

**Calculations executed for the 3-bladed rotor of the VIRYA-4.4 windmill  
( $\lambda_d = 5.25$ , stainless steel blades) using a 34-pole PM-generator for coupling to  
a centrifugal pump with a 1.1 kW asynchronous motor**

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It is allowed to copy this report for private use. It is allowed to use the idea of the 3-bladed rotor and the 34-pole PM-generator. The VIRYA-4.4 has not yet been built and tested and no responsibility is accepted for the correct functioning of it.

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Contains	page
1 Introduction	3
2 Description of the rotor of the VIRYA-4.4 windmill	3
3 Calculations of the rotor geometry	4
4 Determination of the $C_p$ - $\lambda$ and the $C_q$ - $\lambda$ curves	5
5 Determination of the P-n curves, the optimum cubic line and the lines for constant frequencies	7
6 Calculation of the strength of the spoke assembly	10
6.1 Bending stress in the spoke for a rotating rotor and $V = 11$ m/s	10
6.2 Bending stress in the strip for a slowed down rotor	15
7 References	16

## 1 Introduction

The VIRYA-4.4 windmill is developed for manufacture in western countries and in developing countries. The VIRYA-4.4 has a 3-bladed rotor with stainless steel blades and a design tip speed ratio  $\lambda_d = 5.25$ . The generator of the VIRYA-4.4 is made from the housing of a 6-pole, 5.5 kW asynchronous motor frame size IEC132. It has an armature with 34 poles. The same generator is also used for the VIRYA-5 windmill with two wooden blades and is described in report KD 614 (ref. 1). Metal workshops generally don't like it to make wooden blades and therefore it is investigated if it is possible to design a rotor with stainless steel blades for the same generator. The generator uses the original motor shaft which has a shaft with a diameter of 38 mm, a length of 80 mm, a 10 mm key and a central threaded hole M12.

The VIRYA-4.4 is meant to be coupled to the 1.1 kW asynchronous motor of a standard centrifugal pump if the generator has a high voltage winding. So it isn't necessary to design a pump. The generator can also be used for 24 V battery charging if the generator has a low voltage winding rectified in star.

The head is the same as the head of the VIRYA-4.6 which is provided with a vane blade made out of 9 mm water proof plywood. The rated wind speed with this vane blade is about 11 m/s. The 12 m high, 3-legs tower of the VIRYA-4.6 can be used but it is also possible to use the 8.65 m high tubular tower which is described in report KD 582 (ref. 2).

## 2 Description of the rotor of the VIRYA-4.4 windmill

The 3-bladed rotor of the VIRYA-4.4 windmill has a diameter  $D = 4.4$  m and a design tip speed ratio  $\lambda_d = 5.25$ . Advantages of a 3-bladed rotor are that the gyroscopic moment in the rotor shaft isn't fluctuating and that a 3-bladed rotor looks nicer than a 2-bladed rotor.

The rotor has blades with a constant chord and no twist and is provided with a 7.14 % cambered airfoil. A blade is made of a stainless steel strip with dimensions of 200 \* 1500 \* 3 mm and fifteen blades for five rotors can be made from a standard sheet of 1.5 \* 3 m with no material waste. Because the blade is cambered, the chord  $c$  is a little less than the blade width, resulting in  $c = 197.3$  mm = 0.1973 m.

The blades are connected to each other by a spoke assembly which is made of three spokes welded together at the centre at an angle of 120°. The welds are ground flat. Six 1000 mm long spokes can be made out of a standard length of 6 m. A spoke has a width of 80 mm and a thickness of 12 mm. The spokes are rather flexible and this neutralises vibrations caused by wind turbulence.

The overlap in between spoke and blade is 300 mm resulting in a rotor diameter of 4.4 m and in a free blade length of 1.2 m. This free blade length, in combination with a sheet thickness of 3 mm, is expected to be short enough to prevent flutter at high wind speeds. However, the connection in between the blade and the end of the central strip must be very tight. This is realised by bevelling the left and right backside of the spokes under an angle of 3° and by an extra cambered sheet size 4 \* 60 \* 60 mm which is placed under the head of the outer connecting bolt. A sketch of the rotor is given in figure 1.

The spokes are twisted in between the hub and the blade root to realise the correct blade setting angle. Three M12 \* 50 mm bolts are used for connection of a blade to a spoke. These bolts can also be used for connection of the balancing weights.

The stainless steel hub has a diameter of 90 mm and a length of 80 mm and is provided with an internal key groove. The spoke assembly is clamped in between the hub and a 90 mm clamping disk to prevent that the welds are loaded by a bending moment. Four hexagon socket head cap screws M12 \* 40 mm are used for clamping. The mass of the whole rotor including the hub is about 47 kg which seems acceptable for a 3-bladed steel rotor with a diameter of 4.4 m.

### 3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 699) has its own formula numbering. Substitution of  $\lambda_d = 5.25$  and  $R = 2.2$  m in formula (5.1) of KD 35 gives:

$$\lambda_{rd} = 2.3864 * r \quad (-) \quad (1)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \quad (^\circ) \quad (2)$$

$$\phi = 2/3 \arctan 1 / \lambda_{rd} \quad (^\circ) \quad (3)$$

Substitution of  $B = 3$  and  $c = 0.1973$  m in formula (5.4) of KD 35 gives:

$$C_l = 42.461 r (1 - \cos\phi) \quad (-) \quad (4)$$

Substitution of  $V = 6$  m/s and  $c = 0.1973$  m in formula (5.5) of KD 35 gives:

$$R_{e_r} = 0.790 * 10^5 * \sqrt{(\lambda_{rd}^2 + 4/9)} \quad (-) \quad (5)$$

The blade is calculated for six stations A till F which have a distance of 0.3 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for  $C_l$ ,  $\alpha$  and  $\beta$  and next  $\beta$  is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 4). The Reynolds values for the stations are calculated for a wind speed of 6 m/s because this is a reasonable wind speed for a windmill with  $V_{rated} = 11$  m/s (see chapter 5). Those airfoil Reynolds numbers are used which are lying closest to the calculated values.

