blades when the bolts are tightened. The bolts are also used for connection of the balancing weights.

The hub is made of square stainless steel bar 60 * 60 mm with a tapered hole in the centre for connection to the rotor shaft which has a diameter of 30 mm. The two blade strips are clamped in between the hub and a square sheet by means of four bolts M10 and that is why the strip is not loaded by a bending moment at the position of the holes. The hub is pulled on the tapered shaft end by one central bolt M16. The mass of the whole rotor plus the hub is about 36 kg. A sketch of a blade of the rotor is given in figure 1.

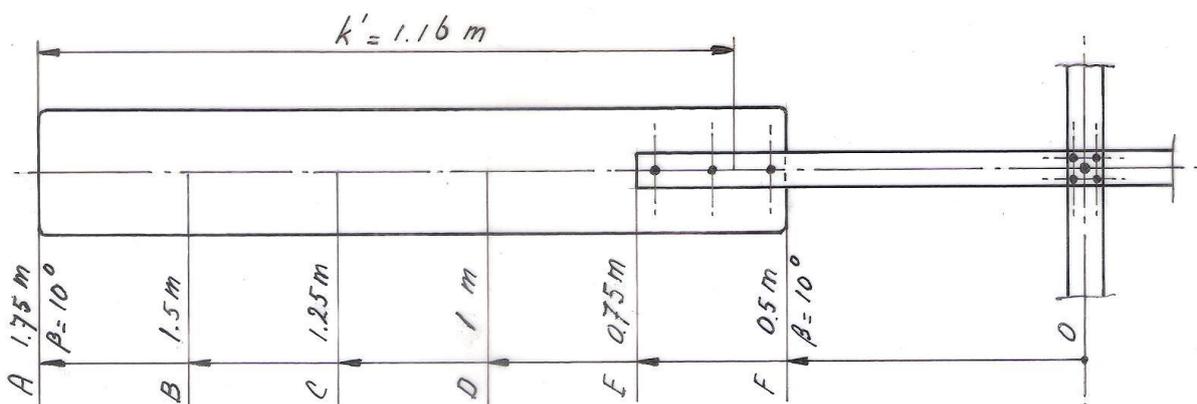


fig. 1 Sketch of a blade of the VIRYA-3.5 rotor

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 1). This report (KD 693) has its own formula numbering. Substitution of $\lambda_d = 4$ and $R = 1.75$ m in formula (5.1) of KD 35 gives:

$$\lambda_{rd} = 2.2857 * r \quad (-) \quad (1)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \quad (^\circ) \quad (2)$$

$$\phi = 2/3 \arctan 1 / \lambda_{rd} \quad (^\circ) \quad (3)$$

Substitution of $B = 4$ and $c = 0.205$ m in formula (5.4) of KD 35 gives:

$$C_l = 30.650 r (1 - \cos\phi) \quad (-) \quad (4)$$

Substitution of $V = 5$ m/s and $c = 0.205$ m in formula (5.5) of KD 35 gives:

$$Re_r = 0.684 * 10^5 * \sqrt{(\lambda_{rd}^2 + 4/9)} \quad (-) \quad (5)$$

The blade is calculated for six stations A till F which have a distance of 0.25 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l , α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 2). The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for regions with moderate wind speeds. Those airfoil Reynolds numbers are used which are lying closest to the calculated values. The effect of the strip on the aerodynamic characteristics of the inner part of the blade is neglected.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	C_{lth} (-)	C_{lin} (-)	$Re_r * 10^{-5}$ V = 5 m/s	$Re * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	C_d/C_{lin} (-)
A	1.75	4	9.4	0.205	0.71	0.60	2.77	2.5	-0.1	-0.6	9.5	10	0.052
B	1.5	3.429	10.8	0.205	0.82	0.83	2.39	2.5	0.7	0.8	10.1	10	0.033
C	1.25	2.857	12.9	0.205	0.96	0.94	1.94	1.7	3.0	2.9	9.9	10	0.041
D	1	2.286	15.8	0.205	1.15	1.23	1.63	1.7	5.1	5.8	10.7	10	0.060
E	0.75	1.714	20.2	0.205	1.41	1.43	1.26	1.2	9.2	10.2	11.0	10	0.14
F	0.5	1.143	27.5	0.205	1.73	1.28	0.91	1.2	-	17.5	-	10	-

table 1 Calculation of the blade geometry of the VIRYA-3.5 rotor

No value for α_{th} and therefore for β_{th} is found for station F because the required C_l value can not be generated. The theoretical blade angle β_{th} for stations A to E varies only in between 9.5° and 11.0° . If a constant blade angle of 10° is taken, the linearised angle of attack α_{lin} differ only a little from the theoretical value α_{th} for the most important outer side of the blade. Each strip is twisted 10° right hand in between the hub and the blade root.

4 Determination of the C_p - λ and the C_q - λ curves

The determination of the C_p - λ and C_q - λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.045. Figure 4.8 of KD 35 (for $B = 4$) and $\lambda_{opt} = 4$ and $C_d/C_l = 0.045$ gives $C_{p\ th} = 0.43$. The blade is stalling in between station E and F and the airfoil is disturbed because of the blade connection to the strip. Therefore not the whole blade length $k = 1.25$ m, but only the part up to 0.09 m outside station F is used for the calculation of the C_p . This gives an effective blade length $k' = 1.16$ m.

Substitution of $C_{p\ th} = 0.43$, $R = 1.75$ m and effective blade length $k' = 1.16$ m in formula 6.3 of KD 35 gives $C_{p\ max} = 0.38$. $C_{q\ opt} = C_{p\ max} / \lambda_{opt} = 0.38 / 4 = 0.095$.

Substitution of $\lambda_{opt} = \lambda_d = 4$ in formula 6.4 of KD 35 gives $\lambda_{unl} = 6.4$.

