Windmills using aerodynamic drag as propelling force;
a hopeless concept

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1 Introduction

To propel the blades of a windmill one can use the aerodynamic lift force or the aerodynamic drag force. The lift force L is the force (in N) which is generated on an airfoil perpendicular to the relative wind speed W. The drag force D is the force (in N) which is generated on a drag body in the direction of the relative wind speed W. The relative wind speed W is the wind speed (in m/s) which is felt by the airfoil or by the drag body. Modern horizontal axis wind turbines make use of the lift force and the blades move in a direction perpendicular to the undisturbed wind speed V (in m/s). Information about horizontal axis windmills can be found for instance in my report KD 35 (ref. 1).

The mechanical power P (in W) which is generated by a windmill of whatever type is given by:

$$P = C_p * ½\rho V^3 * A \qquad (W) \qquad (1)$$

In this formula C_p is the power coefficient (dimensionless). ρ is the air density and ρ is about 1.2 kg/m^3 for air of 20 °C at sea level. V is the undisturbed wind speed (in m/s), so the wind speed which one would measure at the rotor plane if the rotor would not be placed. The real wind speed at the rotor plane of an horizontal axis windmill is lower than V because, power can only be extracted from the wind if the wind speed at the rotor plane is reduced. A is the area swept by the rotor blades (in m^2). So A is much larger than the total blade area. For an horizontal axis windmill, A is the area of a circle with radius R. R (in m) is the blade length from the hart of the rotor up to the blade tip. So in this case $A = \pi * R^2$.

The maximum theoretical power coefficient for any windmill is calculated by Betz and is 16/27 or 0.59. The real power coefficient for an horizontal axis wind turbine is substantially lower than the Betz coefficient because it is reduced by four effects, wake rotation, tip losses, aerodynamic drag of the airfoil and the fact that the effective blade length k is shorter than the blade radius R. A real maximum value $C_p = 0.45$ is realistic for a well designed horizontal axis windmill rotor.

A well known windmill using the drag force is the cup anemometer which is used for measuring wind speeds. It has a vertical axis and is equipped with three cups made of halve hollow spheres which are mounted on three arms which make angles of 120° with respect to each other. One uses halve hollow spheres because this drag body gives the largest difference in drag coefficient depending if the hollow or convex side faces the wind. Assume the distance in between the vertical axis and the hart of the spheres is called R (in m). Assume the halve hollow spheres have a diameter d (in m). So the swept area is now given by $A = \pi/4 * d^2 + 2 * R * d$. So the swept area is much larger than the projected area of an halve sphere. How much larger, depends on the ratio in between R and d.

Cup anemometers are normally not used to generate power. They are running unloaded and the rotational speed is a measure for the wind speed. However, to generate power, very large cup anemometers have been built but the power which can be generated by such a windmill is very low. The maximum power coefficient which can be realised for a drag machine is not higher than about 0.05 and much more material is needed to realise a certain swept area than for an horizontal axis windmill. So to my opinion development of windmills using the drag force as the propelling force is a waste of time and money. But many types of drag machines are invented by people having almost no knowledge of aerodynamics.

This report KD 416 is written to discourage the development of drag machines and to give the warning, that one must be very suspicious if someone says to have developed a drag machine with a high power at moderate wind speeds.

2 Determination of the power coefficient for a drag machine

It is rather difficult to determine the power coefficient for a cup anemometer because a cup moves only about in the direction of the wind during a small part of its revolution.

The problem can be simplified by taking two cups made of halve hollow spheres which are mounted to a string which is running along two wheels with vertical shafts. The distance in between the shafts is taken rather long and both shafts are positioned such that the stretched string parts are parallel to the wind direction. So cup no 1 for which the hollow part is facing the wind is moving in the direction of the wind and cup no 2 for which the convex side is facing the wind is moving against the wind direction (see figure 1) . The problem which arises if the cups meet the wheels is neglected for the description of the system.

