

Determination of C_q for low values of λ
Deriving the C_p - λ and C_q - λ curves of the VIRYA-1.8D rotor

ing. A. Kragten

July 2002
reviewed January 2020

KD 97

It is allowed to copy this report for private use.

Engineering office Kragten Design
Populierenlaan 51
5492 SG Sint-Oedenrode
The Netherlands
telephone: +31 413 475770
e-mail: info@kdwindturbines.nl
website: www.kdwindturbines.nl

Contents	page
1 Introduction	3
2 Determination of C_q for low values of λ	3
3 Calculation of C_q for low values of λ for the VIRYA-1.8D rotor	5
3.1 Determination of C_q for $\lambda = 0$ ($C_{q \text{ start}}$)	5
3.2 Determination of C_q for $\lambda = 1$	5
3.3 Determination of C_q for $\lambda = 2$	7
4 Determination of the C_p - λ and C_q - λ curves	8
5 References	10

1 Introduction

In report KD 35 (ref. 1) a method is given for the design of a windmill rotor. In chapter 6 of KD 35 a simple procedure is given for the determination of the $C_p-\lambda$ and $C_q-\lambda$ curves if only the starting torque coefficient, $C_{q \text{ start}}$, the maximum power coefficient, $C_{p \text{ max}}$ at the optimum tip speed ratio λ_{opt} and the unloaded tip speed ratio λ_{unl} are known. The first part of the $C_q-\lambda$ curve is found by drawing a S-shaped curve in between the point for $C_{q \text{ start}}$ and $C_{q \text{ max}}$. The direction of this curve at $\lambda = 0$, so at $C_{q \text{ start}}$, must be taken horizontal. However this procedure gives only a very rough impression of the shape of this part of the $C_q-\lambda$ curve. The real curve depends on the kind of rotor which is designed and of the kind of airfoil which is used. The shape of the real $C_q-\lambda$ curve for low values of λ is important for the determination of the starting behaviour of the windmill rotor with a load like a PM-generator which has a certain “sticking torque”. In this report (KD 97) a method is given to determine C_q for values of λ in between $\lambda = 0$ and about $\lambda = \frac{1}{2} \lambda_{\text{opt}}$.

2 Determination of C_q for low values of λ

Formula 6.12 of KD 35 gives the starting torque coefficient $C_{q \text{ start}}$ for a blade with a constant chord c and a constant blade angle β . If the blade angle is not constant formula 6.12 can be used with some inaccuracy if the average blade angle is used. If the chord is not constant formula 6.12 can not be used but the blade has to be divided into some sections and the contribution of each section to the starting torque has to be calculated for the average chord and the average blade angle of each section.

If the rotor is rotating, figure 6.4 of KD 35 is no longer valid because now the blade itself has also a certain velocity. Therefore the torque is not only determined by the lift L but also by the drag D . The situation for a rotating blade is given in figure 4.4 and figure 5.1 of KD 35. The components of lift and drag which supply the tangential force ΔF_u are given by formula 4.13. This formula can be written dimensionless as:

$$C_{Fu} = C_l \sin\phi - C_d \cos\phi \quad (-) \quad (1)$$

For low values of λ , C_p will be very low and therefore the wind speed in the rotor plane will be almost equal to the undisturbed wind speed V . In figure 4.4 and figure 5.1 of KD 35, $2/3 V$ has to be replaced by V . If λ_{rd} is replaced by λ_r formula 5.9 of KD 35 changes into:

$$\phi = \arctan 1 / \lambda_r \quad (^\circ) \quad (2)$$

For the previous conditions formula 5.12 of KD 35 changes into:

$$W = \sqrt{(\lambda_r^2 + 1)} \quad (\text{m/s}) \quad (3)$$

The blade is now divided into n sections which are numbered from the blade tip to the blade root. The radius in the middle of a section is called r_n . The chord in the middle of a section is called c_n . Formula 5.7 from KD 35 changes into:

$$\lambda_r = \lambda * r_n / R \quad (-) \quad (4)$$

(2) + (4) gives:

$$\phi = \arctan R / (r_n * \lambda) \quad (^\circ) \quad (5)$$

(3) + (4) gives:

$$W = V \sqrt{(\lambda^2 * r_n^2 / R^2 + 1)} \quad (\text{m/s}) \quad (6)$$

Formula 5.2 of KD 35 can be written as:

$$\alpha = \phi - \beta \quad (^\circ) \quad (7)$$

Formula 6.10 of KD 35 gives the torque Q_{start} for a blade length k . If the blade is divided into a certain number of segments with each a segment length k , formula 6.10 is valid for the contribution ΔQ_{start} of a segment. Formula 6.10 can be modified for a rotating blade for low values of λ , if the influence of the drag is incorporated and if the relative wind speed W is used. The term $R - \frac{1}{2} k$ has to be replaced by r_n . This results in:

$$\Delta Q_n = 0.75 * B * r_n * (C_l \sin\phi - C_d \cos\phi) * \frac{1}{2} \rho V^2 * (\lambda^2 * r_n^2 / R^2 + 1) * c_n * k \quad (\text{Nm}) \quad (8)$$

ΔQ_n can be made dimensionless by dividing by $\frac{1}{2} \rho V^2 * \pi R^3$. This gives:

$$\Delta C_{qn} = 0.75 * B * r_n * (C_l \sin\phi - C_d \cos\phi) * (\lambda^2 * r_n^2 / R^2 + 1) * c_n * k / \pi R^3 \quad (-) \quad (9)$$

C_q is found by taking the sum total of ΔC_{qn} for all chosen segments, so:

$$C_q = \varphi \Delta C_{qn} \quad (-) \quad (10)$$

Formula 9 can be used for values of λ lying in between $\lambda = 0$ en about $\lambda = \frac{1}{2} \lambda_{\text{opt}}$. For $\lambda = 0$ we find that $\phi = 90^\circ$ and therefore the term $(C_l \sin\phi - C_d \cos\phi)$ is replaced by C_l and the term $(\lambda^2 * r_n^2 / R^2 + 1)$ is cancelled. For $\lambda = 0$ formula 9 therefore changes into:

$$\Delta C_{qn \text{ start}} = 0.75 * B * r_n * C_l * c_n * k / \pi R^3 \quad (-) \quad (11)$$

For $\lambda = 0$ formula 7 changes into:

$$\alpha = 90^\circ - \beta \quad (^\circ) \quad (12)$$

So formula 11 and 12 can be used to calculate $C_{q \text{ start}}$ accurately for a rotor blade with a variable chord c and a variable blade angle β .

It is found that C_q is calculated with enough accuracy if the blade length is divided into five equal sections with length k .

3 Calculation of C_q for low values of λ for the VIRYA-1.8D rotor

The geometry calculations of the VIRYA-1.8D rotor are given in chapter 3 of report KD 222 (ref. 2). The VIRYA-1.8D windmill has a three bladed rotor with a diameter $D = 1.8$ m (so $R = 0.9$ m) and a design tip speed ratio $\lambda_d = 4$. Each blade has a constant chord c . Each blade is made from a steel sheet with dimensions $125 * 750 * 1.5$ mm. The blade is 7.14 % cambered which results in a blade chord of $123,3$ mm = $0,1233$ m and a blade length of 0.75 m. So in the middle of the rotor a disk with a radius of 0.15 m is not provided with airfoil.

The blade is connected to the hub by a spoke assembly and the length of the spokes is 0.33 m, so the overlap of the spokes and the blades is 0.18 m. The effect of this overlap on the aerodynamic characteristics of the airfoil is neglected concerning the calculation of C_q for low values of λ because the airfoil is completely stalling (it is not neglected for the calculation of $C_{p\max}$). Each blade has a constant blade angle $\beta = 8^\circ$, so the blade is not twisted.

The blade is divided into five equal sections with a length $k = 0.15$ m. The radius at the centre of each section is r_n , so the radii for the five sections are $r_1 = 0.825$ m, $r_2 = 0.675$ m, $r_3 = 0.525$ m, $r_4 = 0.375$ m and $r_5 = 0.225$ m.