station	r (m)	$\lambda_{rd}$ (-)	$\phi$ (°)	c (m)	$C_{l_{th}}$ (-)	$C_{l_{lin}}$ (-)	$R_{e_r} * 10^{-5}$ V = 6 m/s	$R_{e_r} * 10^{-5}$ 7.14 %	$\alpha_{th}$ (°)	$\alpha_{lin}$ (°)	$\beta_{th}$ (°)	$\beta_{lin}$ (°)	$C_d/C_{l_{lin}}$ (-)
A	2.2	5.25	7.2	0.1973	0.73	0.73	4.18	3.4	-0.3	-0.3	7.5	7.5	0.039
B	1.9	4.534	8.3	0.1973	0.84	0.85	3.62	3.4	0.6	0.8	7.7	7.5	0.035
C	1.6	3.818	9.8	0.1973	0.99	1.00	3.06	3.4	2.1	2.3	7.7	7.5	0.039
D	1.3	3.102	11.9	0.1973	1.19	1.15	2.51	2.5	4.7	4.4	7.2	7.5	0.043
E	1	2.386	15.2	0.1973	1.48	1.36	1.96	1.7	-	7.7	-	7.5	0.115
F	0.7	1.670	20.6	0.1973	1.90	1.37	1.42	1.2	-	13.1	-	7.5	0.190

table 1 Calculation of the blade geometry of the VIRYA-4.4 rotor

No value for  $\alpha_{th}$  and therefore for  $\beta_{th}$  is found for stations E and F because the required  $C_l$  values can't be generated. The theoretical blade angle  $\beta_{th}$  varies only in between  $7.7^\circ$  and  $7.2^\circ$ . If a constant blade angle of  $7.5^\circ$  is taken, the linearised blade angles are lying close to the theoretical values for the most important outer part of the blade. The spokes are twisted  $7.5^\circ$  right hand in between the hub and the blade root to get the correct blade angles. A sketch of the rotor is given in figure 1.

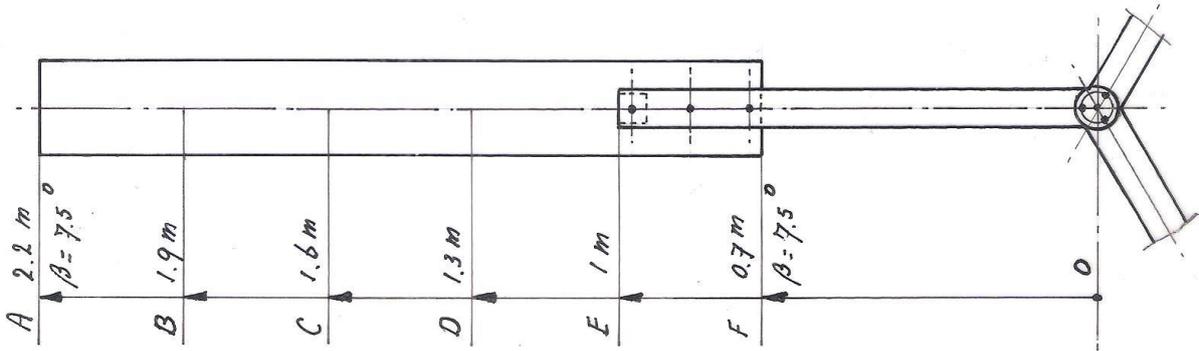


fig. 1 Sketch of the VIRYA-4.4 rotor

#### 4 Determination of the $C_p$ - $\lambda$ and the $C_q$ - $\lambda$ curves

The determination of the  $C_p$ - $\lambda$  and  $C_q$ - $\lambda$  curves is given in chapter 6 of KD 35. The average  $C_d/C_l$  ratio for the most important outer part of the blade is about 0.039. Figure 4.7 of KD 35 (for  $B = 3$ ) and  $\lambda_{opt} = 5.25$  and  $C_d/C_l = 0.039$  gives  $C_{p\ th} = 0.425$ .

Substitution of  $C_{p\ th} = 0.425$ ,  $R = 2.2$  m and blade length  $k = 1.5$  m in formula 6.3 of KD 35 gives  $C_{p\ max} = 0.38$ .

$$C_{q\ opt} = C_{p\ max} / \lambda_{opt} = 0.38 / 5.25 = 0.0724.$$

Substitution of  $\lambda_{opt} = \lambda_d = 5.25$  in formula 6.4 of KD 35 gives  $\lambda_{unl} = 8.4$ .

The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - \frac{1}{2}k) * C_l * c * k / \pi R^3 \quad (-) \quad (6)$$

The blade angle is  $7.5^\circ$  for the whole blade. For a non rotating rotor, the angle of attack  $\alpha$  is therefore  $90^\circ - 7.5^\circ = 82.5^\circ$ . The estimated  $C_l$ - $\alpha$  curve for large values of  $\alpha$  is given as figure 5 of KD 398. For  $\alpha = 82.5^\circ$  it can be read that  $C_l = 0.25$ .

Substitution of  $B = 3$ ,  $R = 2.2$  m,  $k = 1.5$  m,  $C_l = 0.25$  and  $c = 0.1973$  m in formula 6 gives that  $C_{q\ start} = 0.0072$ . For the ratio in between the starting torque and the optimum torque we find that it is  $0.0072 / 0.0724 = 0.1$ . This is acceptable for a rotor with  $\lambda_d = 5.25$ .

The starting wind speed  $V_{start}$  of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left( \frac{Q_s}{C_{q\ start} * \frac{1}{2}\rho * \pi R^3} \right)} \quad (m/s) \quad (7)$$

For the VIRYA-4.6B2 generator with 5RN132M04V housing, a sticking torque  $Q_s$  of 1.6 Nm has been measured if the rotor is not rotating. It is assumed that the sticking torque of the 32-pole generator with IEC132 housing is somewhat higher as the armature volume is larger. Assume  $Q_s = 2$  Nm. Substitution of  $Q_s = 2$  Nm,  $C_{q\ start} = 0.0072$ ,  $\rho = 1.2$  kg/m<sup>3</sup> and  $R = 2.2$  m in formula 7 gives that  $V_{start} = 3.7$  m/s. This is low enough for a rotor with a design tip speed ratio of 5.25 and a rated wind speed of 11 m/s.

Because the generator is not rectified for coupling to an asynchronous motor and rectified in star for 24 V battery charging, the unloaded Q-n curve is rising only a little at increasing rotational speed. The  $C_q$ - $\lambda$  curve of a rotor equipped with cambered steel blades is rising rather fast for  $V = 3.7$  m/s. Therefore, the real starting wind speed will be about the same as the calculated value.

In chapter 6.4 of KD 35 it is explained how rather accurate  $C_p$ - $\lambda$  and  $C_q$ - $\lambda$  curves can be determined if only two points of the  $C_p$ - $\lambda$  curve and one point of the  $C_q$ - $\lambda$  curve are known. The first part of the  $C_q$ - $\lambda$  curve is determined according to KD 35 by drawing a S-shaped line which is horizontal for  $\lambda = 0$ .