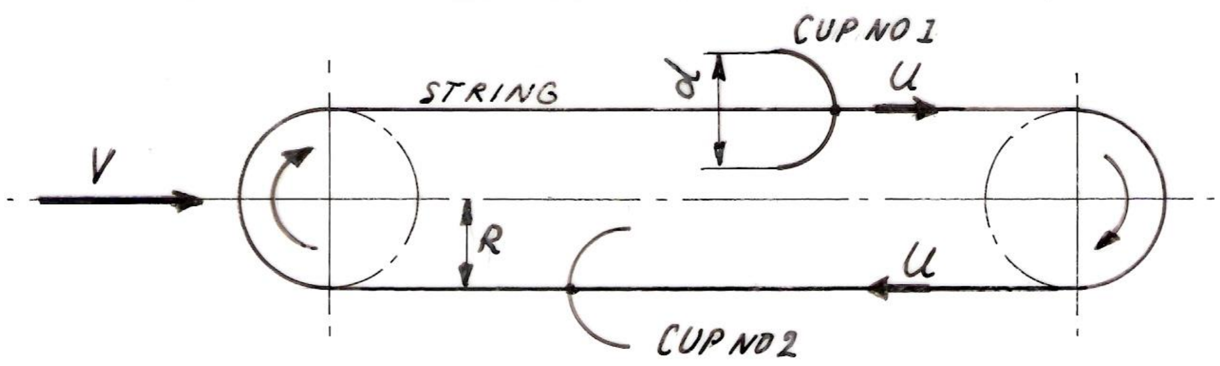

fig. 1 Cup no 1 and cup no 2 mounted on a string parallel to the wind direction

The drag coefficient for the hollow side of the cup is 1.42. The drag coefficient for the convex side of the cup is 0.38, so much lower. It will be clear that cup no 1 which moves in the direction of the wind will produce power but that cup no 2 which moves against the wind direction will consume power. The nett power is the difference of the produced and the consumed powers.

The smallest possible distance in between the strings is such that the spheres just touch which means that R = 1/2 d and so for the minimum swept area A_{min} we now find that:

$$A_{min} = \pi/4 * d^2 + d^2 = d^2 * (1 + \pi/4) \quad (m^2) \qquad (2)$$

The problem of description of the system can be simplified even more, by observing only cup no 1 which moves in the direction of the wind. This situation is comparable to a sail boat with a spinnaker, sailing in the direction of the wind. The power generated by the sail is used to overcome the friction in between the hull and the water. So in the first instance only cup no 1 which is moving in the direction of the wind is taken into account.

The undisturbed wind speed is called V (in m/s). The absolute speed of the string and so of the cup is called U (in m/s). The relative speed of the cup no 1 with respect to the wind speed is called W_1 (in m/s). So for W_1 we find that:

$$W_1 = V - U \quad (m/s) \qquad (3)$$

The maximum relative wind speed W_1 is found when the cup is not moving so for U = 0 m/s. For this situation we will have the maximum drag force D_1 acting on cup no 1 but no power is produced as the speed is zero. The minimum relative wind speed is found when the cup moves with the same speed as the wind speed, so for U = V. For this case, the drag force D_1 will be zero and the power will be zero too. It is assumed that one of the wheel shafts is equipped with a generator and that the generated load can be changed in between unloaded and completely braked.

The ratio in between U and V is called λ analogue to the definition of the tip speed ratio λ for horizontal axis windmills. So

$$\lambda = U / V \quad (-) \tag{4}$$

Formula 4 can be written as:

$$U = \lambda * V \quad (m/s) \tag{5}$$

(3) + (5) gives:

$$W_1 = V (1 - \lambda) \quad (m/s) \tag{6}$$

The mechanical power P_1 generated by cup no 1 is given by:

$$P_1 = D_1 * U \quad (W) \tag{7}$$

(5) + (7) gives:

$$P_1 = D_1 * \lambda * V \quad (W) \tag{8}$$

The drag force D_1 acting on cup no 1 is given by:

$$D_1 = C_{d1} * ½\rho W_1^2 * A \quad (N) \tag{9}$$

The drag coefficient C_{d1} for the cup no 1, for which the hollow side is facing the wind, is 1.42. ρ is the air density (kg/m^3). W_1 is the relative wind speed of cup no 1 (see formula 3 and 6). As we are looking only to one cup, we now take for A, the projected area of only one cup.

