Determination of the starting torque coefficient $C_{q\ start}$
for constant chord and tapered blades

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1 Introduction

Generally it is thought that a rotor with blades which are tapered and twisted has a larger starting torque coefficient $C_{q\ start}$ than a rotor with blades with a constant chord and blade angle if both rotors have the same design tip speed ratio. To investigate which type of rotor has really the largest starting torque coefficient, the same examples no. 1 and no. 2 are taken as given in chapter 5.4.1 and 5.4.2 of my public report KD 35 (ref. 1). Both rotors have the same rotor diameter and the same design tip speed ratio so it is reasonable to compare them.

The formula for the starting torque coefficient is derived in chapter 6.3 of KD 35. If the blades have a constant chord and blade angle, this derivation is rather simple as the lift is the same for every blade section and so it can be assumed that the point of application of the lift lies at a radius r_m half way the blade length. Formula 6.12 of KD 35 finally gives $C_{q\ start}$ for a rotor with blades with a constant chord and blade angle. The starting torque coefficient for the rotor of example no. 2 has been calculated in chapter 6.5 of KD 35. However, no formula is given for a tapered blade and the starting torque coefficient of rotor no. 1 has not been calculated in KD 35. At page 42 of KD 35 it is roughly explained how this must be done but in this report KD 697, the calculations will be made for both rotors. So the starting torque coefficient will also be calculated for the rotor of example no. 1. It is assumed that the blade length of 1.5 m is divided into five blade sections with each a length k = 0.3 m and it is investigated how much every blade section contributes to the starting torque coefficient.

2 Determination of $C_{q\ start}$ for the rotor of example no. 2 of KD 35

It is started with the rotor with constant chord blades as described in example no. 2. A picture of the blade of this rotor is given in figure 5.6 of KD 35. The blade has seven stations A, B, C, D, E, F and G. The distance in between the five outer stations is 0.3 m and in between the three inner stations it is 0.15 m. All stations have a chord c = 0.2 m, a blade angle β = 7° and are provided with the Gö 623 airfoil. The angle of attack α is 90° - 7° = 83° if the rotor isn't rotating. A picture of this blade is given in figure 1.
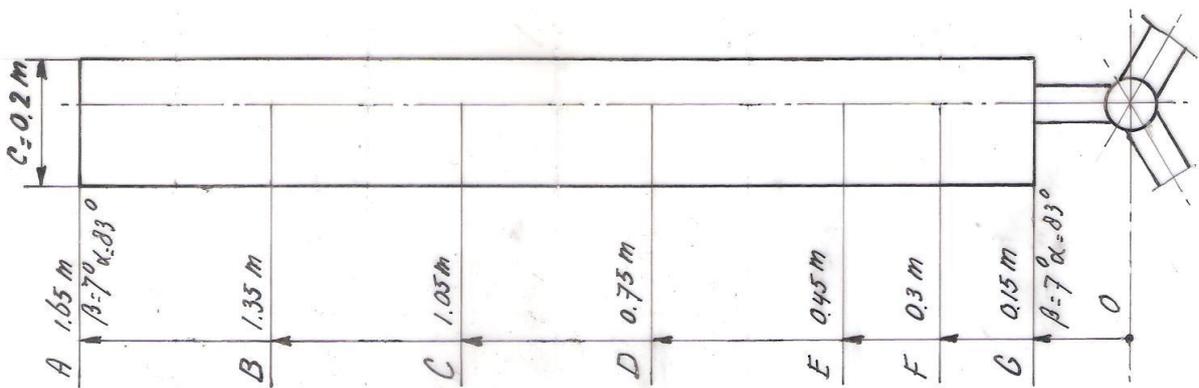

fig. 1 Constant chord blade with 7 stations for the rotor of example no. 2

The starting torque coefficient is given by formula 6.12 of KD 35. This formula is copied as formula 1.

$$C_{q\ start} = \frac{0.75 * B * (R - \frac{1}{2}k) * C_l * c * k}{\pi R^3} \quad (-) \qquad (1)$$

In this formula B is the number of blades (-), R is the tip radius of the rotor (m), k is the blade length (m), C_l is the lift coefficient (-) and c is the chord (m). The lift coefficient C_l depends on the angle of attack α. In figure 6.4 of KD 35 it can be seen that $\phi = 90°$ when the rotor isn't rotating and so it is valid that:

$$\alpha = 90° - \beta \qquad (°) \qquad\qquad\qquad\qquad (2)$$

In this formula α is the angle of attack (°) and β is the blade angle (°). The blade angles β are rather small for fast running rotors and so the angles of attack α are very large during starting. The C_l-α curve for large angles of α for the Gö 623 airfoil is given in figure 5.10 of KD 35.