Kragten Design developed a method with which the value of  $C_q$  for low values of  $\lambda$  can be determined (see report KD 97 ref. 5). With this method, it can be determined that the  $C_q$ - $\lambda$  curve is directly rising for low values of  $\lambda$  if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated  $C_p$ - $\lambda$  and  $C_q$ - $\lambda$  curves for the VIRYA-4.4 rotor are given in figure 2 and 3.

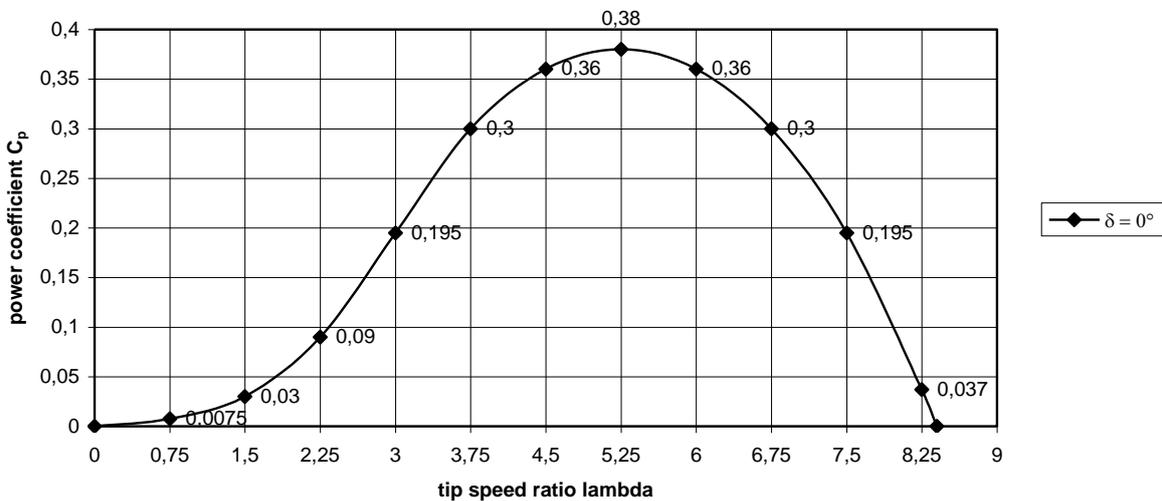


fig. 2 Estimated  $C_p$ - $\lambda$  curve for the VIRYA-4.4 rotor for the wind direction perpendicular to the rotor ( $\delta = 0^\circ$ )

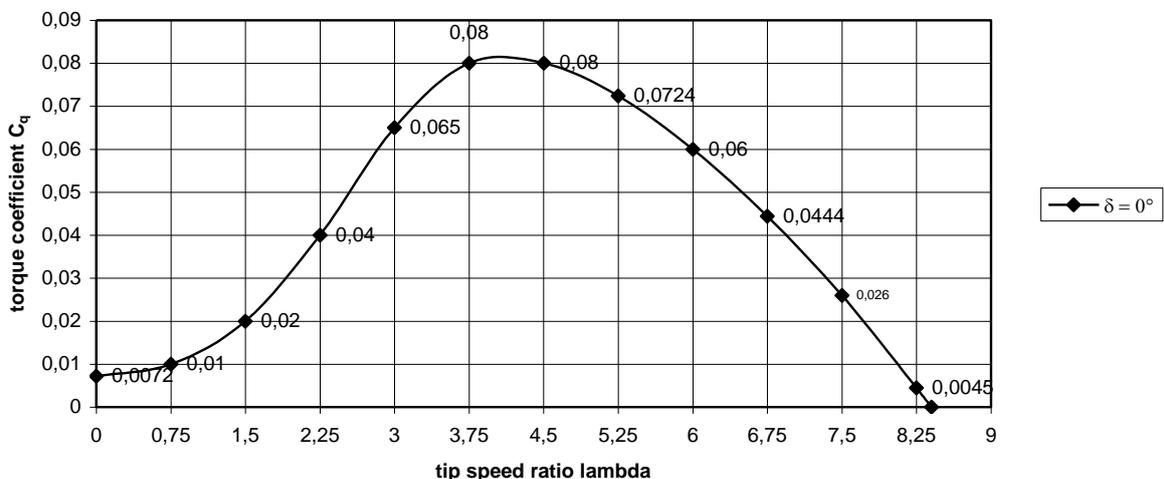


fig. 3 Estimated  $C_q$ - $\lambda$  curve for the VIRYA-4.4 rotor for the wind direction perpendicular to the rotor ( $\delta = 0^\circ$ )

## 5 Determination of the P-n curves, the optimum cubic line and the lines for constant frequencies

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a  $C_p\lambda$  curve of the rotor and a  $\delta$ -V curve of the safety system together with the formulas for the power P and the rotational speed n. The head of the VIRYA-4.4 will be the same as the head of the VIRYA-4.6B2. The VIRYA-4.6B2 has a vane blade made out of 9 mm water proof plywood. The  $\delta$ -V curve has been checked in report KD 223 (ref. 6) for the VIRYA-3.3D windmill. This windmill also has a 9 mm plywood vane blade and steel cambered rotor blades. The rated wind speed for this vane blade is about 11 m/s. The estimated  $\delta$ -V curve is given in figure 4.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 11 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 11 m/s will therefore also be valid for wind speeds higher than 11 m/s.

The P-n curves are used to check the matching with the  $P_{mech}$ -n curve of the generator for a certain gear ratio i (the VIRYA-4.4 has no gearing so  $i = 1$ ). Because we are especially interested in the domain around the optimal cubic line and because the P-n curves for low values of  $\lambda$  appear to lie very close to each other, the P-n curves are not determined for low values of  $\lambda$ . The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9, 10 and 11 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle  $\delta$  and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