(6) + (8) + (9) gives:

$$P_1 = C_{d1} * ½\rho V^3 * (1 - \lambda)^2 * A * \lambda \quad (W) \tag{10}$$

Formula 10 can be written as:

$$P_1 = C_{d1} * ½\rho V^3 * A * (\lambda - 2\lambda^2 + \lambda^3) \quad (W) \tag{11}$$

This function has a maximum for $dP/d\lambda = 0$.

$$dP/d\lambda = C_{d1} * ½\rho V^3 * A * (1 - 4\lambda + 3\lambda^2) \tag{12}$$

So $dP/d\lambda = 0$ for:

$$1 - 4\lambda + 3\lambda^2 = 0 \quad \text{or} \quad 3\lambda^2 - 4\lambda + 1 = 0 \tag{13}$$

This is a quadratic equation which has as roots:

$\lambda_{1,2} = \{4 \pm \sqrt{(16 - 4 * 3)}\} / 6$ or $\lambda_{1,2} = \{4 \pm 2\} / 6$ (14)

This gives $\lambda_1 = 1/3$ and $\lambda_2 = 1$. $\lambda_2 = 1$ means that the cup moves with the same speed as the wind speed so the produced power is zero. So only $\lambda_1 = 1/3$ is relevant. Substitution of $\lambda = 1/3$ in formula 11 gives:

$P_{1max} = 4/27 * C_{d1} * ½\rho V^3 * A$ (W) (for $\lambda = U / V = 1/3$) (15)

This formula is also given in the report Rotors (ref. 2) written by Paul Smulders who was my boss when I was working at the Wind Energy Group of the University of Technology Eindhoven. However, the full derivation of the formula is not given and the report is no longer available. I found it useful to publish the knowledge of drag machines again.

The drag coefficient C_{d1} for a halve sphere with the hollow side facing the wind is 1.42. Substitution of this value in formula 15 gives:

$P_{1max} = 0.210 * ½\rho V^3 * A$ (W) (for $\lambda = U / V = 1/3$) (16)

This is much lower than the maximum C_p value of 0.45 which can be realised for a well designed horizontal axis windmill. But the reality for a drag machine is much worse than what we have calculated up to now, because we have neglected the fact that cup no 2 moves against the wind direction and that for this part of the movement power will be consumed.

Formula 16 is true for $\lambda = U / V = 1/3$, so for $U = 1/3$ V and this means that the relative wind speed W_1 in between the cup and the wind speed is 2/3 V (see formula 6).

Now we go back to the original drag machine with cup no 1 and no 2 mounted on one string as given in figure 1. We assume that the string is moving with the same speed as the speed for which we found that cup no 1 generates the maximum power. The string will have the same absolute speed everywhere which means that the relative wind speed W_2 in between cup no 2, for which the convex side is facing the wind, will be:

$W_2 = V + U$ (m/s) (17)

Substitution of $U = 1/3$ V in formula 17 gives $W_2 = 4/3$ V.

Formula 9 can now be written for cup no 2 as:

$D_2 = C_{d2} * ½\rho W_2^2 * A$ (N) (18)

(18) and $W_2 = 4/3$ V gives:

$D_2 = C_{d2} * ½\rho * 16/9 * V^2 * A$ (N) (19)

Formula 7 can be transformed for cup no 2 and then it is:

$P_2 = D_2 * U$ (W) (20)

(19) + (20) and $U = 1/3$ V gives:

$P_2 = C_{d2} * ½\rho * 16/9 * V^2 * A * 1/3\ V$ or

$P_2 = 16 / 27 * C_{d2} * ½\rho * V^3 * A$ (W) (21)

The drag coefficient C_{d2} for a halve hollow sphere with the convex side facing the wind is 0.38. Substitution of this value in formula 21 gives for cup no 2 that:

$P_2 = 0.225 * ½\rho * V^3 * A$ (W) (22)