For the calculation of C_q , $C_l-\alpha$ and $C_d-\alpha$ curves for large angles of α are needed. These curves are given in figure 3 of report KD 96 (ref. 3). These curves are valid for a 10 % camber but it is assumed that for large angles α , these curves can also be used for 7.14 % camber as the airfoil is stalling for the whole blade length and during stalling the camber isn't relevant.

For the calculation of ΔC_{qn} formulas 5 and 9 are first made more specific by substitution of the values for B , R , c_n and k .

Substitution of $R = 0.9$ m in formula 5 gives:

$$\phi = \arctan 0.9 / (r_n * \lambda) \quad (^\circ) \quad (13)$$

Substitution of $B = 3$, $R = 0.9$ m, $c_n = 0.1233$ m and $k = 0.15$ m in formula 9 gives:

$$\Delta C_{qn} = 0.01817 * r_n * (C_l \sin\phi - C_d \cos\phi) * (1.2346 \lambda^2 * r_n^2 + 1) \quad (-) \quad (14)$$

3.1 Determination of C_q for $\lambda = 0$ ($C_{q\text{ start}}$)

Because the blade is not twisted it is not necessary to use formulas 11 and 12 to calculate $C_{q\text{ start}}$ accurately, but formula 6.12 from KD 35 can be used (now $k = 0.75$ m and not 0.15 m because the blade is not divided into five sections). This formula is:

$$C_{q\text{ start}} = 0.75 * B * (R - \frac{1}{2}k) * C_l * c * k / \pi R^3 \quad (-) \quad (15)$$

Substitution of $\beta = 8^\circ$ in formula 12 gives $\alpha = 82^\circ$. Figure 3 of KD 96 and $\alpha = 82^\circ$ gives $C_l = 0.26$.

Substitution of $B = 3$, $R = 0.9$ m, $k = 0.75$ m, $C_l = 0.26$, $c = 0.1233$ m in formula 15 gives $C_{q\text{ start}} = 0.0124$.

3.2 Determination of C_q for $\lambda = 1$

Substitution of $\lambda = 1$ in formula 13 gives:

$$\phi = \arctan 0.9 / r_n \quad (^\circ) \quad (16)$$

Substitution of $\lambda = 1$ in formula 14 gives:

$$\Delta C_{qn} = 0.01817 * r_n * (C_1 \sin\phi - C_d \cos\phi) * (1.2346 r_n^2 + 1) \quad (-) \quad (17)$$

Section 1

Substitution of $r_n = r_1 = 0.825$ m in formula 16 gives $\phi = 47.5^\circ$

Substitution of $\phi = 47.5^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 39.5^\circ$.

KD 96 figure 3 for $\alpha = 39.5^\circ$ gives $C_1 = 1.41$.

KD 96 figure 3 for $\alpha = 39.5^\circ$ gives $C_d = 1.13$.

Distribution of $r_n = r_1 = 0.825$ m, $C_1 = 1.41$, $C_d = 1.13$, $\phi = 47.5^\circ$ in formula 17 gives:

$$\Delta C_{qn} = 0.00762.$$

Section 2

Substitution of $r_n = r_2 = 0.675$ m in formula 16 gives $\phi = 53.1^\circ$

Substitution of $\phi = 53.1^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 45.1^\circ$.

KD 96 figure 3 for $\alpha = 45.1^\circ$ gives $C_1 = 1.38$.

KD 96 figure 3 for $\alpha = 45.1^\circ$ gives $C_d = 1.33$.

Distribution of $r_n = r_2 = 0.675$ m, $C_1 = 1.38$, $C_d = 1.33$, $\phi = 53.1^\circ$ in formula 17 gives:

$$\Delta C_{qn} = 0.00585.$$

Section 3

Substitution of $r_n = r_3 = 0.525$ m in formula 16 gives $\phi = 59.7^\circ$

Substitution of $\phi = 59.7^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 51.7^\circ$.

KD 96 figure 3 for $\alpha = 51.7^\circ$ gives $C_1 = 1.21$.

KD 96 figure 3 for $\alpha = 51.7^\circ$ gives $C_d = 1.52$.