The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - \frac{1}{2}k) * C_l * c * k / \pi R^3 \quad (-) \quad (6)$$

The blade angle is 10° for the whole blade. For a non rotating rotor, the angle of attack α is therefore $90^\circ - 10^\circ = 80^\circ$. The estimated C_l - α curve for large values of α is given as figure 5 of KD 398. For $\alpha = 80^\circ$ it can be read that $C_l = 0.33$. The whole blade is stalling during starting and therefore now the whole blade length $k = 1.25$ m is taken.

Substitution of $B = 4$, $R = 1.75$ m, $k = 1.25$ m, $C_l = 0.33$ and $c = 0.205$ m in formula 6 gives that $C_{q\ start} = 0.017$. For the ratio in between the starting torque and the optimum torque we find that it is $0.017 / 0.095 = 0.18$. This is good for a rotor with a design tip speed ratio of 4. The ratio is expected to be high enough for combination of the windmill with a centrifugal pump because a centrifugal pump has no starting torque and so the only starting torque of the rotor is needed for the friction of the bearings of the rotor shaft and the vertical shaft and may be for some friction of the Polycord string.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * \frac{1}{2}\rho * \pi R^3} \right)} \quad (\text{m/s}) \quad (7)$$

At this moment the friction torque Q_s measured on the rotor shaft is not yet known, so the starting wind speed can't be calculated accurately. Assume $Q_s = 1$ Nm.

Substitution of $Q_s = 1$ Nm, $C_{q\ start} = 0.017$, $\rho = 1.2$ kg/m³ and $R = 1.75$ m in formula 7 gives that $V_{start} = 2.4$ m/s. This is acceptable low for a 4-bladed rotor with a design tip speed ratio of 4. The Q-n curve of the rotor for $V = 2.4$ m/s is rising rather fast and therefore it is allowed that the pump torque is also rising at increasing rotational speed. The pump torque is rising because the water level in the rising main is rising as the rotor makes more revolutions. Only when the water level has reached the top of the rising main, the pump will have a real output.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p - λ and C_q - λ curves can be determined if only two points of the C_p - λ curve and one point of the C_q - λ curve are known. The first part of the C_q - λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for $\lambda = 0$.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 3). With this method, it can be determined that the C_q - λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p - λ and C_q - λ curves for the VIRYA-3.5 rotor are given in figure 2 and 3.

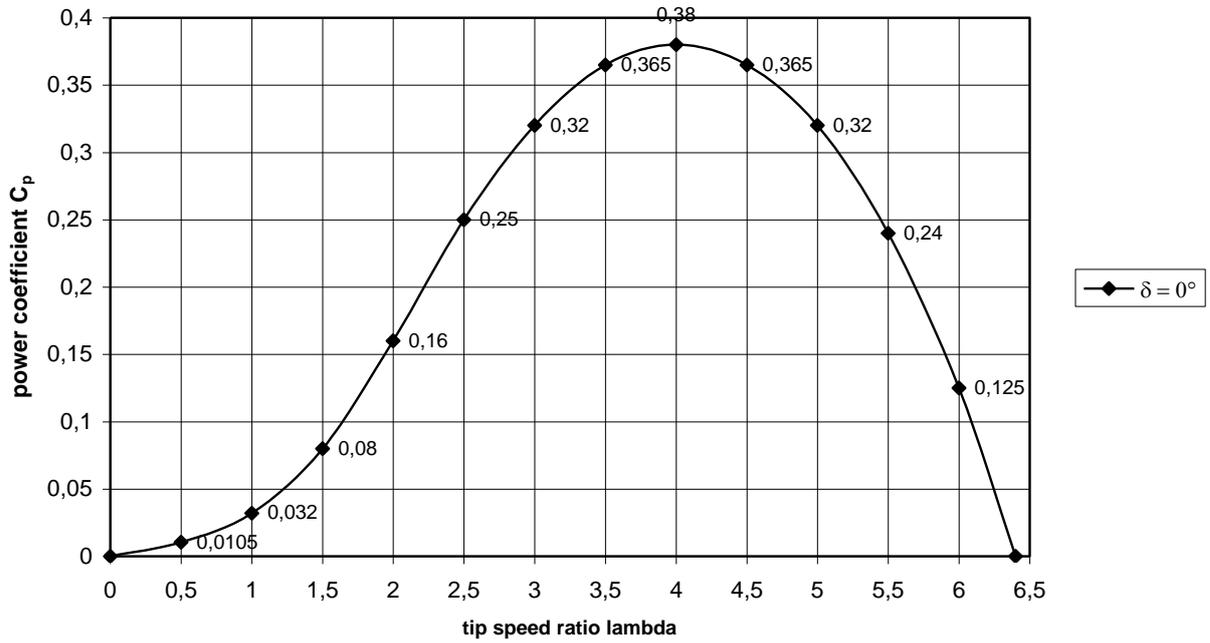


fig. 2 Estimated C_p - λ curve for the VIRYA-3.5 rotor for the wind direction perpendicular to the rotor ($\delta = 0^\circ$)