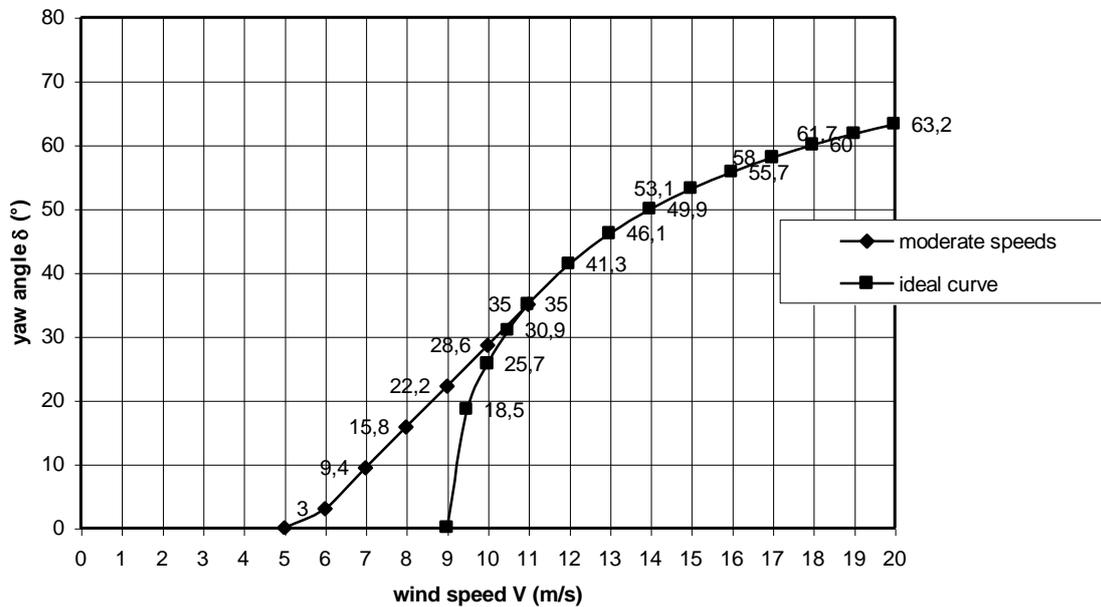


fig. 4 Estimated  $\delta$ -V curve VIRYA-4.4 for a 9 mm water proof plywood vane blade

Substitution of  $R = 2.2$  m in formula 7.1 of KD 35 gives:

$$n_{\delta} = 4.341 * \lambda * \cos\delta * V \quad (\text{rpm}) \quad (8)$$

Substitution of  $\rho = 1.2$  kg / m<sup>3</sup> and  $R = 2.2$  m in formula 7.10 of KD 35 gives:

$$P_{\delta} = 9.123 * C_p * \cos^3\delta * V^3 \quad (\text{W}) \quad (9)$$

The P-n curves are determined for  $C_p$  values belonging to  $\lambda = 3, 3.75, 4.5, 5.25, 6, 6.75, 7.5$  and  $8.4$ . (see figure 2). For a certain wind speed, for instance  $V = 3$  m/s, related values of  $C_p$  and  $\lambda$  are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 4, is taken into account. The result of the calculations is given in table 3.

$\lambda$ (-)	$C_p$ (-)	V = 3 m/s $\delta = 0^\circ$		V = 4 m/s $\delta = 0^\circ$		V = 5 m/s $\delta = 0^\circ$		V = 6 m/s $\delta = 3^\circ$		V = 7 m/s $\delta = 9.4^\circ$		V = 8 m/s $\delta = 15.8^\circ$		V = 9 m/s $\delta = 22.2^\circ$		V = 10 m/s $\delta = 28.6^\circ$		V = 11 m/s $\delta = 35^\circ$	
		n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	$n_\delta$ (rpm)	$P_\delta$ (W)	$n_\delta$ (rpm)	$P_\delta$ (W)	$n_\delta$ (rpm)	$P_\delta$ (W)	$n_\delta$ (rpm)	$P_\delta$ (W)	$n_\delta$ (rpm)	$P_\delta$ (W)	$n_\delta$ (rpm)	$P_\delta$ (W)
3	0.195	39.1	48.0	52.1	113.9	65.1	222.4	78.0	382.7	89.9	585.9	100.2	811.5	108.5	1029	114.3	1204	117.3	1301
3.75	0.3	48.8	73.9	65.1	175.2	81.4	342.1	97.5	588.7	112.4	901.4	125.3	1248	135.6	1584	142.9	1852	146.7	2002
4.5	0.36	58.6	88.7	78.1	210.2	97.7	410.5	117.0	706.5	134.9	1082	150.4	1498	162.8	1900	171.5	2223	176.0	2403
5.25	0.38	68.4	93.6	91.2	221.9	114.0	433.3	136.6	745.7	157.4	1142	175.4	1581	189.9	2006	200.1	2346	205.4	2536
6	0.36	78.1	88.7	104.2	210.2	130.2	410.5	156.1	706.5	179.9	1082	200.5	1498	217.0	1900	228.7	2223	234.7	2403
6.75	0.3	87.9	73.9	117.2	175.2	146.5	342.1	175.6	588.7	202.4	901.4	225.6	1248	244.2	1584	257.3	1852	264.0	2002
7.5	0.195	97.7	48.0	130.2	113.9	162.8	222.4	195.1	382.7	224.8	585.9	250.6	811.5	271.3	1029	285.8	1204	293.4	1301
8.4	0	109.4	0	145.9	0	182.3	0	218.5	0	251.8	0	280.7	0	303.9	0	320.2	0	328.6	0

table 3 Calculated values of n and P as a function of  $\lambda$  and V for the VIRYA-4.4 rotor

The calculated values for n and P are plotted in figure 5. The optimum cubic line which can be drawn through the maximum of all P-n curves is also given in figure 5.