This power is already more than the power which is generated by cup no 1 (see formula 16), so the whole system consumes energy if the string moves with a speed U = 1/3 V. To really produce nett power it is required that the string moves with a speed ratio λ which is much lower than 1/3. Mathematical calculation of this optimum speed ratio is rather complicated but the optimum speed ratio can be found by try and error. This is done as follows. P_1 is given by formula 11. The power coefficient C_{p1} of cup no 1 is therefore given by:

$C_{p1} = C_{d1} * (\lambda - 2\lambda^2 + \lambda^3)$ (-) (23)

Substitution of $C_{d1} = 1.42$ in formula 23 gives:

$C_{p1} = 1.42 * (\lambda - 2\lambda^2 + \lambda^3)$ (-) (24)

P_2 has to be written as a function of λ.

(5) + (17) gives:

$W_2 = V (1 + \lambda)$ (m/s) (25)

(18) + (20) + (25) gives:

$P_2 = C_{d2} * ½\rho V^3 * A * (\lambda + 2\lambda^2 + \lambda^3)$ (W) (26)

So the power coefficient C_{p2} of cup no 2 is given by:

$C_{p2} = C_{d2} * (\lambda + 2\lambda^2 + \lambda^3)$ (-) (27)

Substitution of $C_{d2} = 0.38$ in formula 27 gives:

$C_{p2} = 0.38 * (\lambda + 2\lambda^2 + \lambda^3)$ (-) (28)

Next C_{p1} and C_{p2} are determined for several values of λ using formula 24 and 28 and the nett power coefficient C_{pnett} is determined with:

$C_{pnett} = C_{p1} - C_{p2}$ (-) (29)

The relation in between the power coefficient C_p, the torque coefficient C_q and the tip speed ratio λ is given by formula 4.5 of KD 35 (ref. 1). This formula can be written for the nettt values as:

$C_{qnett} = C_{pnett} / \lambda$ (-) (30)

The result of the calculations of C_{p1}, C_{p2}, C_{pnett} and C_{qnett} for several values of λ is given in table 1.

λ	C_{p1}	C_{p2}	C_{pnett}	C_{qnett}	C_p	C_q
1/3 = 0.333	0.210	0.225	-0.015	-0.045	-0.0095	-0.0285
0.318	0.210	0.210	0	0	0	0
0.25	0.200	0.148	0.052	0.208	0.0329	0.1317
0.2	0.182	0.109	0.073	0.365	0.0462	0.2310
0.175	0.169	0.092	0.077	0.440	0.0487	0.2785
0.15	0.154	0.075	0.079	0.527	0.050	0.3336
0.125	0.136	0.060	0.076	0.608	0.0481	0.3849
0.1	0.115	0.046	0.069	0.690	0.0437	0.4368
0.05	0.064	0.021	0.043	0.86	0.0272	0.5444
0	0	0	0	-	0	0.64

table 1 C_{p1}, C_{p2}, C_{pnett}, C_{qnett}, C_p and C_q as a function of λ

In table 1 it can be seen that a maximum value of $C_{pnett} = 0.079$ is realised for $\lambda = 0.15$. This value of λ is very much lower than for modern horizontal axis wind turbines which have tip speed ratios in between 5 and 10. So a drag machine needs an expensive gear box with a very large accelerating gear ratio if it is used to drive a generator at a reasonable rotational speed.

For the calculation of the maximum value $C_{pnett} = 0.079$ we have used the projected area A of cup no 1 and the same projected area A of cup no 2. So a total area 2 A has been taken into account. So for a cup diameter d we find that a total area $\pi/2 * d^2$ has been taken into account. However, for the determination of the real C_p value one has to use the swept area of the whole rotor. The minimum swept area is given by formula 2 as $A_{min} = d^2 * (1 + \pi/4)$. So to find the real maximum C_p which is possible for a drag machine we have to multiply C_{pnett} by the factor $\pi/2 * d^2 / d^2 * (1 + \pi/4) = \pi/2 / (1 + \pi/4) = 0.880$. So the real maximum C_p for a drag machine is 0.88 * 0.079 = 0.07.

