Calculations executed for an alternative rotor of the VIRYA-4.2 windmill (λ_d = 7.5, Gö 711-10% airfoil)

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Contains page

1 Introduction

The VIRYA-4.2 windmill is developed for use in western countries as well as in developing countries. The windmill has a simple two bladed rotor with wooden blades which are connected to each other by means of a steel strip. The original rotor has a design tip speed ratio of 8, blades with a constant chord of 200 mm and is equipped with a Gö 623 airfoil. This airfoil has a maximum thickness of 12 % of the chord, so the maximum thickness is 24 mm. A blade is made of a hard wood meranti plank with a width of 255 mm and a thickness of 27 mm. The rough outside of this plank is machined to a width of 200 mm and a thickness of 24 mm and the blade is made from this plank using an electric hand plane. Because the airfoil width is only 200 mm, a rather large amount of wood is wasted.

The alternative rotor for the VIRYA-4.2 has a design tip speed ratio of 7.5 and is made of the same plank of 255 * 27 mm but now the chord is taken 240 mm and the thickness is taken 24 mm. This means that the thickness is 10 % of the chord. An advantage of a larger chord is that the rotor will have a larger starting torque coefficient and therefore it will start at a lower wind speed. Another advantage is that the Reynolds number is higher. A 10 % airfoil is not available in the range of the Gö 623 airfoil. Another disadvantage of the Gö 623 airfoil and of most other airfoils is that the airfoil has to be machined on both sides. An exception is the Gö 711 airfoil which is flat over 97.5 % of the lower side. This airfoil is described in report KD 285 (ref. 1). Despite its flat lower side, it has very good aerodynamic characteristics. However, this airfoil has a maximum thickness of 14.85 % of the chord.

In report KD 333 (ref. 2) two new airfoils are derived from the Gö 711 airfoil which have a maximum thickness of 12 % and of 10 % of the chord. These airfoils are called the Gö 711-12% and the Gö 711-10%. The Gö 711-10% airfoil is used for the blades of the alternative rotor of the VIRYA-4.2. The connecting strip is identical to the strip of the original rotor. All other components of the VIRYA-4.2 like hub, generator, head, tower and dump load are also the same.

The windmill is provided with the so called hinged side vane safety system with a 9 mm plywood vane blade. The rated wind speed V_{rated} for this vane blade is about 9.5 m/s.

2 Description of the rotor of the VIRYA-4.2 windmill

The alternative 2-bladed rotor of the VIRYA-4.2 windmill has a diameter D = 4.2 m and a design tip speed ratio λ_d = 7.5. The most important advantages of a 2-bladed rotor above a 3-bladed rotor are that the connection of the blades to the hub is simple and needs no welding, that balancing of the rotor is rather simple and that the rotor can be transported completely mounted.

The rotor has blades with a constant chord and blade angle and is provided with a Gö 711-10% airfoil. The geometry and characteristics of the Gö 711-10% airfoil are given in report KD 333 (ref. 2). The lower side of the Gö 711-10% airfoil is flat over 97.5 % of the chord which simplifies manufacture. It has a maximum lift coefficient ($C_{l\ max}$ = 1.27 for α = 13°) and a low minimum drag-lift ratio (C_d/C_l = 0.017 for C_l = 0.75).

The blades are connected to each other by a 1 m long, wide and thin steel strip and so the blade support is rather elastic. Therefore vibrations which are caused by the gyroscopic moment, by streaming under a certain yaw angle δ and by a non-uniform distribution of the wind speed over the rotor plane, are flattened. The blade length is 1.9 m and the overlap in between blade and strip is 0.3 m which results in a free blade length of 1.6 m. The blade is connected to the strip using three bolts M12. In between the bolt heads and the blade is a thin strip to prevent too strong deformation of the wood. The bolts are also used for connection of the balancing weights.

The strip is clamped in between the hub and a thick clamping disk by means of two bolts M12 and that is why the strip is not loaded by a bending moment at the position of the holes. The hub is pulled on the tapered shaft end of the generator by one bolt M16.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 382) has its own formula numbering. Substitution of λ_d = 7.5 and R = 2.1 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 3.5714 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 2 and c = 0.24 m in formula (5.4) of KD 35 gives:

$C_l = 52.360\ r\ (1 - \cos\phi)$ (-) (4)

Substitution of V = 5 m/s and c = 0.24 m in formula (5.5) of KD 35 gives:

$R_{e\,r} = 0.80 * 10^5 * \sqrt{}\ (\lambda_{r\,d}^2 + 4/9)$ (-) (5)

The blade is calculated for five stations A till E which have a distance of 0.4 m of one to another. Station E corresponds to the edge of the connecting strip. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of the Gö 711-10% airfoil are given in chapter 3 of KD 333 (ref. 2). The Gö 711-10% characteristics are only given for Re = 4 * 10^5. The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for a windmill with V_{rated} = 9.5 m/s.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ Gö 711-10%	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	2.1	7.5	5.1	0.24	0.43	0.41	6.02	4	-0.1	-0.4	5.2	5.5	0.022
B	1.7	6.071	6.2	0.24	0.53	0.51	4.89	4	1.0	0.7	5.2	5.5	0.019
C	1.3	4.643	8.1	0.24	0.68	0.67	3.75	4	2.7	2.6	5.4	5.5	0.018
D	0.9	3.214	11.5	0.24	0.95	0.92	2.63	4	6.4	6.0	5.1	5.5	0.020
E	0.5	1.786	19.5	0.24	1.50	1.27	1.53	4	-	14.0	-	5.5	0.07

table 1 Calculation of the blade geometry of the VIRYA-4.2 rotor

The calculated Reynolds values are only larger or about the same as 4 * 10^5 for stations A, B and C. No value for α_{th} and therefore for β_{th} is found for station E because the required C_l value can not be generated. The variation of the theoretical blade angle β_{th} is only little for station A up to D and varies in between 5.1° en 5.4°. Therefore it is allowed to take a constant value of 5.5° for the whole blade. So the connecting strip is twisted 5.5° in between the hub and the blade root to realise the correct blade angle. This value is the same value as for the original rotor with λ_d = 8, a Gö 623 airfoil and a chord of 200 mm. The alternative VIRYA-4.2 rotor with λ_d = 7.5 is given on drawing 0401-01/A.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.02. Figure 4.6 of KD 35 (for B = 2) and λ_{opt} = 7.5 and C_d/C_l = 0.02 gives $C_{p\ th}$ = 0.465. The blade is stalling at station E and the airfoil is disturbed within station E because of the blade connection. Therefore not the whole blade length k = 1.9 m, but only the part up to 0.15 m outside station E is taken into account for the calculation of the C_p. This gives an effective blade length k' = 1.45 m.
Substitution of $C_{p\ th}$ = 0.465, R = 2.1 m and blade length k = k' = 1.45 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.42. This is a little higher than for the original VIRYA-4.2 rotor for which it was calculated that $C_{p\ max}$ = 0.4. $C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.42 / 7.5 = 0.0560.
Substitution of $\lambda_{opt} = \lambda_d$ = 7.5 in formula 6.4 of KD 35 gives λ_{unl} = 12.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \quad (-) \qquad (6)$$

The blade angle is 5.5° for the whole blade. For a non rotating rotor, the angle of attack α is therefore 90° - 5.5° = 84.5°. The estimated C_l-α curve for large values of α is given as figure 5.10 of KD 35 for the Gö 623 airfoil. It is assumed that this curve can also be used for the Gö 711-10% airfoil for large angles of α. For α = 84.5° it can be read that C_l = 0.2. The whole blade is stalling during starting and therefore now the whole blade length k = 1.9 m is taken.
Substitution of B = 2, R = 2.1 m, k = 1.9 m, C_l = 0.2 and c = 0.24 m in formula 6 gives that $C_{q\ start}$ = 0.0054. For the original VIRYA-4.2 rotor with λ_d = 8 it was found that $C_{q\ start}$ = 0.0045. For the ratio between the starting torque and the optimum torque we find that it is 0.0054 / 0.0560= 0.0964. This is acceptable for a rotor met a design tip speed ratio of 7.5

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3}\right)} \quad (m/s) \qquad (7)$$

For the generator, a sticking torque Q_s has been measured of about 0.9 Nm if the shaft is not rotating. Substitution of Q_s = 0.9 Nm, $C_{q\ start}$ = 0.0054, ρ = 1.2 kg/m^3 and R = 2.1 m in formula 7 gives that V_{start} = 3.1 m/s. This is acceptable low for a 2-bladed rotor with a design tip speed ratio of 7.5. For the original VIRYA-4.2 rotor it was calculated that V_{start} = 3.4 m/s so change of the airfoil and the chord makes sense. Because the generator is rectified in star, the rise of the unloaded generator torque is only little at low rpm. The rise is less than the rise of the Q-n curve of the rotor and therefore the real starting wind speed will be about the same as the calculated value. Rectification in delta is much more unfavourable because the unloaded Q-n curve of the generator is rising rather fast as higher harmonic currents can circulate in the winding.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing a S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 4). With this method, it can be determined that the C_q-λ curve is about straight and horizontal for low values of λ if a Gö 711-10% airfoil is used.