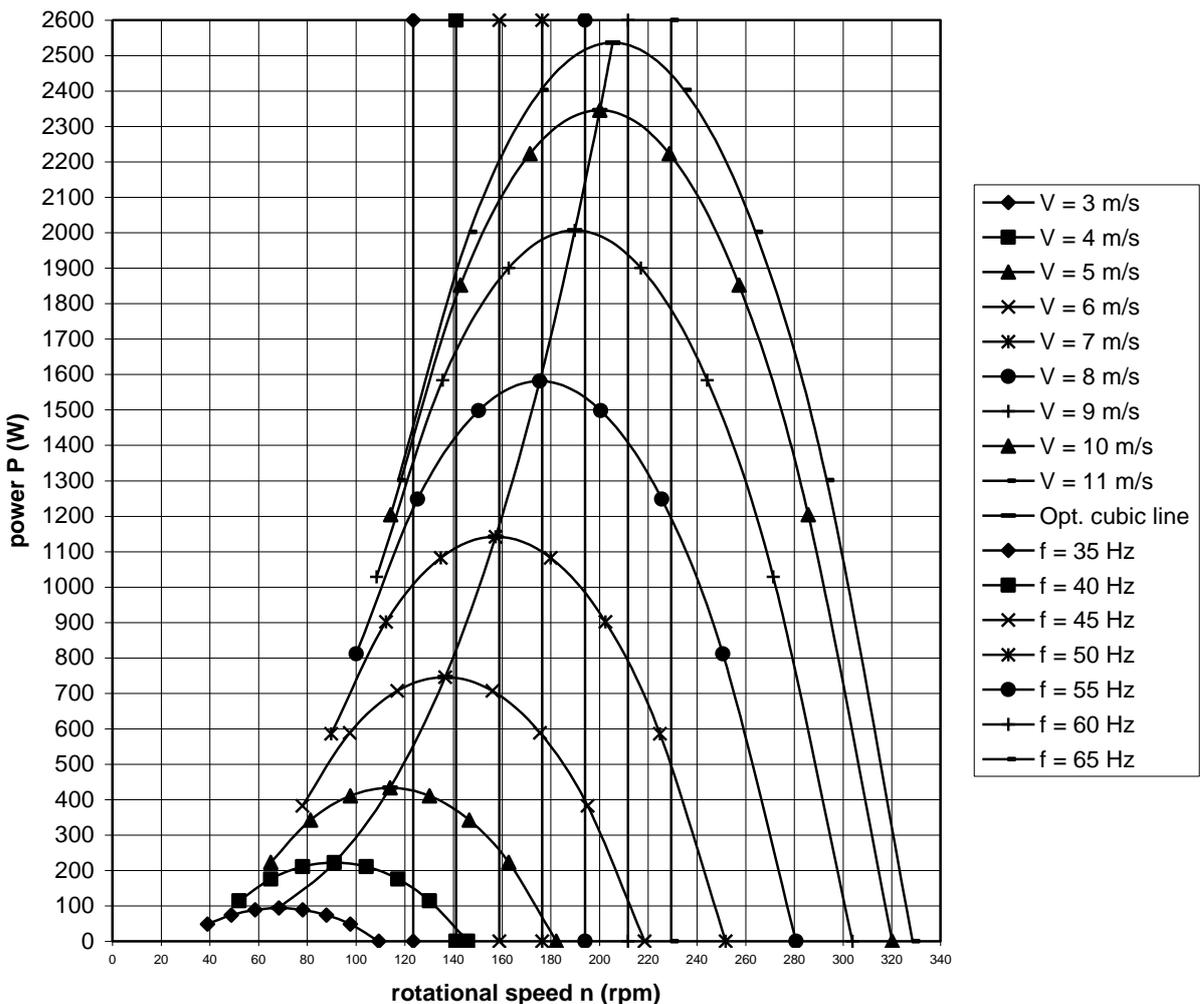


fig. 5 P-n curves and the optimum cubic line of the VIRYA-4.4 rotor. Lines for constant frequencies of 35, 40, 45, 50, 55, 60 and 65 Hz

The 34-pole generator has not yet been built and tested. So measured  $P_{\text{mech-n}}$  and  $P_{\text{el-n}}$  curves for a low voltage winding for 26 V star are not available. Measurements of the generator for a high voltage winding in combination with an asynchronous motor of a centrifugal pump are also not available.

A 2-pole generator has a frequency of 50 Hz at a rotational speed of 3000 rpm. So a 34-pole generator has a frequency of 50 Hz at a rotational speed of  $3000 * 2 / 34 = 176.47$  rpm. The rotational speeds for frequencies of 35, 40, 45, 50, 55, 60 and 65 Hz are respectively 123.53, 141.18, 158.82, 176.47, 194.12, 211.76 and 229.41 rpm. The lines for frequencies of 35, 40, 45, 50, 55, 60 and 65 Hz are also given in figure 5.

In figure 5 it can be seen that the line for a frequency of 50 Hz is intersecting with the optimum cubic line at a mechanical power of about 1600 W belonging to a design wind speed of about 8 m/s. The available electrical power will be lower because of the generator efficiency. Assume the generator efficiency is 0.8, so the electrical power is about 1280 W. The mechanical power which can be generated at  $f = 50$  Hz and  $V = 11$  m/s is about 2400 W and the maximum electrical power at  $f = 50$  Hz will be about 1920 W if the generator efficiency is still 0.8.

A centrifugal pump with a 1.1 kW pump motor used at a factor 0.87 of its nominal power and with a motor efficiency of 0.75 will absorb an electrical power of about 1276 W, so a 1.1 kW pump motor seems an acceptable choice if the pump motor isn't loaded up to its maximum capacity. The real position of the working point depends on the real electric power which the pump motor is asking for a certain head.

In figure 5 it can be seen that the maximum power at a wind speed of 11 m/s is about 2540 W if the optimum cubic line is followed. The frequency is about 58 Hz which is rather high. The load characteristic of a centrifugal pump is about a cubic line which means that the optimum cubic line of the windmill will be followed upwards from the design point if the design point is lying on the optimum cubic line. This means that the maximum frequency will be about 58 Hz at  $V = 11$  m/s. It is expected that this is allowed for the pump and for the pump motor but this must be verified in practice.

Below a frequency of about 35 Hz, belonging to a rotational speed of 123.53 rpm, the pump is no longer able to produce the static water height so no water will be pumped. Probably it is necessary to disconnect the generator and the pump motor by a 3-phase switch below a frequency of about 35 Hz. This makes that the rotor will always start unloaded at low wind speeds. If the connection is broken at  $f = 35$  Hz for a running rotor, this results in acceleration of the rotor. The connection can be made at a frequency of 50 Hz belonging to a rotational speed of 176.47 rpm. This frequency will be reached for an unloaded rotor for a wind speed of about 5 m/s. So the pump will start pumping at this wind speed but it will stop only if the frequency becomes lower than 35 Hz. This means that even at moderate wind speeds, there will be some intermittent output. But the design wind speed is about 8 m/s and so the VIRYA-4.4 can only be used in regions with rather high wind speeds.

If the pump is a centrifugal pump, the system will probably also work if there is no 3-phase switch which disconnects the generator and the pump motor but in this case water is not pumped intermittently if the wind speed is just above 5 m/s. A switch will certainly be needed for a positive displacement pump as such pump demands a torque directly from stand still position.

The generator winding must be chosen such that the loaded voltage is 230 V at a frequency of 50 Hz. This means that the unloaded voltage at 50 Hz must be a lot higher. I expect about 280 V but this must be tested for a prototype of the generator.