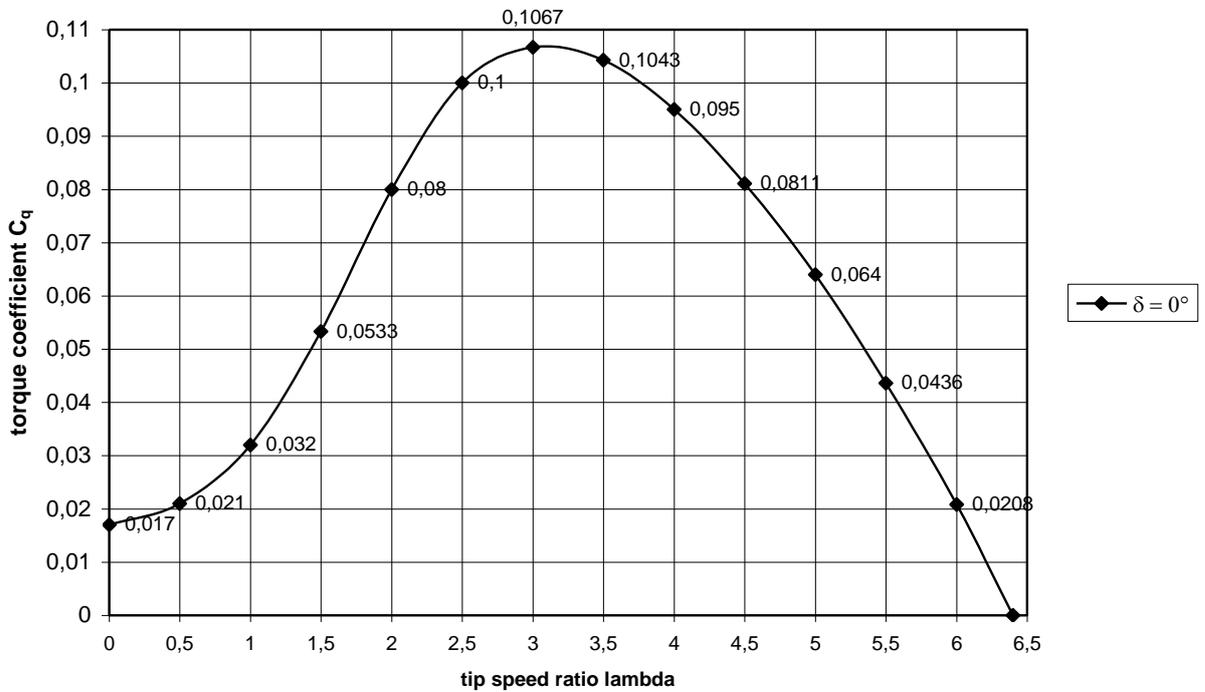


fig. 3 Estimated C_q - λ curve for the VIRYA-3.5 rotor for the wind direction perpendicular to the rotor ($\delta = 0^\circ$)

5 Determination of the Q_v - n_v curves

The determination of the Q - n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_q - λ curve of the rotor and a δ - V curve of the safety system together with the formulas for the torque Q and the rotational speed n . The C_q - λ curve is given in figure 3. The δ - V curve of the safety system depends on the safety system. The estimated δ - V curve of the hinged side vane safety system with a 2 mm aluminium vane blade is given in figure 4.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 8 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The Q - n curve for 8 m/s will therefore also be valid for wind speeds higher than 8 m/s.

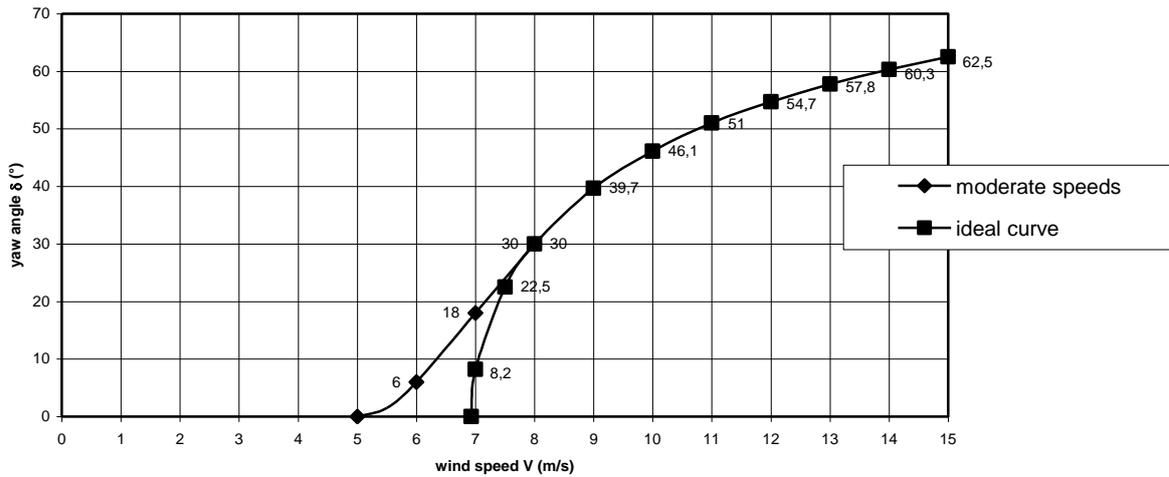


fig. 4 Estimated δ - V curve VIRYA-3.5 safety system with $V_{\text{rated}} = 8$ m/s

The Q - n curves are the curves as felt on the rotor shaft. The Q_v - n_v curves are the curves as felt on the vertical shaft. The Q_v - n_v curves are used to check the matching with the Q - n curve of the pump. The formulas for Q and n are determined first and next the formulas for Q_v and n_v are determined taking into account the accelerating gear ratio $i = 2.5$. The Q_v - n_v curves are determined for wind the speeds 2, 3, 4, 5, 6, 7 and 8 m/s. At high wind speeds, the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for Q and n are used which are given in chapter 7 of KD 35.