Distribution of $r_n = r_3 = 0.525$ m, $C_1 = 1.21$, $C_d = 1.52$, $\phi = 59.7^\circ$ in formula 17 gives:

$$\Delta C_{qn} = 0.00355.$$

Section 4

Substitution of $r_n = r_4 = 0.375$ m in formula 16 gives $\phi = 67.4^\circ$

Substitution of $\phi = 67.4^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 59.4^\circ$.

KD 96 figure 3 for $\alpha = 59.4^\circ$ gives $C_1 = 0.98$.

KD 96 figure 3 for $\alpha = 59.4^\circ$ gives $C_d = 1.62$.

Distribution of $r_n = r_4 = 0.375$ m, $C_1 = 0.98$, $C_d = 1.62$, $\phi = 67.4^\circ$ in formula 17 gives:

$$\Delta C_{qn} = 0.00226.$$

Section 5

Substitution of $r_n = r_5 = 0.225$ m in formula 16 gives $\phi = 76.0^\circ$

Substitution of $\phi = 76.0^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 68.0^\circ$.

KD 96 figure 3 for $\alpha = 68.0^\circ$ gives $C_1 = 0.72$.

KD 96 figure 3 for $\alpha = 68.0^\circ$ gives $C_d = 1.77$.

Distribution of $r_n = r_5 = 0.225$ m, $C_1 = 1.41$, $C_d = 1.13$, $\phi = 47.5^\circ$ in formula 17 gives:

$$\Delta C_{qn} = 0.00117.$$

Substitution of the calculated values for ΔC_{qn} in formula 10 gives:

$$C_q = 0.00762 + 0.00585 + 0.00355 + 0.00226 + 0.00117 = 0.02035 \text{ rounded to } 0.020.$$

This is much larger than the value found for $\lambda = 0$ which was 0.0124. It can be seen that due to the larger radius, the outer sections distribute most to the final C_q value.

3.3 Determination of C_q for $\lambda = 2$

Substitution of $\lambda = 2$ in formula 13 gives:

$$\phi = \arctan 0.45 / r_n \quad (^\circ) \quad (18)$$

Substitution of $\lambda = 2$ in formula 14 gives:

$$\Delta C_{qn} = 0.01817 * r_n * (C_l \sin \phi - C_d \cos \phi) * (4.9384 r_n^2 + 1) \quad (-) \quad (19)$$

Section 1

Substitution of $r_n = r_1 = 0.825$ m in formula 18 gives $\phi = 28.6^\circ$

Substitution of $\phi = 28.6^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 20.6^\circ$.

KD 96 figure 3 for $\alpha = 20.6^\circ$ gives $C_l = 1.21$.

KD 96 figure 3 for $\alpha = 20.6^\circ$ gives $C_d = 0.43$.

Distribution of $r_n = r_1 = 0.825$ m, $C_l = 1.21$, $C_d = 1.43$, $\phi = 28.6^\circ$ in formula 19 gives:

$$\Delta C_{qn} = 0.01318.$$

Section 2

Substitution of $r_n = r_2 = 0.675$ m in formula 18 gives $\phi = 33.7^\circ$

Substitution of $\phi = 33.7^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 25.7^\circ$.

KD 96 figure 3 for $\alpha = 25.7^\circ$ gives $C_l = 1.22$.

KD 96 figure 3 for $\alpha = 25.7^\circ$ gives $C_d = 0.57$.

Distribution of $r_n = r_2 = 0.675$ m, $C_l = 1.22$, $C_d = 0.57$, $\phi = 33.7^\circ$ in formula 19 gives:

$$\Delta C_{qn} = 0.00808.$$

Section 3

Substitution of $r_n = r_3 = 0.525$ m in formula 18 gives $\phi = 40.6^\circ$

Substitution of $\phi = 40.6^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 32.6^\circ$.

KD 96 figure 3 for $\alpha = 32.6^\circ$ gives $C_l = 1.39$.

KD 96 figure 3 for $\alpha = 32.6^\circ$ gives $C_d = 0.86$.

Distribution of $r_n = r_3 = 0.525$ m, $C_l = 1.39$, $C_d = 0.86$, $\phi = 40.6^\circ$ in formula 19 gives:

$$\Delta C_{qn} = 0.00567.$$

Section 4

Substitution of $r_n = r_4 = 0.375$ m in formula 18 gives $\phi = 50.2^\circ$

Substitution of $\phi = 50.2^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 42.2^\circ$.