A scale model of a three bladed rotor with constant chord and blade angle and with a design tip speed ratio λ_d = 6 has been measured in the wind tunnel already on 20-11-1980. It has been found that the maximum C_p was more than 0.4 and that the C_q-λ curve for low values of λ was not horizontal but somewhat rising. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the alternative VIRYA-4.2 rotor are given in fig. 1 and 2.

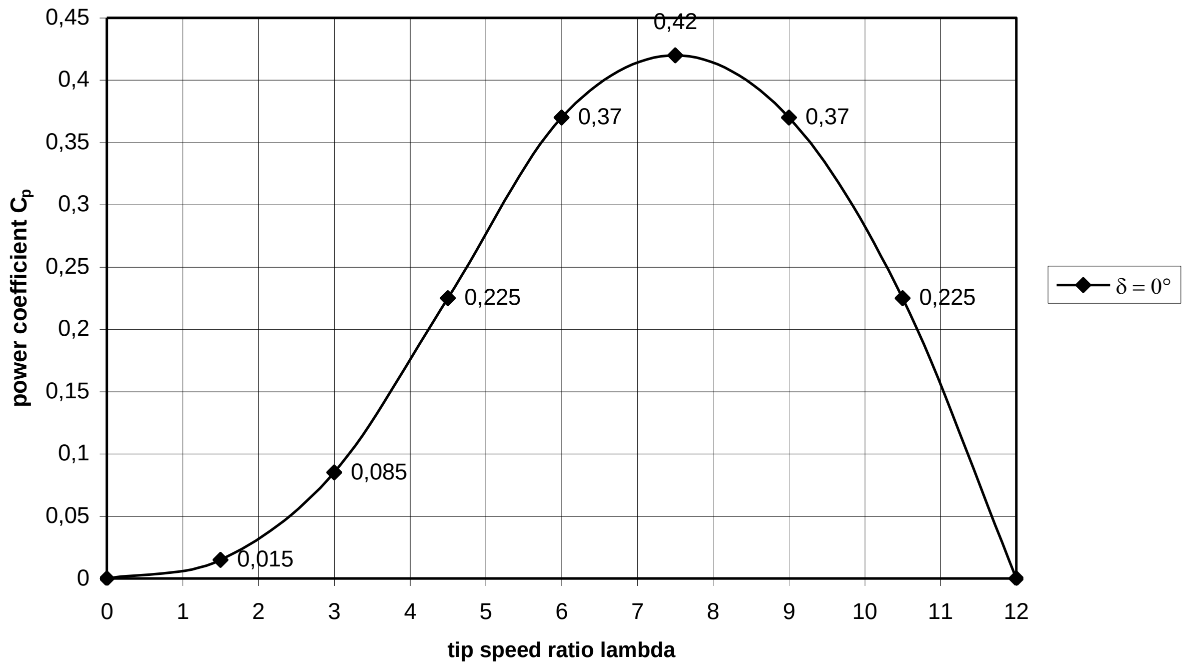

fig. 1 Estimated C_p-λ curve for the alternative VIRYA-4.2 rotor with λ_d = 7.5 for the wind direction perpendicular to the rotor (δ = 0°)

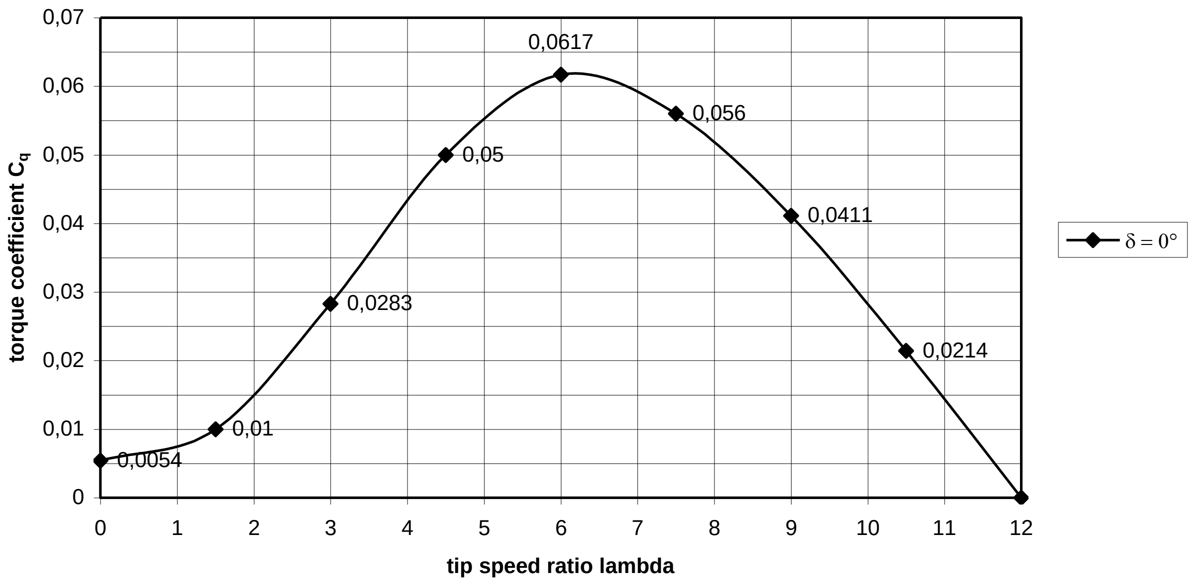

fig. 2 Estimated C_q-λ curve for the alternative VIRYA-4.2 rotor with λ_d = 7.5 for the wind direction perpendicular to the rotor (δ = 0°)

5 Determination of the P-n curves and P_{el}-V curve for 48 V battery charging

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_p-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The C_p-λ curve is given in figure 1. The δ-V curve of the safety system depends on the vane blade mass per area. The vane blade is made of 9 mm meranti plywood with a density of about 0.6 * 10^3 kg/m^3. In report KD 213 (ref. 4) a method is given to check the estimated δ-V curve and the estimated δ-V curve of the VIRYA-4.2 windmill is checked as an example. The estimated and calculated curves appear to lie close to each other so it is allowed to use the estimated curve. This windmill has a 9 mm plywood vane blade and for this vane blade the rated wind speed is about 9.5 m/s. The alternative VIRYA-4.2 has also a 9 mm vane blade so the rated wind speed will be the same. The estimated δ-V curve is given in figure 3.

The head starts to turn away at a wind speed of about 6 m/s. For wind speeds above 9.5 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The P-n curve for 9.5 m/s will therefore also be valid for wind speeds higher than 9.5 m/s.