Substitution of $R = 1.75$ m in formula 7.1 of KD 35 gives:

$$n_{\delta} = 5.4567 * \lambda * \cos\delta * V \quad (\text{rpm}) \quad (8)$$

Substitution of $\rho = 1.2$ kg / m³ and $R = 1.75$ m in formula 7.7 of KD 35 gives:

$$Q_{\delta} = 10.1022 * C_q * \cos^2\delta * V^2 \quad (\text{Nm}) \quad (9)$$

$n_{v\delta}$ is a factor 2.5 higher than n_{δ} for $i = 2.5$. So formula 8 becomes:

$$n_{v\delta} = 13.6418 * \lambda * \cos\delta * V \quad (\text{rpm}) \quad (10)$$

$Q_{v\delta}$ is a factor 2.5 lower than Q_{δ} for $i = 2.5$ if the transmission efficiency η_{tr} is 1. Assume that $\eta_{tr} = 0.95$. So $Q_{v\delta}$ is a factor $2.5 / 0.95 = 2.6316$ lower than Q_{δ} . So formula 9 becomes:

$$Q_{v\delta} = 3.8388 * C_q * \cos^2\delta * V^2 \quad (\text{Nm}) \quad (11)$$

The Q_v - n_v curves are determined for C_q values belonging to λ is 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6 and 6.4 (see figure 3). For a certain wind speed, for instance $V = 2$ m/s, related values of C_q and λ are substituted in formula 10 and 11 and this gives the Q_v - n_v curve for that wind speed. For the higher wind speeds the yaw angle δ as given by figure 4, is taken into account. The result of the calculations is given in table 2.

	V = 2 m/s $\delta = 0^\circ$	V = 3 m/s $\delta = 0^\circ$	V = 4 m/s $\delta = 0^\circ$	V = 5 m/s $\delta = 0^\circ$	V = 6 m/s $\delta = 6^\circ$	V = 7 m/s $\delta = 18^\circ$	V = 8 m/s $\delta = 30^\circ$
λ (-)	C_q (-)	n_v (rpm)	Q_v (Nm)	n_v (rpm)	Q_v (Nm)	$n_{v\delta}$ (rpm)	$Q_{v\delta}$ (Nm)
0	0.017	0	0.26	0	0.59	0	1.04
0.5	0.021	13.6	0.32	20.5	0.73	27.3	1.29
1	0.032	27.3	0.49	40.9	1.11	54.6	1.97
1.5	0.0533	40.9	0.82	61.4	1.84	81.9	3.27
2	0.08	54.6	1.23	81.9	2.76	109.1	4.91
2.5	0.1	68.2	1.54	102.3	3.45	136.4	6.14
3	0.1067	81.9	1.64	122.8	3.69	163.7	6.55
3.5	0.1043	95.5	1.60	143.2	3.60	191.0	6.41
4	0.095	109.1	1.46	163.7	3.28	218.3	5.83
4.5	0.0811	122.8	1.25	184.2	2.80	245.6	4.98
5	0.064	136.4	0.98	204.6	2.21	272.8	3.93
5.5	0.0436	150.1	0.67	225.1	1.51	300.1	2.68
6	0.0208	163.7	0.32	245.6	0.72	327.4	1.28
6.4	0	174.6	0	261.9	0	349.2	0

table 2 Calculated values of n_v and Q_v as a function of λ and V for the VIRYA-3.5 rotor

The calculated values for n_v and Q_v are plotted in figure 5. The optimum parabola which is going through the points with $\lambda = 4$, where C_p is maximum, is also drawn in figure 5.

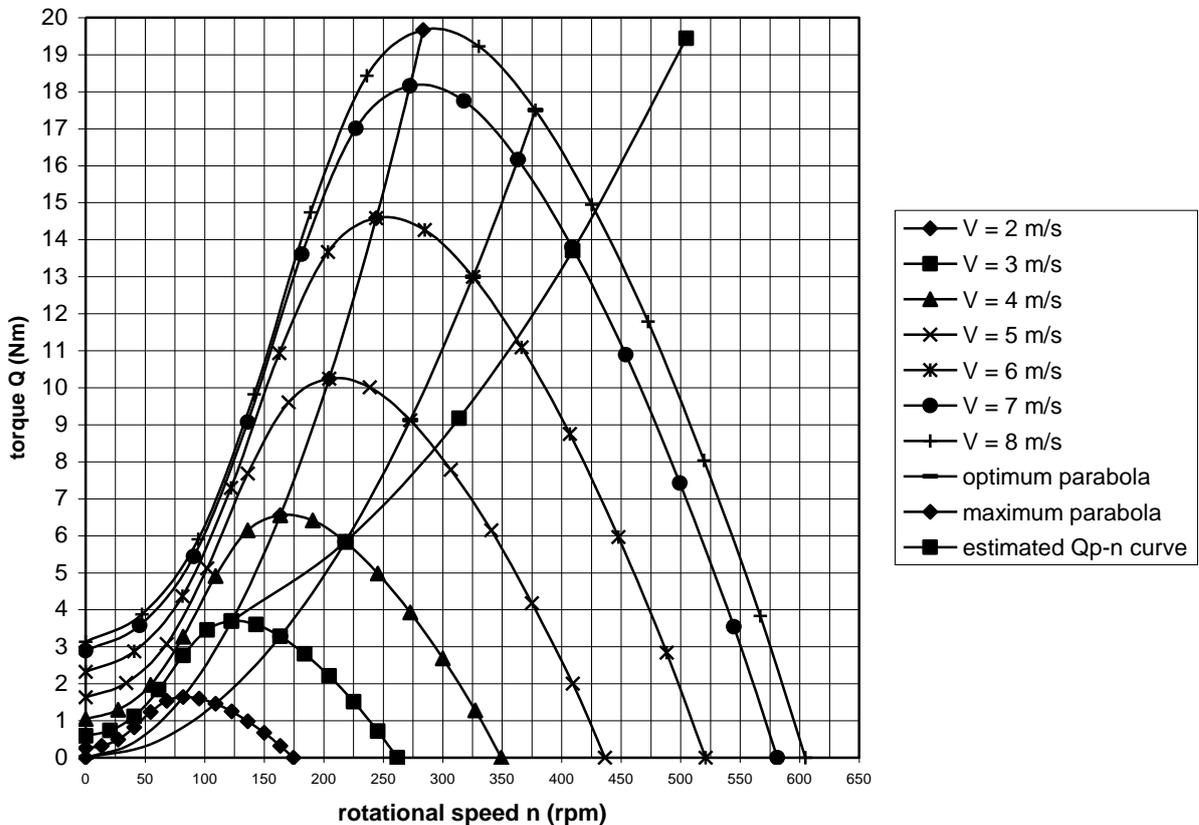


fig. 5 Q_v - n_v curves of the VIRYA-3.5 rotor for $i = 2.5$. Optimum and maximum parabola.