## 6 Calculation of the strength of the spoke assembly

The blades are connected to each other by the spoke assembly. Each spoke has a length  $l = 1000$  mm, a width  $b = 80$  mm and a height  $h = 12$  mm. The blade has a thickness of 3 mm but the blade is cambered and the moment of resistance therefore increases substantially (see formula 12 report KD 398, ref. 4). The bending moment in a spoke at the hub is also much higher than in the blade at the end of the spoke. The spoke is therefore the weakest component. The spoke is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The spoke is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because a spoke is long and rather thin, it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1° The load which appears for a rotating rotor at  $V_{\text{rated}} = 11$  m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is  $35^\circ$  for  $V_{\text{rated}} = 11$  m/s.

2° The load which appears for a slowed down rotor. The rotor is slowed down by making short-circuit in the generator winding. The rotor will turn at a very low rotational speed and for this very low rotational speed the effect of compensation by the centrifugal moment is negligible. Therefore it is assumed that the rotor stands still.

### 6.1 Bending stress in the spoke for a rotating rotor and $V = 11$ m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade  $F_{t \delta \text{ bl}}$  is the rotor thrust  $F_{t \delta}$  divided by the number of blades  $B$ . This gives:

$$F_{t \delta \text{ bl}} = C_t * \cos^2 \delta * \frac{1}{2} \rho V^2 * \pi R^2 / B \quad (\text{N}) \quad (10)$$

For the rotor theory it is assumed that every small area  $dA$  which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation, the wind speed in the rotor plane has to be slowed down till  $2/3$  of the undisturbed wind speed  $V$ . This results in a pressure drop over the rotor plane which is the same for every value of  $r$ . It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient  $C_t$  for the whole rotor is  $8/9 = 0.889$  for the optimal tip speed ratio. In practice  $C_t$  is lower because of the tip losses and because the blade is not effective up to the rotor centre. The blade length  $k$  of the VIRYA-4.4 rotor is only 1.5 m but the rotor radius  $R = 2.2$  m. Therefore there is a disk in the centre with an area of 0.1 of the rotor area on which no thrust is working. This results in a theoretical thrust coefficient  $C_t = 8/9 * 0.9 = 0.8$ . Because of the tip losses, the real  $C_t$  value is substantially lower. Assume this results in a real practical value of  $C_t = 0.75$ . It is assumed that the thrust coefficient is constant for values of  $\lambda$  in between  $\lambda_d$  and  $\lambda_{\text{unloaded}}$ .

Substitution of  $C_t = 0.75$ ,  $\delta = 35^\circ$ ,  $\rho = 1.2$  kg/m<sup>3</sup>,  $V = 11$  m/s,  $R = 2.2$  m and  $B = 3$  in formula 10 gives  $F_{t \delta \text{ bl}} = 185$  N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at  $2/3 R = 1.467$  m.

Because the blade length is only  $k$ , there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part  $R - k$  falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about  $r_1 = 1.6$  m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at  $r_2 = 0.045$  m for a hub with a diameter of 90 mm. At this edge we find a bending moment  $M_{b_t}$  caused by the thrust which is given by:

$$M_{b_t} = F_{t \delta_{bl}} * (r_1 - r_2) \quad (\text{Nm}) \quad (11)$$

Substitution of  $F_{t \delta_{bl}} = 185$  N,  $r_1 = 1.6$  m and  $r_2 = 0.045$  m gives  $M_{b_t} = 288$  Nm = 288000 Nmm.

For the stress we use the unit  $\text{N/mm}^2$  so the bending moment has to be given in Nmm. The bending stress  $\sigma_b$  is given by:

$$\sigma_b = M / W \quad (\text{N/mm}^2) \quad (12)$$

The moment of resistance  $W$  of a strip is given by:

$$W = 1/6 bh^2 \quad (\text{mm}^3) \quad (13)$$

(12) + (13) gives:

$$\sigma_b = 6 M / bh^2 \quad (\text{N/mm}^2) \quad (\text{M in Nmm}) \quad (14)$$

Substitution of  $M = 288000$  Nmm,  $b = 80$  mm and  $h = 12$  mm in formula 14 gives  $\sigma_b = 150$   $\text{N/mm}^2$ . For this stress, the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the spoke is bending in between the hub and the inner connection bolt of blade and spoke. This bolt is lying at  $r_3 = 0.73$  m = 730 mm. So the length of the spoke  $l$  which is loaded by bending is given by:

$$l = r_3 - r_2 \quad (\text{mm}) \quad (15)$$

The load from the blade on the strip at  $r_3$  can be replaced by a moment  $M$  and a point load  $F$ .  $F$  is equal to  $F_{t \delta_{bl}}$ .  $M$  is given by:

$$M = F * (r_1 - r_3) \quad (\text{Nmm}) \quad (16)$$

The bending angle  $\phi$  (in radians) at  $r_3$  for a strip with a length  $l$  is given by (combination of the standard formulas for a moment plus a point load):

$$\phi = l * (M + 1/2 Fl) / EI \quad (\text{rad}) \quad (17)$$

The bending moment of inertia  $I$  of a strip is given by:

$$I = 1/12 bh^3 \quad (\text{mm}^4) \quad (18)$$

(15) + (16) + (17) + (18) gives:

$$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + \frac{1}{2} (r_3 - r_2)\} / (E * bh^3) \quad (\text{rad}) \quad (19)$$

Substitution of  $F = 185 \text{ N}$ ,  $r_3 = 730 \text{ mm}$ ,  $r_2 = 45 \text{ mm}$ ,  $r_1 = 1600 \text{ mm}$ ,  $E = 2.1 * 10^5 \text{ N/mm}^2$ ,  $b = 80 \text{ mm}$  and  $h = 12 \text{ mm}$  in formula 19 gives:  $\phi = 0.06351 \text{ rad} = 3.64^\circ$ . This is an angle which can not be neglected. In report KD 684 (ref. 7) a formula is derived for the angle  $\varepsilon$  with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-4.4 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin \left( \frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2} \right) \quad (^\circ) \quad (20)$$

In this formula  $A_{pr}$  is the cross sectional area of the airfoil (in  $\text{m}^2$ ) and  $\rho_{pr}$  is the density of the used airfoil material (in  $\text{kg/m}^3$ ). For a plate width of 200 mm and a plate thickness of 3 mm it is found that  $A_{pr} = 0.0006 \text{ m}^2$ . The blade is made of stainless steel sheet with a density  $\rho_{pr}$  of about  $\rho_{pr} = 7.8 * 10^3 \text{ kg/m}^3$ . It is assumed that the optimum cubic line is followed so the rotor runs at the design tip speed ratio  $\lambda = 5.25$ . Substitution of  $C_t = 0.75$ ,  $\rho = 1.2 \text{ kg/m}^3$ ,  $R = 2.2 \text{ m}$ ,  $B = 3$ ,  $A_{pr} = 0.0006 \text{ m}^2$ ,  $\rho_{pr} = 7.8 * 10^3 \text{ kg/m}^3$  and  $\lambda = 5.25$  in formula 20 gives:  $\varepsilon = 2.03^\circ$ . This angle is smaller than the calculated angle of  $3.64^\circ$  with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than  $2.03^\circ$ .