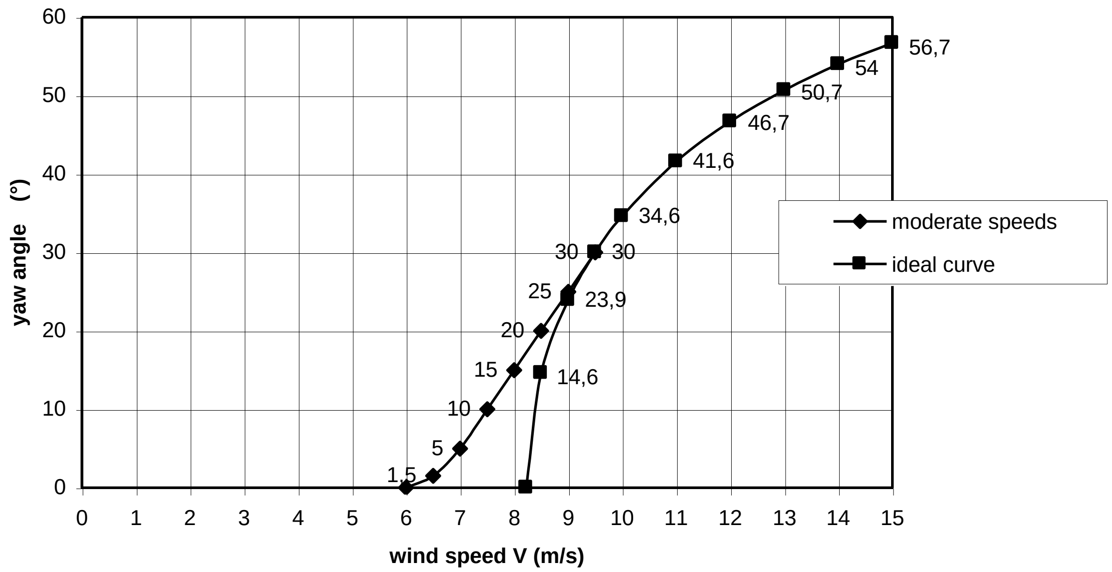

fig. 3 δ-V curve VIRYA-4.2 safety system with V_{rated} = 9.5 m/s

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator for a certain gear ratio i (the VIRYA-4.2 has no gearing so i = 1). Because we are especially interested in the domain around the optimal cubic line and because the P-n curve for low values of λ appear to lie very close to each other, the P-n curves are not determined for low values of λ. The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8 and 9.5 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of R = 2.1 m in formula 7.1 of KD 35 gives:

$n_\delta = 4.5473 * \lambda * \cos\delta * V$ (rpm) (8)

Substitution of ρ = 1.2 kg / m^3 and R = 2.1 m in formula 7.10 of KD 35 gives:

$P_\delta = 8.3127 * C_p * \cos^3\delta * V^3$ (W) (9)

The P-n curves are determined for C_p values belonging to λ is 3, 4.5, 6, 7.5, 9, 10.5 and 12 (see figure 1). For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 3, is taken into account. The result of the calculations is given in table 2.

		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 0°		V = 7 m/s δ = 5°		V = 8 m/s δ = 15°		V = 9.5 m/s δ = 30°	
λ (-)	C_p (-)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)
3	0.085	40.9	19.1	54.6	45.2	68.2	88.3	81.9	152.6	95.1	239.6	105.4	326.0	112.2	393.5
4.5	0.225	61.4	50.5	81.9	119.7	102.3	233.8	122.8	404.0	142.7	634.2	158.1	863.0	168.4	1042
6	0.37	81.9	83.0	109.1	196.8	136.4	384.5	163.7	664.4	190.3	1043	210.8	1419	224.5	1713
7.5	0.42	102.3	94.3	136.4	223.4	170.5	436.4	204.6	754.1	237.8	1184	263.5	1611	280.6	1944
9	0.37	122.8	83.0	163.7	196.8	204.6	384.5	245.6	664.4	285.4	1043	316.2	1419	336.7	1713
10.5	0.225	143.2	50.5	191.0	119.7	238.7	233.8	286.5	404.0	333.0	634.2	369.0	863.0	392.8	1042
12	0	163.7	0	218.3	0	272.8	0	327.4	0	380.5	0	421.7	0	448.9	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-4.2 rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which is going through the tops of the P_{mech}-n curves is also given in figure 4. For charging of a 48 V battery, the generator has to be rectified in star. The generator measurements are given in report KD 200 (ref. 6). The measured P_{mech}-n and P_{el}-n curves of the generator for 52 V star are also plotted in figure 4. A voltage of 52 V is about the average charging voltage for a 48 V battery.

The generator has been measured for short-circuit in delta because the maximum torque level for short-circuit in delta is higher than for short-circuit in star. Short-circuit in delta is identical to short-circuit in star if the star point is short-circuited too. The P-n curve for short-circuit in delta is also plotted in figure 4.

The point of intersection of the P_{mech}-n curve of the generator with the P-n curve of the rotor for a certain wind speed, gives the working point for that wind speed. The electrical power P_{el} for that wind speed is found by going down vertically from the working point up to the point of intersection with the P_{el}-n curve. The values of P_{el} found this way for all wind speeds, are plotted in the P_{el}-V curve (see figure 5). The charging voltage at high powers will be somewhat higher than the average charging voltage of 52 V and therefore the generator efficiency will be somewhat higher too. This results in a somewhat higher electrical power. The P_{el}-V curve is corrected for this effect for high wind speeds.

The matching of rotor and generator is good because the P_{mech}-n curve of the generator is lying close to the optimum cubic line for wind speeds in between 4 and 9.5 m/s. In the P_{el}-V curve it can be seen that the maximum power is 1120 W. This is 20 W higher than for the original rotor of the VIRYA-4.2. The whole P_{el}-V curve is lying somewhat higher which is caused by the higher maximum C_p. The supply of power starts already at a wind speed of 3 m/s ($V_{cut\ in}$ = 3 m/s). This is rather low and therefore the windmill can be used in regions with low wind speeds. In chapter 4 it was calculated that V_{start} = 3 m/s, so there is no hysteresis in the P_{el}-V curve.

The P-n curve for short-circuit in delta is lying left from the P-n curve of the rotor for V = 9.5 m/s and higher. This means that the rotor will slow down to almost stand still for every wind speed if short-circuit is made.