In figure 3 it can be seen that the maximum torque coefficient is reached at about $\lambda = 3$. The curve through the tops of the Q_v - n_v curves called maximum parabola is also given in figure 5.

The Q - n curve of a centrifugal pump depends on the pump geometry and the static height H . It is assumed that the pump is used to pump water in a reservoir and that the outlet opening of the rising main is lying at a height H above the water level in the well, the river or the lake from which the water is pumped. For low rotational speeds, the pump isn't able to supply the static height but at a certain critical rotation speed n_{crit} , the water column in the rising main has just reached the pipe outlet. The Q - n curve of a centrifugal pump for $0 < n < n_{crit}$ is a parabola as the static height which can be supplied increases quadratic to the rotational speed. Assume that the pump is designed such that this parabola coincides to the curve maximum parabola. For rotational speeds higher than n_{crit} , water will be pumped and the flow will increase strongly at increasing rotational speed. The Q - n curve of the pump for $n > n_{crit}$ will also be about a parabola but with a lesser strong slope as for $n < n_{crit}$.

Next it is assumed that n_{crit} is reached at a wind speed of 3 m/s. So the curve maximum parabola will be followed up to the critical rotational speed $n_{crit} = 122.8$ rpm (see table 2). For $n > n_{crit}$, a new parabola has been estimated such that it intersects with the optimum parabola at a wind speed of 4 m/s. This estimated parabola of the pump is called the Q_p - n curve and this curve is also given at figure 5. In figure 5 it can be seen that at high wind speeds, the rotor is running faster than the design tip speed ratio $\lambda_d = 4$ but even at $V = 8$ m/s, the real tip speed ratio is only a little higher than 4.5. So the matching is rather good for wind speed higher than about 3.5 m/s. The pump has to be designed such that the Q_p - n curve intersects with the optimum parabola of the windmill at a wind speed of 4 m/s. So this means that $Q_p = 5.83$ Nm for $n = 218.3$ rpm (see table 2). Design of the correct centrifugal pump for a certain height H is out of the scope of this report.

6 Description of the safety system and the Polycord transmission

Five different safety systems for which the rotor is turned out of the wind, are described in report KD 485 (ref. 4). The hinged side vane safety system which is described in chapter 3.2 of KD 485, is used for all electricity generating VIRYA windmills. It will also be used for the VIRYA-3.5. For the electricity generating VIRYA windmills, the rotor is positioned at the right side of the tower and the vane is positioned at the left side. For the VIRYA-3.5 this is done just opposite because of the direction of the reaction torque of the vertical shaft.

The main advantages of the hinged side vane safety system are that it is rather simple, that the hinges of the vane blade are loaded only lightly and that the whole vane has a large moment of inertia around the tower axis. This makes that the head yaws only slowly and this limits the gyroscopic moment in the rotor blades and in the rotor shaft. A disadvantage is that the vane arm and the vane blade must be rather stiff to prevent flutter of the vane blade at high wind speeds. Flutter is oppressed effectively by two elastic vane blade stops which prevent that the angle of attack of the vane blade can become negative at high wind gusts.

The torque to drive the pump as given by the Q_v - n curve in figure 5, gives a reaction moment M_{react} on the head in the opposite direction. The direction of this reaction moment depends on the direction of rotation of the vertical shaft. The rotor is rotating right hand. The direction of rotation of the vertical shaft is also chosen right hand if seen from above. The wheels of the Polycord transmission are positioned such that this is realized for a rotor which is rotating right hand. If the vertical shaft is rotating right hand, it means that the reaction moment which is executed by the vertical shaft on the head, is working left hand if seen from above. If the rotor is mounted at the left side of the tower, it means that the rotor moment M_{rot} caused by the rotor thrust, is working right hand if seen from above. So the direction M_{react} is opposite the direction of M_{rot} . This is the safest direction of M_{react} because it means that the rotor will turn out of the wind more if the reaction moment disappears, for instance if the well becomes empty or if the pump is removed.

The Polycord transmission is described in report KD 320 (ref. 5) for the VIRYA-2.8B4 windmill. It makes use of a round string which has the advantage that it can be bent in any direction. The string runs in V-shaped grooves of the transmission wheels. A big wheel is mounted on the rotor shaft. A small wheel is mounted on the vertical shaft. Only one small auxiliary wheel is used to make that the string is running exactly tangential at the points where it enters and leaves the wheels. The smallest allowable wheel diameter is ten times the string diameter. For the VIRYA-2.8B4, a string diameter of 12 mm is chosen so the smallest wheel diameter is 120 mm. The biggest wheel must have a diameter of at least 2.5 times the diameter of the smallest wheel to prevent that the auxiliary wheel touches the big wheel. For the VIRYA-2.8B4 it was chosen that the diameter of the big wheel is $2.5 * 120 = 300$ mm. This wheel diameter results in an accelerating gear ratio of 2.5 : 1.

For the VIRYA-3.5 it is chosen to use a string with a diameter of 15 mm (which is the largest diameter available). All dimensions of the transmission are scaled up with a factor $5/4$ which results in a diameter d of the small wheels of 150 mm and in a diameter of 375 mm of the big wheel. The string path is given in figure 3 of KD 320. This figure is copied as figure 6.