However, this value of C_p can only be realised if R = ½ d so for the situation that both cups of figure 1 touch each other. In reality R will always be taken larger than ½ d which results in a larger swept area without increase of the generated power. So this results in decrease of the maximum C_p value and a maximum C_p value of about 0.05 for drag machines, as mentioned in chapter 1 of this report, is therefore a realistic value. This means that the real C_p and C_q values are a factor 0.05 / 0.079 = 0.633 lower than the nett values. The real values for C_p and C_q are found by multiplying all nett values by a factor 0.633 and are given in the last two columns of table 1.

Cup anemometers normally have three cups in stead of two cups to reduce the fluctuation of the torque in one revolution. The ratio R / d is chosen that large that there is place for a third cup without overlapping cups. But the ratio in between the total area of three cups and the swept area is certainly not that small that a larger maximum C_p than 0.05 is obtained.

The values of C_p and C_q as given in the last two columns of table 1 are put into the C_p-λ and C_q-λ curves of figure 2 and figure 3. The value of C_{qnett} for $\lambda = 0$ can't be derived from the value of C_{pnett} because it isn't allowed to divide by zero. But the value of C_q for $\lambda = 0$ was found by extension of the C_q-λ curve to the left.

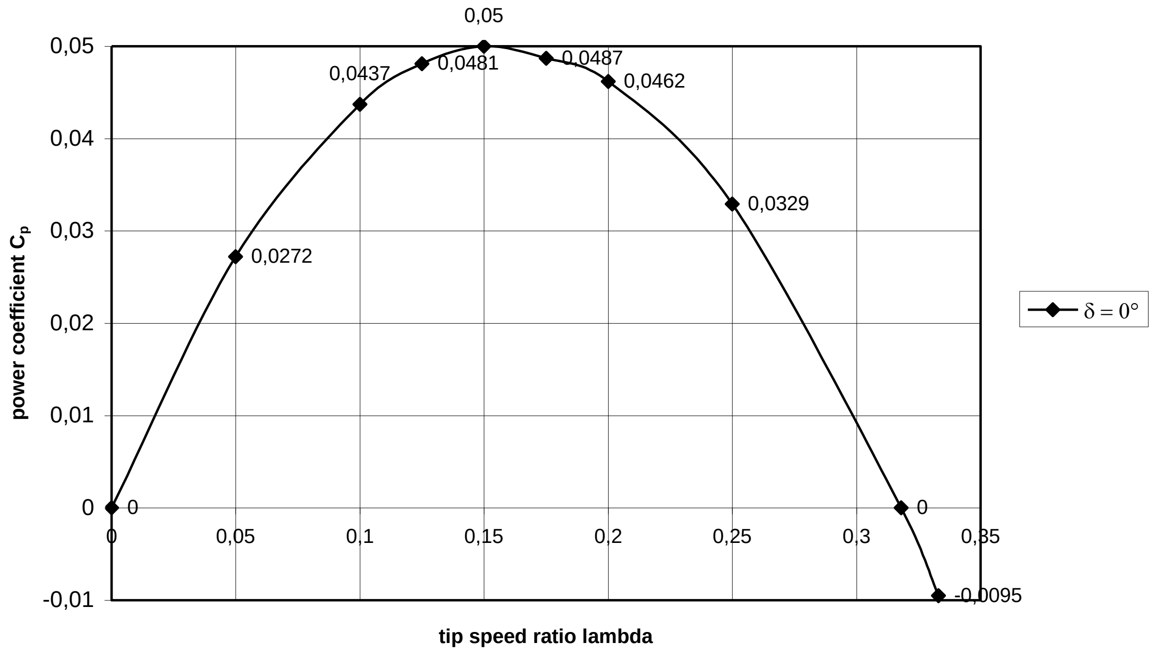

fig. 2 Calculated C_p-λ curve for a drag machine with half hollow spheres

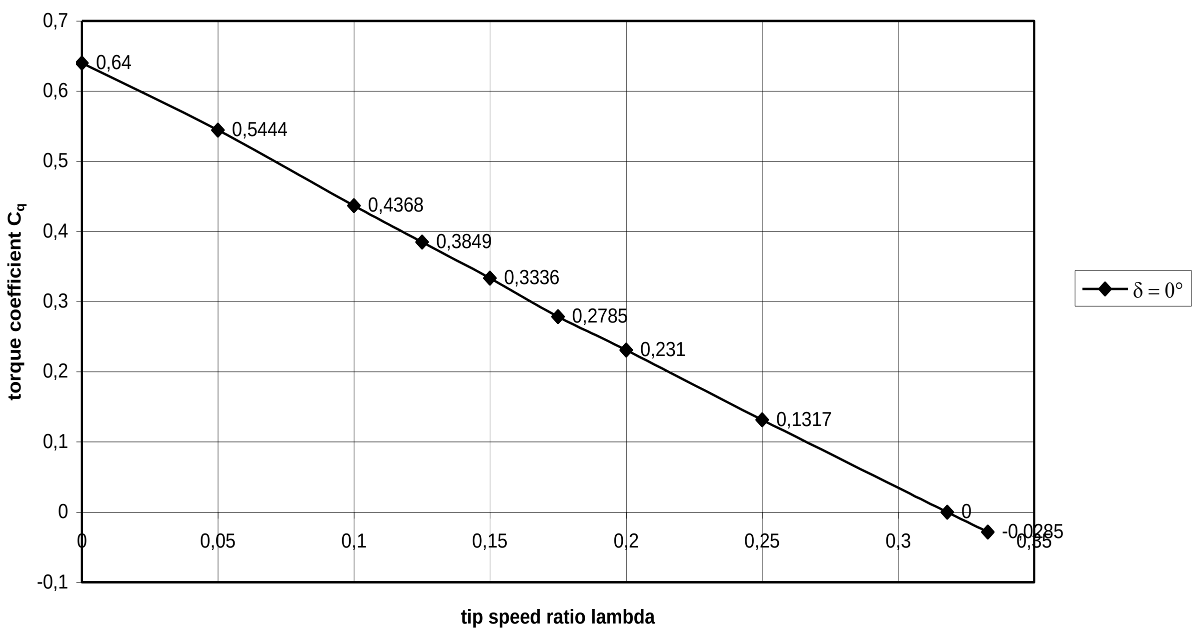

fig. 3 Calculated C_q-λ curve for a drag machine with half hollow spheres

In figure 3 it can be seen that the C_q-λ curve is about a straight line. This means that the C_p-λ curve is about a parabola.