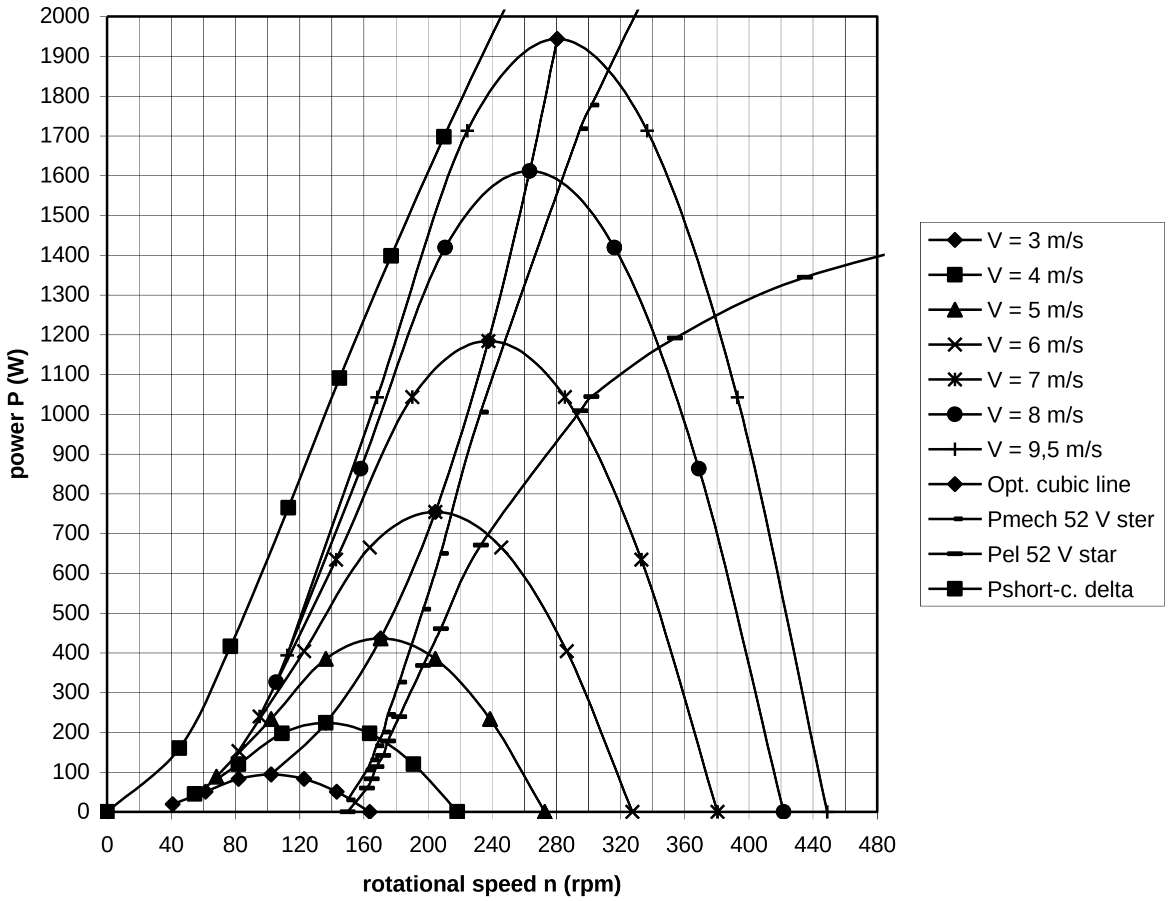

fig. 4 P-n curves of the alternative VIRYA-4.2 rotor and the generator for 52 V star

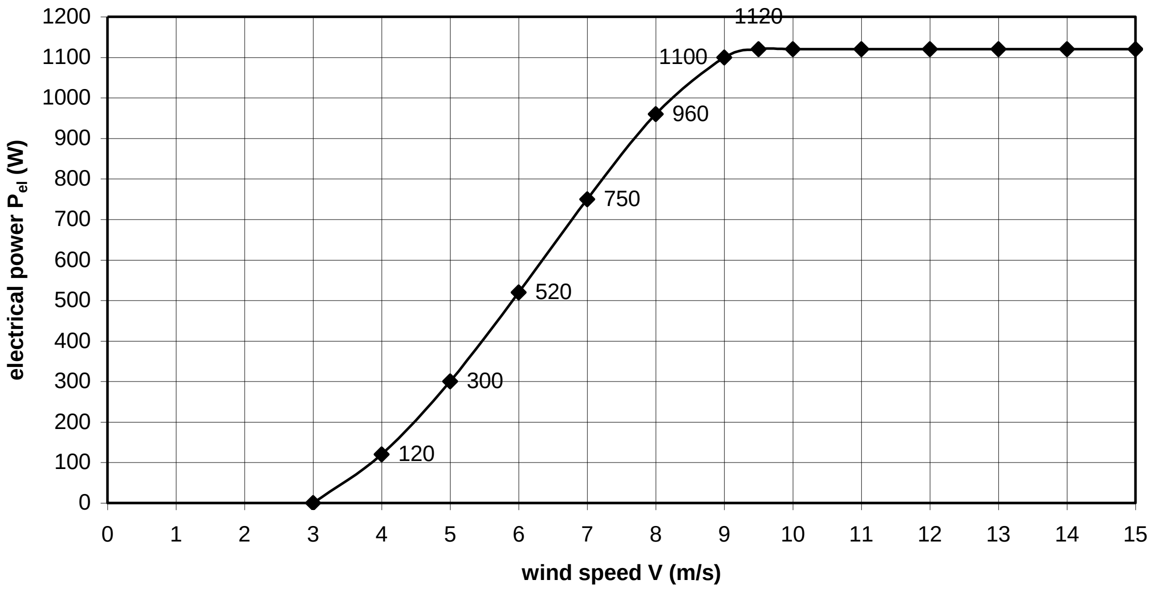

fig. 5 P_{el}-V curve of the VIRYA-4.2 windmill with alternative rotor for 48 V battery charging and rectification in star

6 Calculation of the strength of the strip which connects the blades

The two blades are connected to each other by a 1 m long, steel strip with a width b = 120 mm and a height h = 8 mm. The strip is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The strip is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because the strip is thin and long it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1[e] The load which appears for a rotating rotor at V_{rated} = 9.5 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 30° for V_{rated} = 9.5 m/s.

2[e] The load which appears for a slowed down rotor. The rotor is slowed down by making short-circuit in the generator winding. A graph has been made in which the Q-n curve of the rotor for V = 9.5 m/s has been plotted together with the Q-n curve of the generator for short-circuit in delta. For the working point it is found that the rotor rotates at a rotational speed of about 8 rpm and has a tip speed ratio of about 0.2. For this very low rotational speed the effect of compensation by the centrifugal moment is negligible and a tip speed ratio of 0.2 is very low. Therefore it is assumed that the rotor stands still.

6.1 Bending stress in the strip for a rotating rotor and V = 9.5 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * \tfrac{1}{2}\rho V^2 * \pi R^2 / B \quad \text{(N)} \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The effective blade length k’ of the VIRYA-4.2 rotor is only 1.6 m but the rotor radius R = 2.1 m. Therefore there is a disk in the centre with an area of about 0.057 of the rotor area on which almost no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.943 = 0.838. Because of the tip losses the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.7.

Substitution of C_t = 0.7, δ = 30°, ρ = 1.2 kg/m^3, V = 9.5 m/s, R = 2.1 m and B = 2 in formula 10 gives $F_{t\,\delta\,bl}$ = 197 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 1.4 m. Because the effective blade length is only k', there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k' falls off. The centre of gravity of the trapezium has been determined graphically and is laying at about r_1 = 1.46 m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is laying at r_2 = 0.04 m. At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$M_{b\,t} = F_{t\,\delta\,bl} * (r_1 - r_2)$ (Nm) (11)

Substitution of $F_{t\,\delta\,bl}$ = 197 N, r_1 = 1.46 m and r_2 = 0.04 m gives $M_{b\,t}$ = 280 Nm = 280000 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$\sigma_b = M / W$ (N/mm^2) (12)

The moment of resistance W of a strip is given by:

$W = 1/6\ bh^2$ (mm^3) (13)

(12) + (13) gives:

$\sigma_b = 6\ M / bh^2$ (N/mm^2) (M in Nmm) (14)

Substitution of M = 280000 Nmm, b = 120 mm and h = 8 mm in formula 14 gives σ_b = 219 N/mm^2. For this stress the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the strip is bending only in between the hub and the inner connection bolt of blade and strip. So it is assumed that the blade itself is not bending. The inner connection bolt is laying at r_3 = 0.23 m = 230 mm. So the length of the strip l which is loaded by bending is given by:

$l = r_3 - r_2$ (mm) (15)

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$M = F * (r_1 - r_3)$ (Nmm) (16)

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$\phi = l * (M + ½\ Fl) / EI$ (rad) (17)

The bending moment of inertia I of a strip is given by:

$I = 1/12\ bh^3$ (mm^4) (18)

(15) + (16) + (17) + (18) gives:

$\phi = 12 * F * (r_3 – r_2) * \{(r_1 – r_3) + ½ (r_3 – r_2)\} / (E * bh^3)$ (rad) (19)

Substitution of F = 197 N, r_3 = 230 mm, r_2 = 40 mm, r_1 = 1460 mm, E = 2.1 * 10^5 N/mm^2, b = 120 mm and h = 8 mm in formula 19 gives: ϕ = 0.04613 rad = 2.64°. This is an angle which can not be neglected. In report KD 684 (ref. 7) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-4.2 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin \left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2} \right) \quad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For the Gö 711-10% airfoil A_{pr} is about given by A_{pr} = 0.7 * c * t. For a chord c = 240 mm = 0.24 m and a maximal airfoil thickness t = 24 mm = 0.024 m we find that A_{pr} = 0.00403 m^2. The blade is made of hard wood with a density ρ_{pr} of about ρ_{pr} = 0.65 * 10^3 kg/m^3. In figure 4 it can be seen that for high wind speeds, the rotor is running at about λ = 8.5. Substitution of C_t = 0.7, ρ = 1.2 kg/m^3, R = 2.1 m, B = 2, A_{pr} = 0.00403 m^2, ρ_{pr} = 0.65 * 10^3 kg/m^3 and λ = 8.5 in formula 20 gives: ε = 1.76°. This angle is smaller than the calculated angle of 2.64° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.76°.