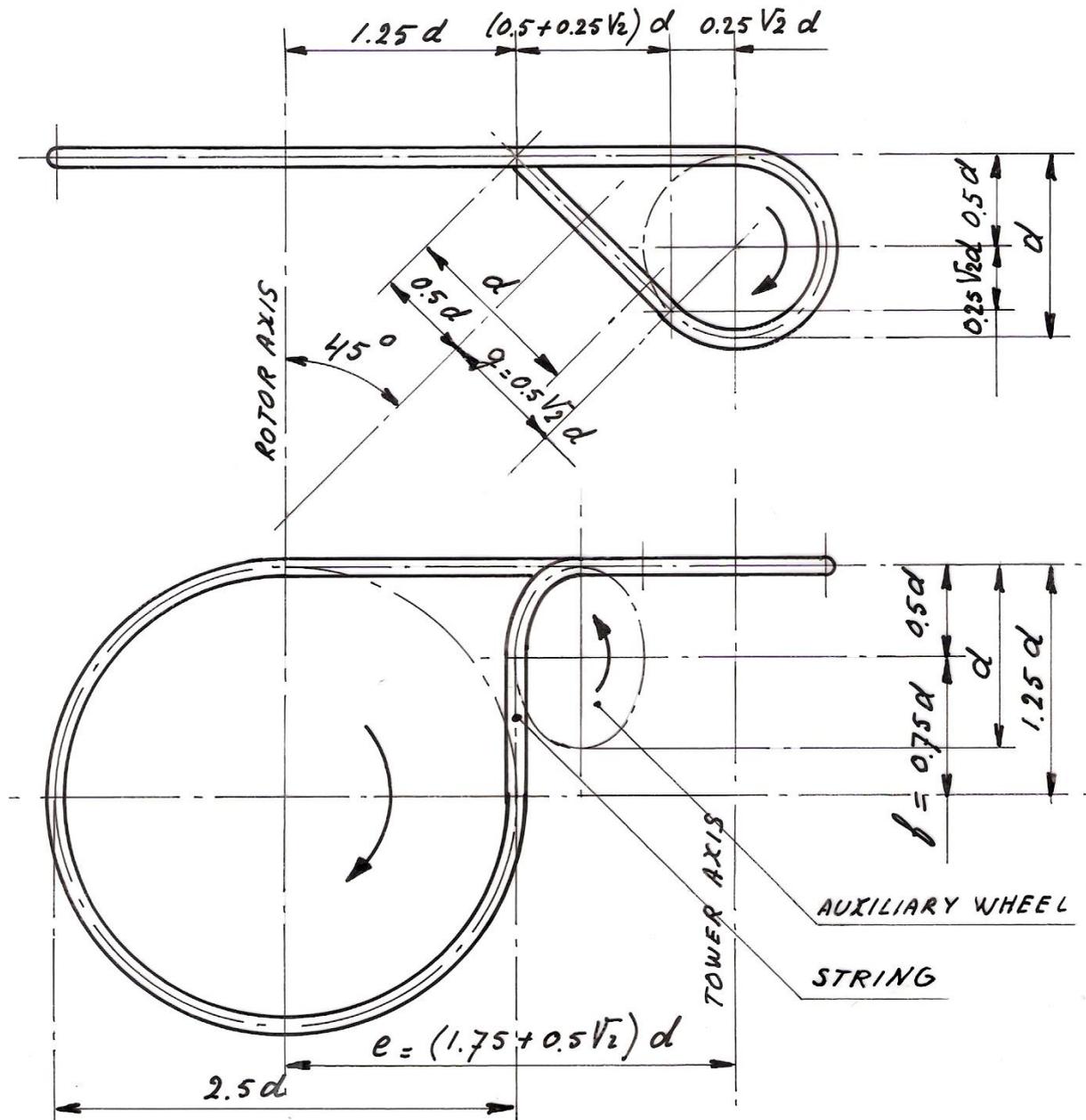


fig. 6 String path and wheels for VIRYA-3.5 transmission with $i = 2.5$

The string is made by melting a certain length together by a so called quick-melt mirror which can be supplied by the manufacturer of the string Habasit. But it is also possible to buy a melted string of a certain length. A pre-tension in the string is realized by taking the string length 8 % shorter than the theoretical length of the string path. So the shafts can have a fixed position. The calculation of the string is given in chapter 3 and 4 of KD 320 (ref. 5). Only some basic calculations will be made for the VIRYA-3.5 to check if the safety system works properly. The eccentricity e is calculated by formula 16 of KD 320. This formula is copied as formula 12.

$$e = (1.75 + 0.5 \sqrt{2}) * d = 2.45711 d \quad (\text{mm}) \quad (12)$$

Substitution of $d = 150 \text{ mm}$ in formula 12 gives that $e = 368.6 \text{ mm} = 0.3686 \text{ m}$. The rotor diameter $D = 3.5 \text{ m}$ so the ratio $e / D = 0.1053$. So e is 10.53 % of the rotor diameter which is rather large and which makes that the side force on the rotor and the self orientating moment will have only a small influence on the rotor moment if the rotor is yawing. If the rotor is perpendicular to the wind, the contributions of these moments are zero.

However, the reaction moment of the vertical shaft M_{react} , has an influence on M_{rot} . Next it is assumed that the rotor is perpendicular to the wind for a wind speed $V = 4 \text{ m/s}$. It is also assumed that the vertical shaft is loaded such by the pump that the rotor runs at its design tip speed ratio $\lambda_d = 4$. This means that $Q_{\text{react}} = 5.83 \text{ Nm}$ left hand (see table 2 and figure 5).