Most inventions about drag machines have to do with ways to reduce the power loss caused by the cups or the blades which are moving against the wind. This was already done by the Persians at about 600 A.D. by covering these blades by a kind of shield. But even a shield can't completely prevent that a certain drag force is formed. Another disadvantage of a shield is that now the shield hast to be orientated with respect to the wind direction such that the driving blades feel the highest wind speed. Formula 16 shows that even if the negative drag force can completely be eliminated, only a rather low maximum power coefficient is possible.

As mentioned earlier the amount of material needed for the cups to realise a certain swept area is much larger than for modern horizontal axis wind turbines which makes a drag machine expensive. Another problem with drag machines is that it is almost impossible to limit the rotational speed and the thrust at very high wind speeds. Horizontal axis wind turbines can be turned out of the wind or one can use pitch control systems to limit rotational speed and thrust. But these options are not possible for drag machines which means that they can be dangerous at very high wind speeds. Because of all these arguments it can be concluded that development of drag machines is a waste of time and money.

3 Determination of the C_p-λ curve of only cup no 1

A real wind turbine has always cups moving against the wind and these cups strongly reduce the power which is generated by the cups which are moving in the direction of the wind. But a sail boat using a spinnaker and driving in the direction of the wind can be seen as a drag machine with only one cup. So it seems useful to determine the C_p-λ curve for such a device.

The power coefficient of only cup no 1 is given by formula 24. The value of C_{p1} is now determined for a range of λ values increasing by 0.1 and including λ = 1/3 = 0.33333. The result of the calculation is given in table 2.