The real bending angle ε is determined a follows. A thrust moment M_t = 280 Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. M_b = 280 Nm for ε = 2.64°. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. M_c = 280 Nm for ε = 1.76°. The path of these three moments is given in figure 6. The sum total of M_b + M_c is determined and the line M_b + M_c is also given in figure 6.

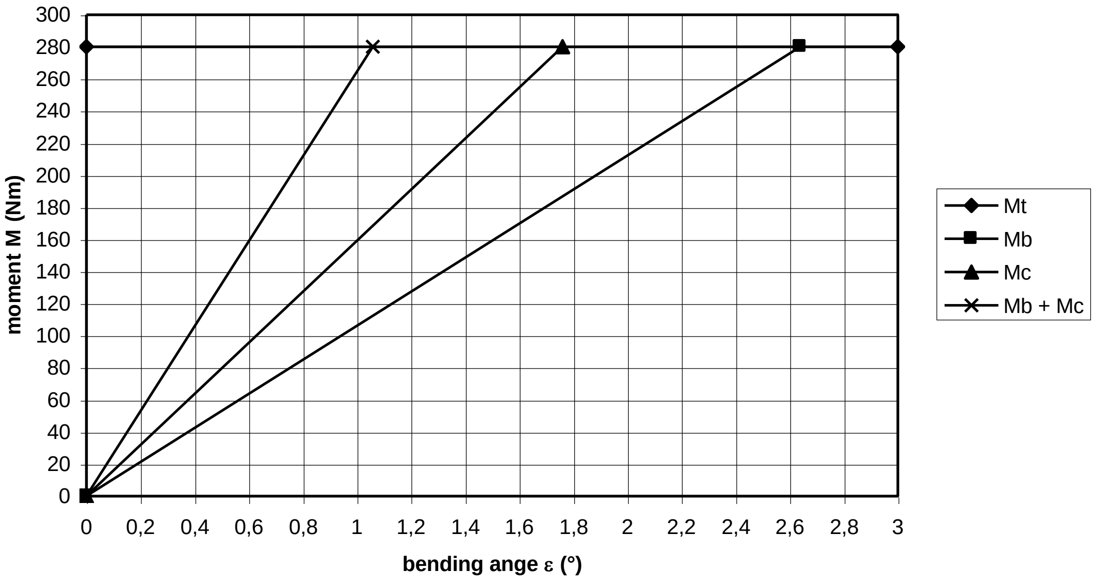

fig. 6 Path of M_t, M_b, M_c, and M_b + M_c as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 6 it can be seen that ε = 1.06°. This is a factor 0.402 of the calculated angle of 2.64°. Because the bending stress is proportional to the bending angle it will also be a factor 0.402 of the calculated stress of 219 N/mm^2 resulting in a stress of about 88 N/mm^2. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with sinα with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment dF * r which is exerted by this force with respect to the blade is therefore varying sinusoidal too.

However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (21)$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a three bladed rotor because adding three $\sin^2\alpha$ functions which make an angle of 120° which each other, appear to result in a constant value. The resulting moment for a two bladed rotor fluctuates just as it is does for one blade because the moments of both blades are strengthening each other. Formula 21 is still valid for the average value of the moment but the momentary value is given by:

$$M_{gyr\ x\text{-}as} = 2 \sin^2\alpha * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (22)$$

This function is given in figure 7.

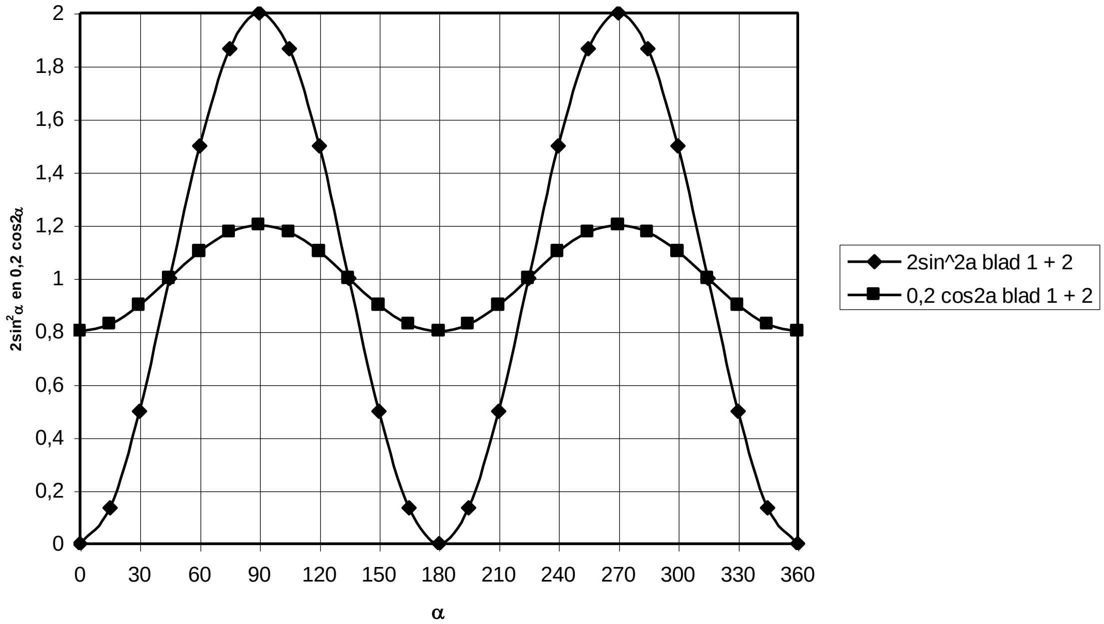

fig. 7 Path of $2 \sin^2\alpha$ and $(1 – 0.2 \cos 2\alpha)$ for a two bladed rotor

Formula 22 can also be written as:

$$M_{gyr\ x\text{-}as} = (1 - \cos 2\alpha) * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (23)$$

Because the average of $\cos 2\alpha$ is zero, the average of formula 23 is the same as formula 21.

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 22 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $2 \sin^2\alpha$ function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened curve can be given by:

$$M_{gyr\ x\text{-}as\ flattened} = (1 - 0.2 \cos 2\alpha) * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (24)$$

The function $(1 – 0.2 \cos 2\alpha)$ is also plotted in figure 7. This function has a maximum for $\alpha = 90°$ and for $\alpha = 270°$. The maximum is $1.2 * I_{rot} * \Omega_{rot} * \Omega_{head}$.

For the calculation of the blade strength we are not interested in the variation of the gyroscopic moment with respect to a fixed axis frame but in variation of the moment in the blade itself so with respect to a rotation axis frame for which it was explained earlier that the moment is varying sinusoidal. If the blade is vertical both axis frames coincide and the moment for both axis frames is the same. The maximum moment in one blade is then halve the value of the moment for the whole rotor.
Therefore the maximum moment in one blade is given by:

$$M_{gyr\ bl\ max} = 0.6 * I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (25)$$

For a two bladed rotor, the moment of inertia of the whole rotor I_{rot} is twice the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$$M_{gyr\ bl\ max} = 1.2\ I_{bl} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (26)$$

For the chosen blade geometry it is calculated that I_{bl} = 8.09 kgm^2. The maximum loaded rotational speed of the rotor can be read in figure 4 and it is found that n_{max} = 320 rpm. This gives $\Omega_{rot\ max}$ = 33.5 rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-4.2 is provided with the hinged side vane safety system which has a light van blade and a large moment of inertia of the whole head around the tower axis. This is because the vane arm is a part of the head. For sudden variations in wind speed and wind direction the vane blade will therefore react very fast but the head will follow only slowly. It is assumed that the maximum angular velocity of the head can be 0.3 rad/s at very high wind speeds.