The moment of the rotor executed by the rotor thrust F_t is called M_{rot} . M_{rot} is given by:

$$M_{\text{rot}} = F_t * e \quad (\text{Nm}) \quad (13)$$

F_t for a rotor perpendicular to the wind is given by formula 4.12 of KD 35. This formula is copied as formula 14.

$$F_t = C_t * \frac{1}{2} \rho V^2 * \pi R^2 \quad (\text{N}) \quad (14)$$

(13) + (14) gives:

$$M_{\text{rot}} = C_t * \frac{1}{2} \rho V^2 * \pi R^2 * e \quad (\text{Nm}) \quad (15)$$

C_t is the thrust coefficient which is about 0.75 for a rotor with cambered steel blades. ρ is the density of air and $\rho = 1.2 \text{ kg/m}^3$ for air of 20° C at sea level. V is the undisturbed wind speed and it was assumed that $V = 4 \text{ m/s}$. It was chosen that $R = 1.75 \text{ m}$ and that $e = 0.3686 \text{ m}$. Substitution of these values in formula 15 gives that $M_{\text{rot}} = 25.53 \text{ Nm}$ left hand. So the net left hand moment $M_{\text{net}} = 25.53 - 5.83 = 19.7 \text{ Nm}$. M_{rot} is a factor $25.52 / 5.83 = 4.38$ larger than M_{react} which seems large enough to make that the safety system works also well if the moment coefficient becomes smaller than $C_{q\text{opt}}$ or if the load disappears completely.

In figure 5 it can be seen that the Q_p - n curve of the pump is lying below the optimum parabola for wind speeds higher than 4 m/s but even at $V = 8 \text{ m/s}$, the Q_p value isn't very much lower than the Q_v value which belongs to the optimum parabola. So the ratio $M_{\text{rot}} / M_{\text{react}}$ is therefore becoming only somewhat larger than 4.38 at high wind speeds.

The rotor turns out of the wind at high wind speeds because the vane blade is moving from the almost vertical position at low wind speeds to the almost horizontal position at very high wind speeds. The vane geometry has to be chosen such that the rotor is about perpendicular to the wind at low wind speeds. This situation is shown by the upper picture of figure 6 of KD 485 (ref. 4) for the normal hinge side vane safety system with the rotor at the right side of the tower. For this situation, there is no side force and no self orientating moment acting on the rotor. A sketch of the rotor, the head and the tower pipe is given in appendix 1.

It is assumed that the friction moment of the head bearings can be neglected. It is assumed that the aerodynamic force acting on the vane arm can be neglected too. So the balance of moments is only determined by the forces acting on the vane blade and the rotor and by the reaction moment.

Next the geometry of the vane blade and the vane arm has to be chosen such that the rotor is about perpendicular to the wind at low wind speeds. For low wind speeds it is assumed that the main vane blade is almost vertical so this means that the angle θ in between the vane blade and the vertical is small. The horizontal component of the normal force N acting on the vane blade is $N * \cos \theta$ (see KD 485 figure 6). So $\cos \theta$ is almost 1 if θ is small.

The normal force N is acting at a distance i_1 from the leading edge of the vane blade if the vane blade is almost vertical. i_1 depends on the angle of attack α . The vane blade makes an angle $\alpha = 30^\circ$ with the wind if the rotor is perpendicular to the wind. The vane blade has a width w and a height h . The ratio i_1 / w is given by figure 6 of KD 551. In this figure it can be seen that $i_1 / w = 0.37$ for $\alpha = 30^\circ$. So i_1 is $0.37 * w$ for $\alpha = 30^\circ$.

Assume that the vane blade is made from 2 mm thick aluminium sheet size $0.75 * 0.75$ m. Eight vane blades can be made from a standard sheet size $1.5 * 3$ m. So $w = 0.75$ m and $h = 0.75$ m. As $i_1 = 0.37 w$ for $\alpha = 30^\circ$, it is found that $i_1 = 0.278$ m.

The vane moment M_v around the tower axis is given by:

$$M_v = N \cos \theta * (R_v + i_1) \quad (\text{Nm}) \quad (16)$$

The normal force N acting perpendicular to the vane blade is given by:

$$N = C_n * \frac{1}{2} \rho V^2 * w * h \quad (\text{N}) \quad (17)$$

(16) + (17) gives:

$$M_v = C_n * \frac{1}{2} \rho V^2 * w * h \cos \theta * (R_v + i_1) \quad (\text{Nm}) \quad (16)$$

C_n is the normal coefficient of a square plate. The C_n - α curve is given in figure 5 of KD 551 (ref. 6). In this figure it can be read that $C_n = 1.37$ for $\alpha = 30^\circ$. ρ is the density of air and $\rho = 1.2 \text{ kg/m}^3$ for air of 20° C at sea level. V is the undisturbed wind speed and it was assumed that $V = 4 \text{ m/s}$. It was assumed that $w = 0.75 \text{ m}$ and that $h = 0.75 \text{ m}$. It was also assumed that θ is small and so $\cos \theta = 1$. The value of R_v depends on the length of the vane arm. It is assumed that the vane arm is made of 2" gas pipe with a length of 3 m. A part of this pipe length lies at the front side of the head bearing. This results in a value of R_v of about 2.5 m. It was assumed that $i_1 = 0.278 \text{ m}$. So $R_v + i_1 = 2.778 \text{ m}$. Substitution of these values in formula 16 gives that $M_v = 20.55 \text{ Nm}$. This is only a little larger than the calculated difference M_{net} in between M_{rot} and M_{react} for which it was found that $M_{\text{net}} = 19.7 \text{ Nm}$. So the rotor will have a small negative yaw angle at $V = 4 \text{ m/s}$. If there is no reaction moment, the rotor will have a positive yaw angle of about 8° as now a larger C_n value of about 1.7 is needed to get a vane moment which is 25.53 Nm . This point is lying close to the peak of the C_n - α curve so it might be that, if the vane blade stalls, the rotor turns much further out of the wind if the vertical shaft isn't loaded. But I think that the chosen geometry is rather optimal. To find the exact geometry of the head and the transmission, it will be necessary to make a composite drawing like it was also done for the head of the VIRYA-2.8B4 but I won't do this.