KD 96 figure 3 for $\alpha = 42.2^\circ$ gives $C_l = 1.40$.

KD 96 figure 3 for $\alpha = 42.2^\circ$ gives $C_d = 1.23$.

Distribution of $r_n = r_4 = 0.375$ m, $C_l = 1.40$, $C_d = 1.23$, $\phi = 50.2^\circ$ in formula 19 gives:

$$\Delta C_{qn} = 0.00333.$$

Section 5

Substitution of $r_n = r_5 = 0.225$ m in formula 18 gives $\phi = 63.4^\circ$

Substitution of $\phi = 63.4^\circ$ and $\beta = 8^\circ$ in formula 7 gives $\alpha = 55.4^\circ$.

KD 96 figure 3 for $\alpha = 55.4^\circ$ gives $C_1 = 1.18$.

KD 96 figure 3 for $\alpha = 55.4^\circ$ gives $C_d = 1.56$.

Distribution of $r_n = r_5 = 0.225$ m, $C_1 = 1.18$, $C_d = 1.56$, $\phi = 63.4^\circ$ in formula 19 gives:

$$\Delta C_{qn} = 0.00182.$$

Substitution of the calculated values for ΔC_{qn} in formula 10 gives:

$$C_q = 0.01318 + 0.00808 + 0.00567 + 0.00333 + 0.00182 = 0.03208 \text{ rounded to } 0.032.$$

This again is larger than the value found for $\lambda = 1$ which was 0.020. So C_q is increasing at increasing λ which favours the starting behaviour. The C_p for $\lambda = 2$ can be calculated with formula 4.5 of KD 35. It is found that $C_p = 0.064$ which is very low and therefore it is acceptable to assume that the windspeed in the rotor plane is equal to the undisturbed wind speed V .

4 Determination of the C_p - λ and C_q - λ curves

In report KD 94 it has been calculated that $C_{p \max} = 0.38$ for $\lambda_{opt} = 4$ and that $\lambda_{unl} = 6.4$. The parts of the C_p - λ and C_q - λ curves between about $\lambda = 3$ and $\lambda = 6.4$ are drawn in accordance to the method as given in chapter 6.4 of KD 35. However the calculated values for $\lambda = 0$, $\lambda = 1$ and $\lambda = 2$ are used to draw the first part of the C_q - λ line. The first part of the C_p - λ line is calculated using formula 4.5 of KD 35 which can be written as:

$$C_p = C_q * \lambda \quad (-) \tag{20}$$

The C_p - λ and C_q - λ curves found this way are given in figure 1 and 2.