Substitution of I_{bl} = 8.09 kgm^2, $\Omega_{rot\ max}$ = 33.5 rad/s en $\Omega_{head\ max}$ = 0.3 rad/s in formula 26 gives: $M_{gyr\ bl\ max}$ = 97.6 Nm = 97600 Nmm.

Substitution of M = 97600 Nmm, b = 120 mm and h = 8 mm in formula 14 gives $\sigma_{b\ max}$ = 76 N/mm^2. This value has to be added to the bending stress of 88 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max}$ = 164 N/mm^2. The minimum stress is 88 – 76 = 12 N/mm^2. So the stress is always positive and therefore it is probably not necessary to take the load as a fatigue load.

For the strip material bright drawn strip Fe360 is chosen. For hot rolled strip the allowable stress for a load in between zero and maximum is about 190 N/mm^2 and for a fatigue load it is about 140 N/mm^2. For bright drawn strip these values are higher because the rolling skin is removed and because the material is strengthened by the deformation. Assume the allowable stress for a load in between zero and maximum is about 240 N/mm^2 and for a fatigue load is 190 N/mm^2. The calculated stress is even lower than the allowable fatigue stress so the strip is strong enough.

In reality the blade is not extremely stiff and will also bend. This reduces the bending of the strip and therefore the stress in the strip will be lower than the calculated value.

6.2 Bending stress in the strip for a slowed down rotor

The rotational speed for a rotor which is slowed down by making short-circuit of the generator is very low. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 197 N for V = V_{rated} = 9.5 m/s and δ = 30°. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 30° if the force on one blade is 197 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at r_4 = 1.15 m because the relative wind speed is almost constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 11 changes into:

$$M_{b\ t} = F_{t\ \delta\ bl} * (r_4 - r_2) \quad \text{(Nm)} \qquad (27)$$

Substitution of $F_{t\ \delta\ bl}$ = 197 N, r_4 = 1.15 m en r_2 = 0.04 m in formula 27 gives $M_{b\ t}$ = 218.7 Nm = 218700 Nmm. Substitution of M = 218700 Nmm, b = 120 mm and h = 8 mm in formula 14 gives σ_b = 171 N/mm^2. This is somewhat higher than the calculated stress for a rotating rotor. But the load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable stress is 240 N/mm^2 for bright drawn strip Fe360, so the strip is strong enough.

Because the strip and the blade are rather flexible it has to be checked if a slowed down rotor can't hit the tower. In chapter 6.1 it has been calculated, for no compensation of the gyroscopic moment, that the bending angle is 2.64° for a stress of 197 N/mm^2. So for a stress of 171 N/mm^2 the bending angle will be 2.64 * 171 / 197 = 2.29°. For a rotor radius of R = 2.1 m this results in a movement at the tip of about 0.084 m. Because the blade itself will bend too, the movement will be larger and it is expected that it will be about 0.15 m. The minimum distance in between the blade tip and a tower leg is about 0.56 m (for the modified head with a tilt angle of the rotor shaft of 5°) if the blade is not bending. So there is no chance that the blade hits the tower for a slowed down rotor.

7 Determination of the P_{el}-V curve for 24 V battery charging and delta rectification

The VIRYA-4.2 can be used for 24 V battery charging if the generator is rectified in delta in stead of in star. The generator measurements are given in report KD 200 (ref. 6). Figure 4 is now copied as figure 8 but the P_{mech}-n and P_{el}-n curves for 52 V star are replaced by the P_{mech}-n and P_{el}-n curves for 26 V delta. A voltage of 26 V is about the average charging voltage of a 24 V battery.

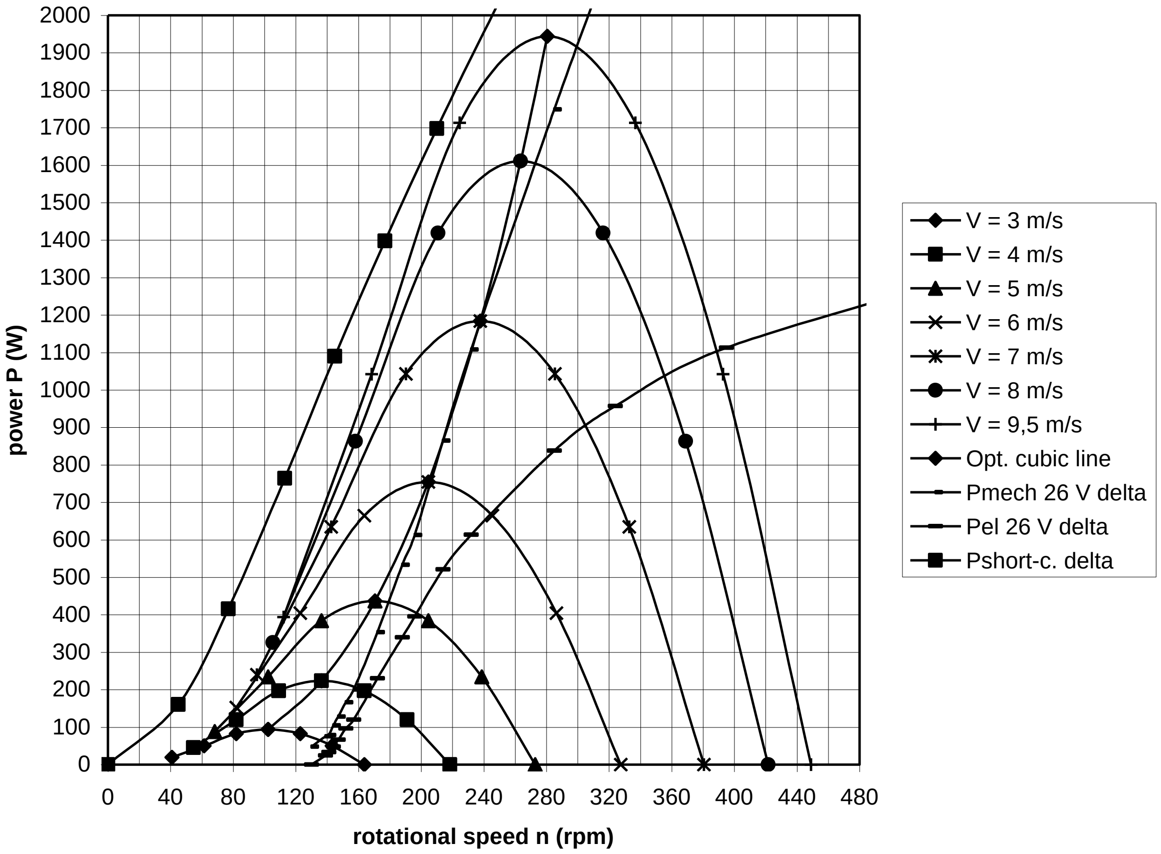

fig. 8 P-n curves of the alternative VIRYA-4.2 rotor and the generator for 26 V delta

If figure 8 is compared to figure 4 it can be seen that the matching for 26 V delta is better than for 52 V star because the P_{mech}-n curve for 26 V delta is lying closer to the optimum cubic line. The P_{el}-V curve for 26 V delta and so for 24 V battery charging is determined in the same way as for 52 V star and is given in figure 9.