λ (-)	C_{p1}
0	0
0.1	0.1150
0.2	0.1818
0.3	0.2087
0.33333	0.2104
0.4	0.2045
0.5	0.1775
0.6	0.1363
0.7	0.0895
0.8	0.0454
0.9	0.0128
1	0

table 2 Calculated values of C_{p1} as a function of λ for only cup no 1

The C_{p1}-λ curve as found using the values of table 2, is given in figure 4.

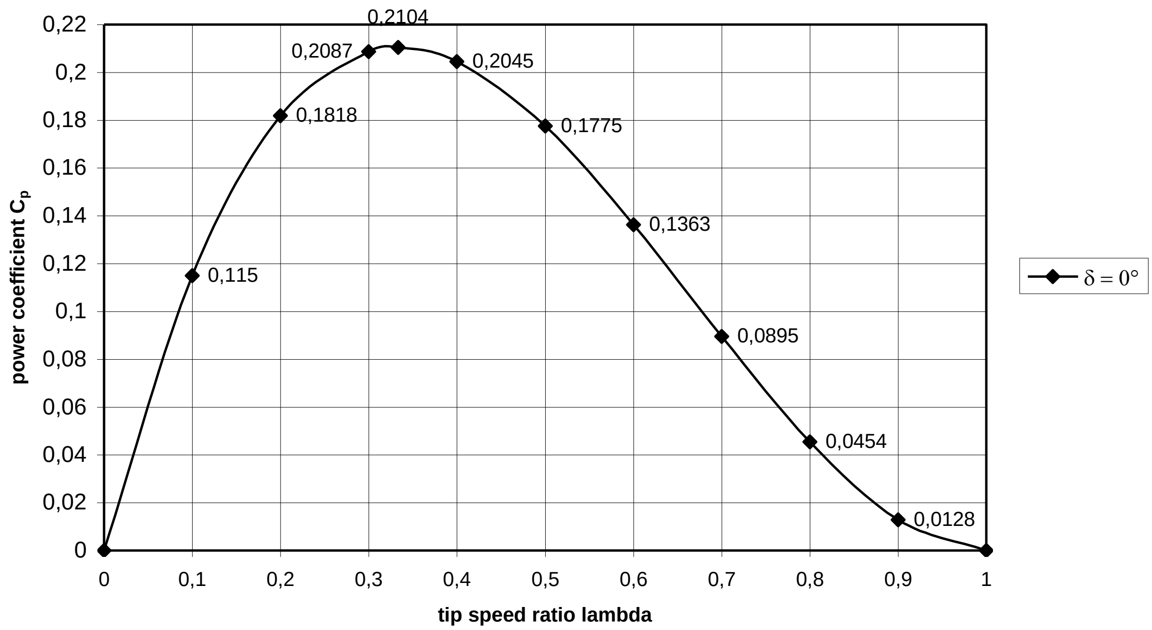

fig. 4 Calculated C_p-λ curve for a half hollow sphere moving in the direction of the wind

The power generated by the spinnaker of a sail boat is used to overcome the friction of the hull with the water. This friction increases proportional to the square of the speed of the boat with respect to the water. The power to overcome this friction increases proportional to the cube of the speed of the boat with respect to the water. So the final speed of the boat is the speed for which the generated power is equal to the needed power. In the C_{p1}-λ curve it can be seen that the generated power decreases strongly as λ approaches the value $\lambda = 1$. So the tip speed ratio at maximum speed of the boat will be substantial lower than 1 even if the boat hull has a low drag coefficient.

If figure 4 is compared to figure 2, it can be seen that the maximum C_p for figure 4 is reached at a tip speed ratio of 0.333. However, for figure 2, the maximum C_p is reached at a tip speed ratio of about 0.15. This effect and the much lower maximum C_p value of figure 2, demonstrates the strong negative effect of cup no 2.

4 References

1 Kragten A. Rotor design and matching for horizontal axis wind turbines, January 1999, reviewed February 2017, free public rapport KD 35, engineering office Kragten Design, Populierenlaan 51, 5492 SG Sint-Oedenrode, The Netherlands.

2 Smulders P. T. Rotors, October 1991, (has no report number), (former) Wind Energy Group, University of Technology Eindhoven (no longer available).