The real bending angle  $\varepsilon$  is determined as follows. A thrust moment  $M_t = 288 \text{ Nm}$  is working backwards and  $M_t$  is independent of  $\varepsilon$  for small values of  $\varepsilon$ . A bending moment  $M_b$  is working forwards and  $M_b$  is proportional with  $\varepsilon$ .  $M_b = 288 \text{ Nm}$  for  $\varepsilon = 3.64^\circ$ . A centrifugal moment  $M_c$  is working forwards and  $M_c$  is also proportional with  $\varepsilon$ .  $M_c = 288 \text{ Nm}$  for  $\varepsilon = 2.03^\circ$ . The path of these three moments is given in figure 6. The sum total of  $M_b + M_c$  is determined and the line  $M_b + M_c$  is also given in figure 6.

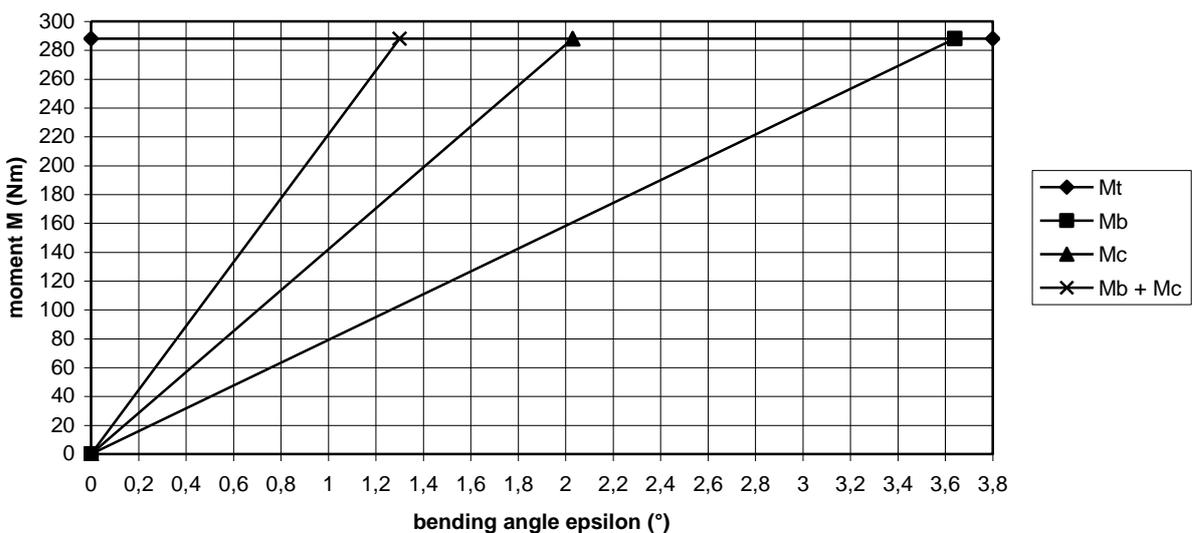


fig. 6 Path of  $M_t$ ,  $M_b$ ,  $M_c$ , and  $M_b + M_c$  as a function of  $\varepsilon$

The point of intersection of the line of  $M_t$  with the line of  $M_b + M_c$  gives the final angle  $\varepsilon$ . In figure 6 it can be seen that  $\varepsilon = 1.3^\circ$ . This is a factor 0.357 of the calculated angle of  $3.64^\circ$ . Because the bending stress is proportional to the bending angle, it will also be a factor 0.357 of the calculated stress of  $150 \text{ N/mm}^2$  resulting in a stress of about  $54 \text{ N/mm}^2$ . This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft, and so on the head. On a rotating mass element  $dm$  at a radius  $r$ , a gyroscopic force  $dF$  is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with  $\sin\alpha$  with respect to a rotating axis frame.  $\alpha$  is the angle with the blade axis and the horizon. So it is valid that  $dF = dF_{\max} * \sin\alpha$ . The direction of  $dF$  depends on the direction of rotation of both axis and  $dF$  is working forwards or backwards. The moment  $dF * r$  which is exerted by this force with respect to the blade is therefore varying sinusoidal too. However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with  $dF_{\max} * r \sin^2\alpha$  with respect to the horizontal x-axis and with  $dF_{\max} * \sin\alpha * \cos\alpha$  with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with  $\sin^2\alpha$ , so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{\text{gyr x-as}} = I_{\text{rot}} * \Omega_{\text{rot}} * \Omega_{\text{head}} \quad (\text{Nm}) \quad (21)$$

In this formula  $I_{\text{rot}}$  is the mass moment of inertia of the whole rotor,  $\Omega_{\text{rot}}$  is the angular velocity of the rotor and  $\Omega_{\text{head}}$  is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three  $\sin^2\alpha$  functions which make an angle of  $120^\circ$  which each other, appear to result in a constant value. The three functions are given in figure 7. It can be proven for a three bladed rotor that the sum value of the three blades is equal to  $3/2$  of the peak value of one blade.