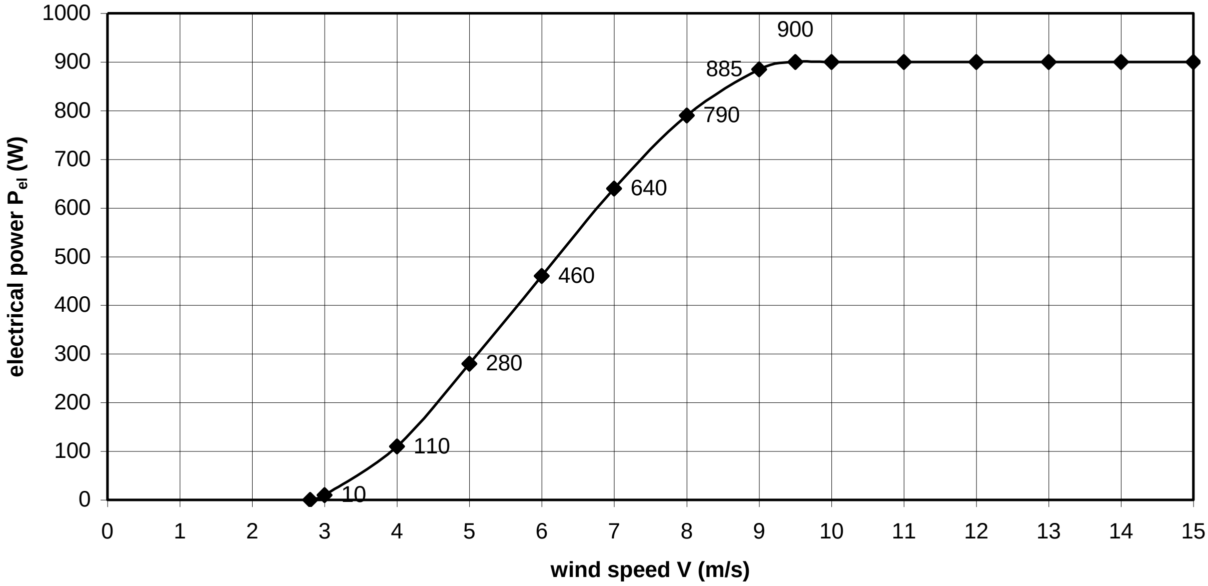

fig. 9 P_{el}-V curve of the VIRYA-4.2 windmill with alternative rotor for 24 V battery charging and rectification in delta

In figure 9 is compared to figure 5 it can be seen that the cut-in wind speed for 26 V delta is a little lower than for 52 V star. However, the P_{el}-V curve for 52 V star is lying higher especially for wind speeds above 5 m/s. The peak power is even 220 W higher for 52 V star. This is because the generator efficiency for 52 V star is a lot better than for 26 V delta. But there are situations where a 24 V battery is preferred as a nominal voltage of 24 is more common than a nominal voltage of 48 V.

It is possible to modify a 230/400 V winding into a 115/200 V winding by connecting the first and the second layer in parallel in stead of in series. If this is done, the same P_{el}-V curve for 52 V star is gained for 26 V star and so one has the advantage of a higher maximum power.

Delta connection has as advantage that the maximum short-circuit torque is higher but as disadvantage that the unloaded sticking torque at low rotational speeds is also higher. This has a negative influence on the starting wind speed. This effect is investigated in chapter 9.

8 Determination of the P_{el}-V curve for 24 V battery charging and star rectification

It is possible to use the VIRYA-4.2 for 24 V battery charging if the generator is rectified in star. However, the matching will be much worse at high wind speeds than for 48 V battery charging and rectification in star as the P_{mech}-n curve starts at a much lower rotational speed. The matching at very low wind speeds will be better and the cut in wind speed will be lower for 24 V battery charging and star rectification is therefore an option for areas with very low wind speeds.

The generator measurements are given in report KD 200 (ref. 6). However, the generator hasn’t been measured for 26 V star. The characteristics for 26 V star are estimated in chapter 5 of report KD 200 using the measurements for 30 V star. Figure 8 is now copied as figure 10 but the P_{mech}-n and P_{el}-n curves for 26 V delta are replaced by the P_{mech}-n and P_{el}-n curves for 26 V star. A voltage of 26 V is about the average charging voltage of a 24 V battery.

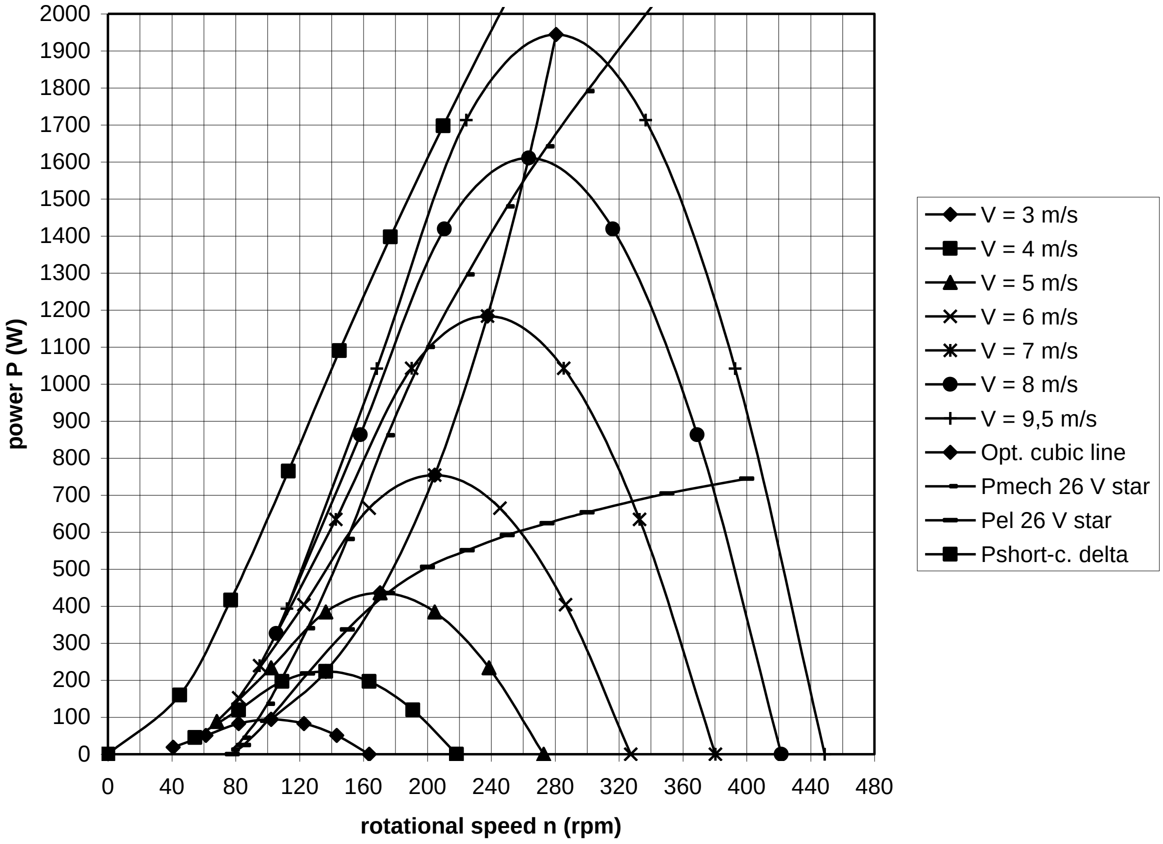

fig. 10 P-n curves of the alternative VIRYA-4.2 rotor and the generator for 26 V star

If figure 10 is compared to figure 8 it can be seen that at high wind speeds, the matching for 26 V delta is much better than for 26 V star because the P_{mech}-n curve for 26 V delta is lying closer to the optimum cubic line. However, at a wind speed of 3 m/s, the matching for 26 V star is better. The P_{el}-V curve for 26 V star and so for 24 V battery charging is determined in the same way as for 26 V delta. Figure 9 is copied as figure 11 and the P_{el}-V curve for 26 V star is added.