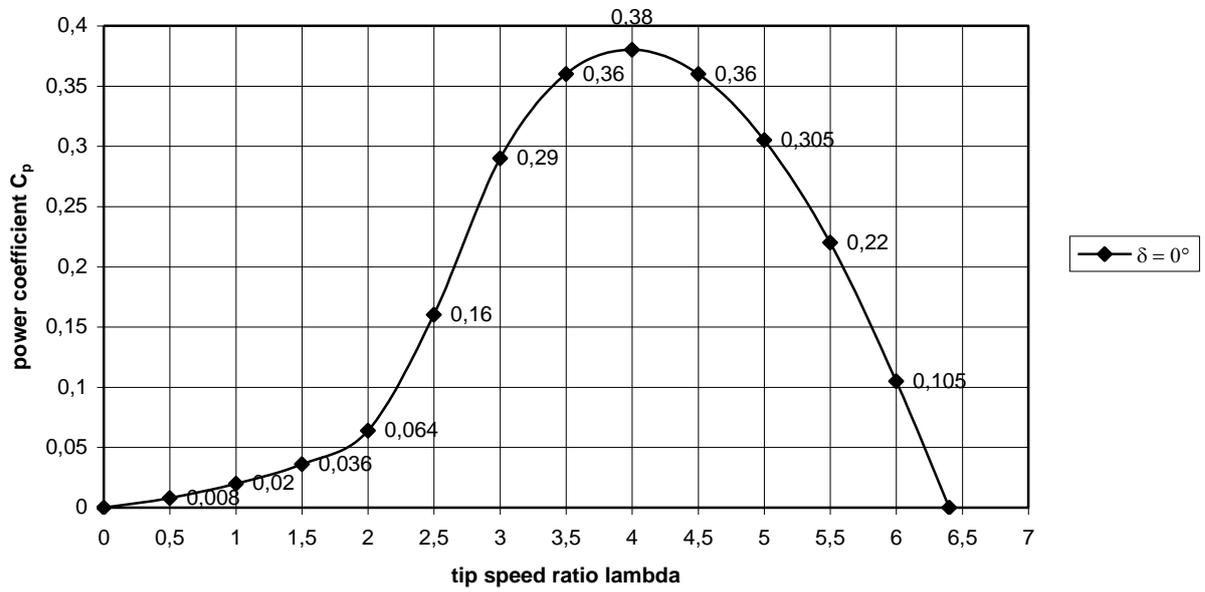


fig. 1 C_p - λ curve for VIRYA-1.8D rotor for wind speed perpendicular to rotor plane ($\delta = 0^\circ$)

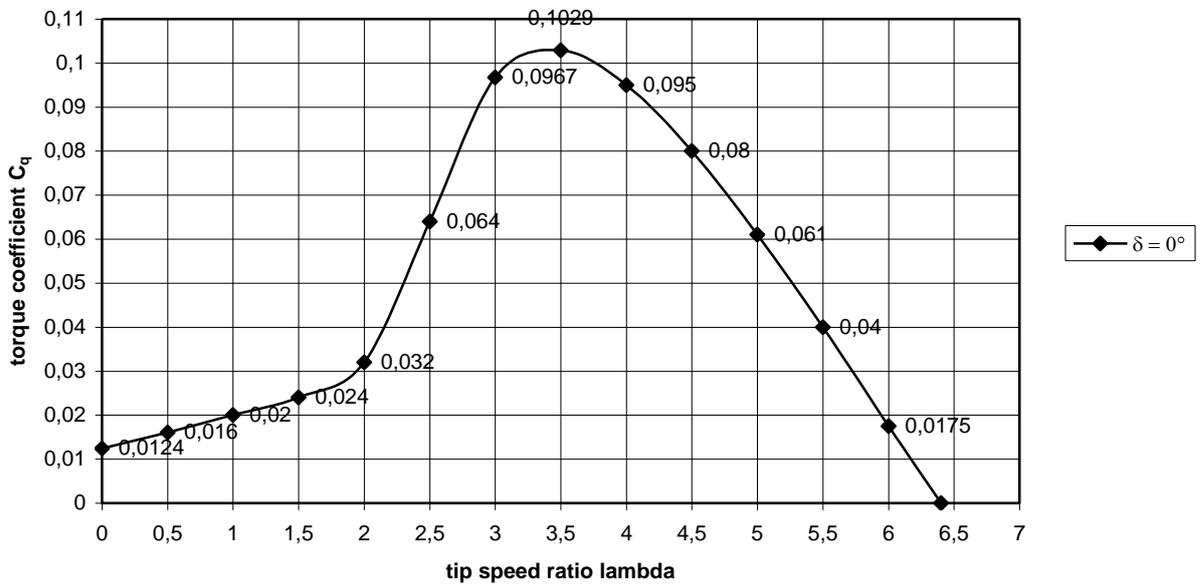


fig. 2 C_q - λ curve for VIRYA-1.8D rotor for wind speed perpendicular to rotor plane ($\delta = 0^\circ$)

5 References

- 1 Kragten A. Rotor design and matching for horizontal axis wind turbines, free public report KD 35, January 1999, reviewed February 2017, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.
- 2 Kragten A. Calculations executed for the 3-bladed rotor of the VIRYA-1.8D windmill ($\lambda_d = 4$, galvanised steel baldes), February 2005, report KD 222, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.
- 3 Kragten A. The 7.14 % cambered plate as airfoil for windmill rotor blades. Aerodynamic characteristics, geometry, moment of inertia I and moment of resistance W. July 2002, report KD 96 (see also free public report KD 398) engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.