$C_{q\ start}$ is now calculated for the rotor of example no. 2 out of KD 35. For this rotor it is valid that B = 3. For this rotor it is valid that R = 1.65 m (see figure 1). For this rotor it is valid that k = 1.65 − 0.15 = 1.5 m (see figure 1). In table 5.3 of KD 35 and in figure 1 it can be seen that β = 7° for the whole blade. So α = 90° - 7° = 83° for the whole blade. In figure 5.10 of KD 35 it can be read that $C_l = 0.24$ for α = 83°. In figure 1 it can be seen that c = 0.2 m. Substitution of these values in formula 1 gives that $C_{q\ start} = 0.010$. This is the same value as found in chapter 6.5 of KD 35.

3 Determination of $C_{q\ start}$ for the rotor of example no. 1 of KD 35

Next the value of $C_{q\ start}$ for the rotor of example no. 1 will be determined. The contribution of each 0.3 m wide blade section is calculated for the heart of each section. So four more stations are added. Station a lies half way station A and B at r = 1.5 m. Station b lies half way station B and C at r = 1.2 m. Station c lies half way station C and D at r = 0.9 m. Station d lies half way station D and E at r = 0.6 m. The chord of this rotor was linearised, so the chords for the four new stations can be calculated easily from figure 5.3 of KD 35. For the blade angle, the average of the adjacent blade angles is taken. The blade of this rotor is given in figure 2. The chords, the blade angles β and the angles of attack α for the heart of each blade section are also given in figure 2. For all stations, the Gö 623 airfoil is used.
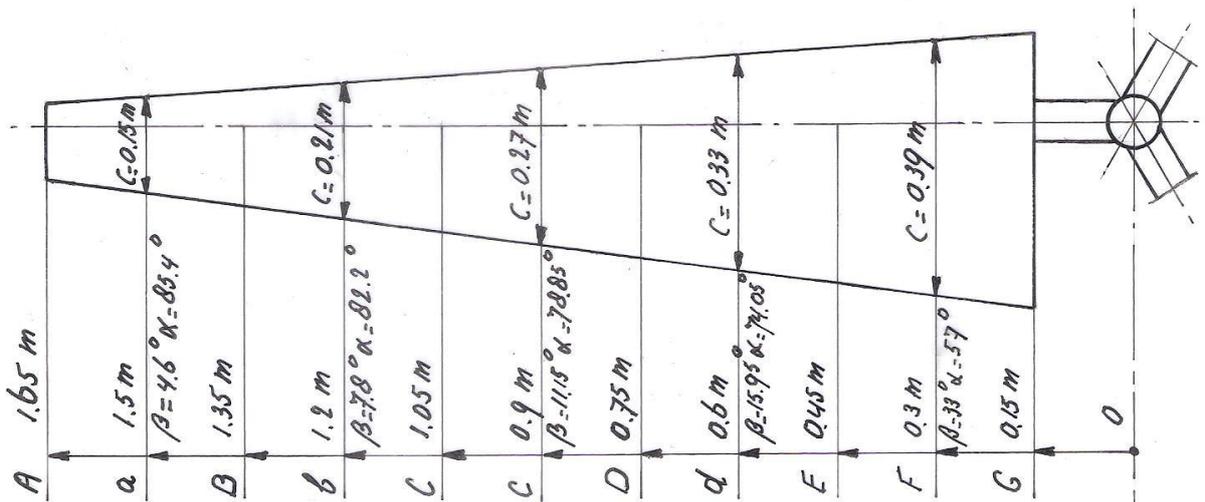

fig. 2 Tapered blade with 11 stations for the rotor of example no. 1

The lift of each blade section differs for a tapered blade. The lift depends on the average chord and on the average blade angle. $C_{q\ start}$ is now calculated for the rotor of example no. 1 out of KD 35. The contribution of the starting torque coefficient of each blade section $\Delta C_{q\ start}$ to the total starting torque coefficient $C_{q\ start}$ for this rotor can be calculated using a formula which differs only slightly from formula 1.