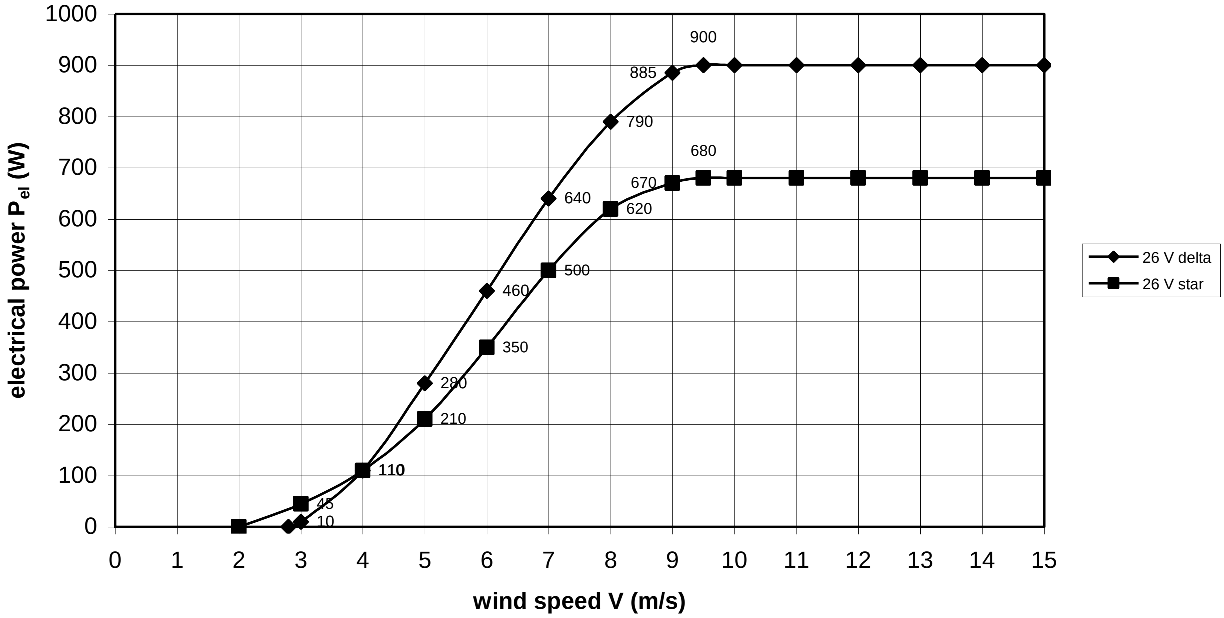

fig. 11 P_{el}-V curve of the VIRYA-4.2 windmill with alternative rotor for 24 V battery charging and rectification in delta and in star

In figure 11 it can be seen that the P_{el}-V curve for 26 V star starts at a wind speed of 2 m/s and that it is lying higher than the P_{el}-V curve for 26 V delta up to a wind speed of 4 m/s. But for wind speeds above 4 m/s, the P_{el}-V curve for 26 V delta is much better. The maximum power at a wind speed of 9.5 m/s and higher is 900 W for 26 V delta and only 680 W for 26 V star. This is because at high rotational speeds, the generator efficiency for 26 V delta is a lot better than for 26 V star.

The consequence of the much lower generator efficiency for 26 V star is also that more heat is generated in the winding at high wind speeds and I don't know if the winding will survive this if the windmill is running for a long time at high rotational speeds for 26 V star. So use of the generator for 26 V star is only advised at low wind speeds.

It is possible to switch from star rectification to delta rectification if a star-delta switch is mounted at the tower foot. However, this requires a 6-wire cable from the generator to this switch as no coil ends can be connected to each other at the terminal of the generator. The rectifier can also be mounted at the tower foot, just behind the star-delta switch. The short-circuit switch must be mounted before the star-delta switch and all six wires coming from the generator have to be connected to each other for short-circuit in delta. All components can be mounted in a box at the tower foot. A 2-wire cable can be used from this box to the batteries and the battery charge controller.

9 Checking of the starting behaviour for delta and star rectification

The sticking torque is increasing at increasing rotational speed because of eddy currents in the stator stamping. At stand still position, the sticking torque is only determined by the bearing friction and the seal on the rotor shaft. It has been measured to be about 0.9 Nm. The unloaded torque for delta rectification rises faster than for star rectification because higher harmonic currents can circulate in the winding for delta rectification. The measurements are given in figure 1 of KD 200 (ref. 1). The unloaded Q-n curves for delta and star rectification are copied in figure 12.

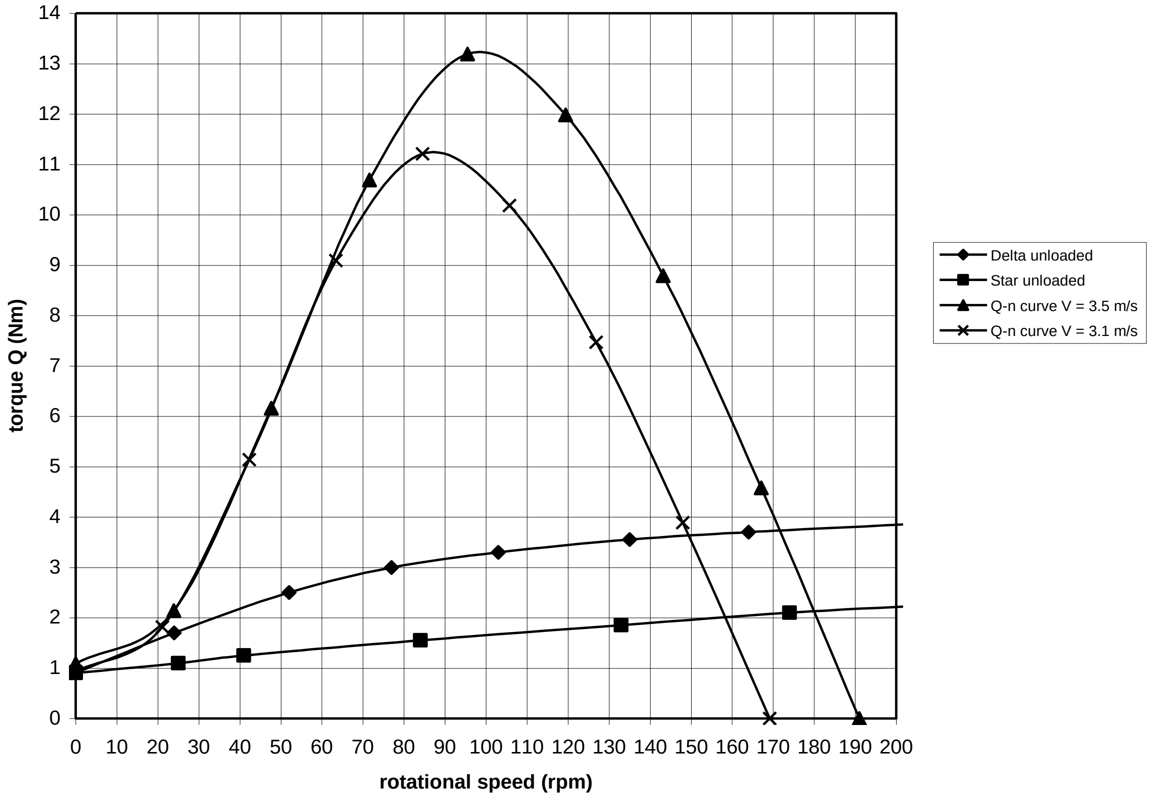

fig. 12 Measured unloaded Q-n curve of the generator for delta and star rectification and Q-n curves of the rotor for V = 3.5 m/s and V = 3.1 m/s

In chapter 4 it was determined that the starting wind speed is 3.1 m/s for a sticking torque Q_s = 0.9 Nm. The Q-n curves of the rotor are determined for V = 3.5 m/s and for V = 3.1 m/s. The Q-n curves of the rotor can be determined in the same way as the P-n curves but only one has to use the formula for the torque Q instead of the formula for P. The formula for Q is given as formula 4.3 in KD 35 (ref. 3). The Q-n curves for V = 3.5 m/s and V = 3.1 m/s are also given in figure 12.

In figure 12 it can be seen that the Q-n curve for V = 3.5 m/s is lying above the unloaded Q-n curve of the generator for delta rectification and so the rotor will start at V = 3.5 m/s if the generator is connected in delta. In figure 12 it can be seen that the Q-n curve for V = 3.1 m/s is lying above the unloaded Q-n curve of the generator for star rectification and so the rotor will start at V = 3.1 m/s if the generator is connected in star.

So the starting behaviour for star rectification will be better than for delta rectification but the starting behaviour for delta rectification is certainly acceptable.

10 References

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