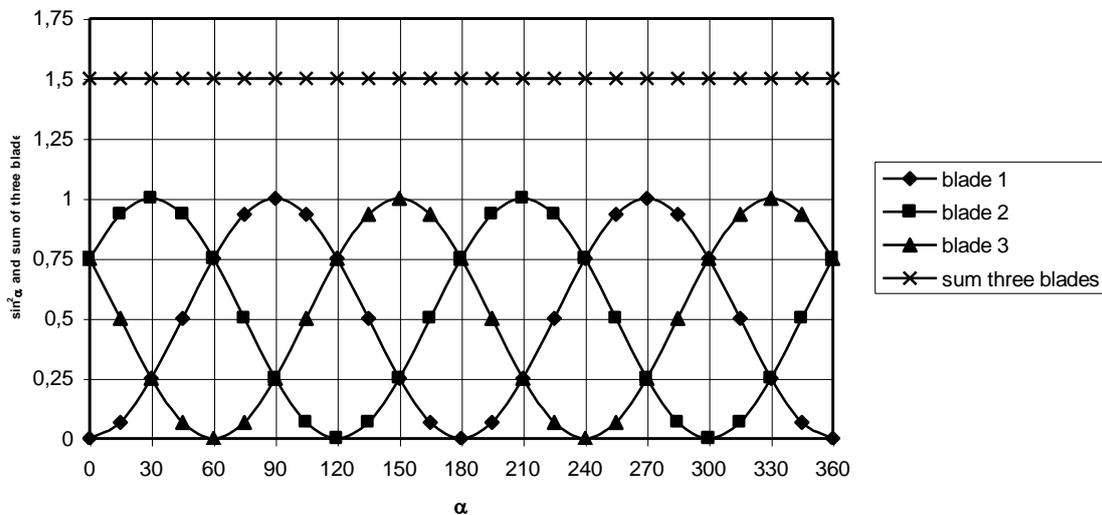


fig. 7 Path of  $\sin^2\alpha$  and the sum of three blades

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then 2/3 of the sum moment as given by formula 21. The variation of the moment in the blade with respect to a rotating axis frame is therefore given by:

$$M_{\text{gyr bl}} = 2/3 \sin\alpha * I_{\text{rot}} * \Omega_{\text{rot}} * \Omega_{\text{head}} \quad (\text{Nm}) \quad (22)$$

For a three bladed rotor, the moment of inertia of the whole rotor  $I_{\text{rot}}$  is three times the moment of inertia of one blade  $I_{\text{bl}}$ . Therefore it is valid that:

$$M_{\text{gyr bl}} = 2 \sin\alpha * I_{\text{bl}} * \Omega_{\text{rot}} * \Omega_{\text{head}} \quad (\text{Nm}) \quad (23)$$

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 22 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the  $\sin^2\alpha$  function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened peak value is given by:

$$M_{\text{gyr bl max}} = 1.2 * I_{\text{bl}} * \Omega_{\text{rot}} * \Omega_{\text{head}} \quad (\text{Nm}) \quad (24)$$

For the chosen blade geometry it is calculated that  $I_{\text{bl}} = 18.6 \text{ kgm}^2$ . The maximum loaded rotational speed of the rotor for the optimum cubic line can be read in figure 5 and it is found that  $n_{\text{max}} = 205 \text{ rpm}$ . This gives  $\Omega_{\text{rot max}} = 21.5 \text{ rad/s}$  (because  $\Omega = \pi * n / 30$ ).

It is not easy to determine the maximum yawing speed. The VIRYA-4.4 is provided with the hinged side vane safety system which has a light van blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be  $0.15 \text{ rad/s}$  at very high wind speeds.

Substitution of  $I_{\text{bl}} = 18.6 \text{ kgm}^2$ ,  $\Omega_{\text{rot max}} = 21.5 \text{ rad/s}$  and  $\Omega_{\text{head max}} = 0.15 \text{ rad/s}$  in formula 24 gives:  $M_{\text{gyr bl max}} = 72 \text{ Nm} = 72000 \text{ Nmm}$ .

Substitution of  $M = 72000 \text{ Nmm}$ ,  $b = 80 \text{ mm}$  and  $h = 12 \text{ mm}$  in formula 14 gives  $\sigma_{\text{b max}} = 38 \text{ N/mm}^2$ . This value has to be added to the bending stress of  $54 \text{ N/mm}^2$  which was the result of the thrust because there is always a position where both moments are strengthening each other. This gives  $\sigma_{\text{b tot max}} = 92 \text{ N/mm}^2$ . The minimum stress is  $54 - 36 = 16 \text{ N/mm}^2$ . So the stress is not becoming negative and therefore it is probably not necessary to take the load as a fatigue load.

For the strip material, bare drawn stainless steel AISI 304 is chosen. The allowable stress for a load in between zero and maximum is about  $190 \text{ N/mm}^2$  and for a fatigue load it is about  $140 \text{ N/mm}^2$ . However, these are the stresses for a tensile stress. The allowable bending stresses are higher. It is assumed that the allowable bending stress for a load in between zero and maximum is  $230 \text{ N/mm}^2$  and for a fatigue load is  $170 \text{ N/mm}^2$ .

The calculated stress is even much lower than the allowable fatigue stress, so the strip is strong enough. In reality the blade is not extremely stiff and will also bend somewhat. This reduces the bending of the strip and therefore the stress in the strip will be somewhat lower.

## 6.2 Bending stress in the strip for a slowed down rotor

The rotational speed for a rotor which is slowed down by making short-circuit of the generator is very low. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 185 N for  $V = V_{\text{rated}} = 11 \text{ m/s}$  and  $\delta = 35^\circ$ . The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above  $V_{\text{rated}}$ . A slowed down rotor will therefore also turn out of the wind by  $35^\circ$  if the force on one blade is 185 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at  $r_4 = 1.45 \text{ m}$  because the relative wind speed is almost constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 16 changes into:

$$M_{bt} = F_{t \delta bl} * (r_4 - r_2) \quad (\text{Nm}) \quad (25)$$

Substitution of  $F_{t \delta bl} = 185 \text{ N}$ ,  $r_4 = 1.45 \text{ m}$  and  $r_2 = 0.045 \text{ m}$  in formula 25 gives  $M_{bt} = 260 \text{ Nm} = 260000 \text{ Nmm}$ . Substitution of  $M = 260000 \text{ Nmm}$ ,  $b = 80 \text{ mm}$  and  $h = 12 \text{ mm}$  in formula 14 gives  $\sigma_b = 135 \text{ N/mm}^2$ . This is higher than the calculated stress for a rotating rotor. However, this load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable bending stress is about  $230 \text{ N/mm}^2$  for bare drawn stainless steel strip, so the strip is strong enough.

Because the strip and the blade are rather flexible it has to be checked if a slowed down rotor can't hit the tower. In chapter 6.1 it has been calculated, for no compensation of the gyroscopic moment, that the bending angle is  $3.64^\circ$  for a stress of  $150 \text{ N/mm}^2$ . So for a stress of  $135 \text{ N/mm}^2$  the bending angle will be  $3.64 * 135 / 150 = 3.28^\circ$ . For a rotor radius of  $R = 2.2 \text{ m}$  this results in a movement at the tip of about  $0.126 \text{ m}$ . Because the blade itself will bend too, the movement will be larger and it is expected that it will be about  $0.18 \text{ m}$ . The rotor shaft has a tilt angle of  $5^\circ$  and the minimum distance in between the blade tip and the tower is much larger if the blade is not bending. So there is no chance that the blade hits the tower for a slowed down rotor.

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