It can be derived that the formula for $\Delta C_{q\ start}$ is given as:

$$\Delta C_{q\ start} = \frac{0.75 * B * r_m * C_l * c * k}{\pi R^3} \qquad (-) \qquad\qquad (3)$$

In this formula, r_m is the radius at the heart of the blade section (m). C_l is the lift coefficient for the heart of the blade section. c is the average chord at the heart of the blade section. The other parameters are the same as for formula 1. So one has to use the correct values of B, r_m, C_l, c, k and R. The values of r_m, c, k and R are derived from figure 2. The values of C_l are read in figure 5.10 of KD 35 for the given values of α.

Blade section a

B = 3, r_m = 1.5 m, α = 85.4° so C_l = 0.2, c = 0.15 m, k = 0.3 m and R = 1.65 m. Substitution of these values in formula 3 gives that $\Delta C_{q\ start\ a}$ = 0.00215.

Blade section b

B = 3, r_m = 1.2 m, α = 82.2° so C_l = 0.27, c = 0.21 m, k = 0.3 m and R = 1.65 m. Substitution of these values in formula 3 gives that $\Delta C_{q\ start\ b}$ = 0.00325.

Blade section c

B = 3, r_m = 0.9 m, α = 78.85° so C_l = 0.39, c = 0.27 m, k = 0.3 m and R = 1.65 m. Substitution of these values in formula 3 gives that $\Delta C_{q\ start\ c}$ = 0.00453.

Blade section d

B = 3, r_m = 0.6 m, α = 74.05° so C_l = 0.51, c = 0.33 m, k = 0.3 m and R = 1.65 m. Substitution of these values in formula 3 gives that $\Delta C_{q\ start\ d}$ = 0.00483.

Blade section F

B = 3, r_m = 0.3 m, α = 57° so C_l = 0.95, c = 0.39 m, k = 0.3 m and R = 1.65. Substitution of these values in formula 3 gives that $\Delta C_{q\ start\ F}$ = 0.00532.

The total starting torque coefficient $C_{q\ start}$ is given by:

$$C_{q\ start} = \Delta C_{q\ start\ a} + \Delta C_{q\ start\ b} + \Delta C_{q\ start\ c} + \Delta C_{q\ start\ d} + \Delta C_{q\ start\ F} \qquad (-) \qquad\qquad (4)$$

Substitution of $\Delta C_{q\ start\ a}$ = 0.00215, $\Delta C_{q\ start\ b}$ = 0.00325, $\Delta C_{q\ start\ c}$ = 0.00453, $\Delta C_{q\ start\ d}$ = 0.00483 and $\Delta C_{q\ start\ F}$ = 0.00532 in formula 4 gives that $C_{q\ start}$ = 0.02008.

So $C_{q\ start}$ for the tapered and twisted blade is about a factor two higher than for the constant chord blade.

The starting torque coefficient is calculated for a non rotating rotor. If the blade is rotating slowly, the torque coefficient will change. In report KD 97 (ref. 2), I have determined how the C_q value changes if the rotor is rotating slowly. This was done for the VIRYA-1.8D rotor with cambered stainless steel constant chord blades and a design tip speed ratio of 4 and it appeared that C_q is rising directly if λ becomes larger than zero. This isn't necessary the case for rotors with a normal airfoil and a higher design tip speed ratio. But already 20-11-1980, a scale model of a 3-bladed rotor with constant chord and blade angle and with a design tip speed ratio $\lambda_d = 6$ has been measured in the wind tunnel. It has been found that the maximum C_p was more than 0.4 and that the C_q-λ curve for low values of λ was not horizontal but somewhat rising if the λ becomes larger than zero.

4 References

1   Kragten A. Rotor design and matching for horizontal axis wind turbines, January 1999, reviewed November 2015, free public rapport KD 35, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.

2   Kragten A. Determination of C_q for low values of λ. Deriving the C_p-λ and C_q-λ curves of the VIRYA-1.8D rotor, July 2002, reviewed January 2020, free public report KD 97, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.