Stopping of the rotor might be possible by putting the vane blade in the horizontal position but this requires a rather complicated mechanism as it has to be activated from ground level. Another option is to use a disk or drum brake on the vertical shaft. This brake can be activated manually. It might also be possible to use the water level of the well. A braked down rotor will also turn out of the wind but at a much higher wind speed as the thrust coefficient of a braked down rotor is much lower than for a rotating rotor.

7 Checking of the strength of the Polycord string

The strength calculations of the 12 mm string of the VIRYA-2.8B4 are given in chapter 4 of KD 320 (ref. 5). The calculations for the VIRYA-2.8B4 were executed for the torque Q on the rotor shaft but for the 15 mm string of the VIRYA-3.5, it will be done for the torque Q_v on the vertical shaft.

The maximum moment which can be passed through for a certain string diameter and a certain wheel diameter depends on the angle of the spanned bow. The smallest angle for a loaded wheel is 225° for the small wheel on the vertical shaft.

The allowable net pulling force F_{net} is given in the first table (Tafel 1) of annex 1 of KD 320 (in German) as Nennumfangskraft F_{un} . F_{net} for a 15 mm diameter string is 225 N. This value is valid for a spanned bow (Umschlingungswinkel) $\beta = 180^\circ$. In figure 1 of appendix 1 of KD 320, the bow factor (Winkelfaktor) c_1 is given as a function of the spanned bow β . For a spanned bow of 225° we can read that c_1 is 0.87 (by interpolation just below $\beta = 220^\circ$). The given value of F_{net} for 180° has to be divided by c_1 to find the corrected value $F_{\text{net cor}}$ or in formula:

$$F_{\text{net cor}} = F_{\text{net}} / c_1 \quad (\text{N}) \quad (17)$$

Substitution of $F_{\text{net}} = 225 \text{ N}$ and $c_1 = 0.87$ in formula 17 gives $F_{\text{net cor}} = 259 \text{ N}$.

The force for 8 % pre-tension = 350 N (see upper table appendix 1, KD 320). This means that the total pulling force for the loaded part of the string is $350 + \frac{1}{2} * 259 = 479.5 \text{ N}$ and that the pulling force for the unloaded part of the string is $350 - \frac{1}{2} * 259 = 220.5 \text{ N}$. So the unloaded part has still a considerable large pulling force and will therefore not run out of the wheels. This is an indication that even for a higher torque than the allowable torque, there is a large reserve. Maybe the string will slip over the smallest wheel if the torque is too large. The string will never break because the breaking force is 7000 N.

The maximum allowable rotor torque Q_{max} on the vertical shaft for the corrected net maximum pulling force $F_{\text{net cor}}$ is given by:

$$Q_{\text{max}} = F_{\text{net cor}} * d / 2 \quad (\text{Nm}) \quad (18)$$

Substitution of $F_{\text{net cor}} = 259 \text{ N}$ and $d = 150 \text{ mm} = 0.15 \text{ m}$ in formula 18 gives $Q_{\text{max}} = 19.4 \text{ Nm}$. The real maximum torque is found for a wind speed of 8 m/s and belongs to the point of intersection of the Q_p - n curve with the Q - n curve of the rotor for $V = 8 \text{ m/s}$. In figure 5 it can be seen that this point lies about at $Q_v = 14.7 \text{ Nm}$ and $n = 430 \text{ rpm}$. So the string is strong enough for this torque.

In figure 5 it can be seen that the maximum torque at $V = 8 \text{ m/s}$ is 19.66 Nm for $n = 283.5 \text{ rpm}$. This is only a little higher than the allowable maximum torque of 19.4 Nm. So the string transmission seems to be just strong enough to brake the rotor to stand still using a disk brake on the vertical shaft at $V = 8 \text{ m/s}$.

In figure 5 it can be seen that the starting torque for $V = 8 \text{ m/s}$ is 3.13 Nm. So once the rotor has braked down, the torque is much lower than the allowable torque. However, the rotor thrust for a braked down rotor is much lower than for a rotating rotor and therefore a much higher wind speed is needed to turn the rotor enough out of the wind. So the Q_v - n_v curves as given in figure 5 are too low for low rotational speeds and the real torque for a braked down rotor can be much higher than 3.13 Nm. However, there is a large reserve as the maximum allowable torque is 19.4 Nm. It must be tested in practice if the string is strong enough at high wind speeds for a braked down rotor.

8 References

- 1 Kragten A. Rotor design and matching for horizontal axis wind turbines, January 1999, reviewed February 2017, free public report KD 35, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.
- 2 Kragten A. The 7.14 %, 10 % and 12.5 % cambered plate as airfoil for windmill rotor blades, Aerodynamic characteristics, geometry, moment of inertia I and moment of resistance W, November 2008, reviewed April 2018, free public report KD 398, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.
- 3 Kragten A. Determination of C_q for low values of λ . Deriving the C_p - λ and C_q - λ curves of the VIRYA-1.8D rotor, July 2002, free public rapport KD 97, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.
- 4 Kragten A. Safety systems for small wind turbines which turn the rotor out of the wind at high wind speeds, February 2012, reviewed November 2016, free public report KD 485, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands
- 5 Kragten A. Development of the VIRYA-2.8B4, a windmill with a Polycord transmission in between the rotor and the vertical shaft to drive a rope pump or an Archimedian screw pump, February 2007, free public report KD 320, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands
- 6 Kragten A. Aerodynamic characteristics of rectangular flat plates with aspect ratios 5 : 1, 2 : 1, 1 : 1, 1 : 2 and 1 : 5 for use as windmill vane blades, March 2014, reviewed April 2016, free public report KD 551, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.

9 Appendix 1 Sketch of the rotor, the head and the tower pipe

