Calculations executed for the 4-bladed rotor of the VIRYA-3.6L2 windmill (λ_d = 2, galvanised steel blades) with a Polycord transmission in between the rotor shaft and the vertical shaft to drive a rotating positive displacement pump

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1 Introduction

The VIRYA-3.6L2 windmill is designed to drive a rotating positive displacement pump mounted at the tower foot using an accelerating Polycord transmission in the head and a vertical shaft in the tower. The pump has a non-fluctuating flow and therefore the driving torque is non-fluctuating too. The windmill is used for low head pumping for irrigation from a river to a reservoir positioned at the highest point of the field. The windmill is positioned half way the river and the reservoir, so it has some meters suction height and some meters pressure height. For irrigation, normally long pipes are needed in between the water source and the reservoir and therefore it is very favourable if the water flow isn't fluctuating because this prevents high acceleration forces. The transmission and the pump are described in chapter 7.

The VIRYA-3.6L2 is meant for use in western countries but also for use in developing countries. The VIRYA-3.6L2 windmill has a simple 4-bladed rotor with galvanised steel blades. The windmill will be provided with the hinged side vane safety system with an eccentrically placed rotor which is used in all VIRYA windmills. This safety system will be equipped with a relatively light vane blade resulting in a rated wind speed of about 8 m/s.

The 6 metre tower has four legs made out of pipe which are connected to each other by only horizontal strips as it is also done for the VIRYA-4.2 tower. The tower has hinges at the foot. The pump is mounted about 0.5 m above the foundation and therefore the vertical shaft can be made out of one piece with a length of about 6 metre. It has two bearings in the tower.

2 Description of the rotor of the VIRYA-3.6L2 windmill

As a positive displacement pump has a constant torque, a windmill rotor with a high starting torque coefficient is needed. This is only possible if the rotor has a rather low design tip speed ratio. A large flow is needed for irrigation and this requires a rather large rotor diameter if this flow has to be supplied at a low wind speed. The VIRYA-3.6L2 windmill has therefore a rotor with a design tip speed ratio $\lambda_d = 2$ and a rotor diameter D = 3.6 m (L2 is added to the name to distinguish it from other 3.6 m VIRYA windmills). The rotor has only four blades and two opposite blades are connected to each other by means of a twisted strip. Advantages of this construction are that no large welded spoke assembly is required, that the rotor can be balanced easily and that the assembly of two opposite blades can be transported mounted.

The rotor has blades with a constant chord and is provided with a 7.14 % cambered airfoil. A blade is made of a strip with dimensions of 625 * 1250 * 2 mm and 4 blades can be made from a standard sheet of 1.25 * 2.5 m. Because the blade is cambered, the chord c is a little less than the blade width, resulting in c = 617 mm = 0.617 m.

Two opposite blades are connected to each other by a 2 m long twisted central strip size 80 * 8 mm. The overlap in between blade and strip is 0.45 m which results in a free blade length of 0.8 m. This blade length in combination with a design tip speed ratio of 2 and a blade thickness of 2 mm must be enough to prevent flutter of the blade at high wind speeds. A connecting strip size 40 * 6 * 500 mm is welded square to each end of a central strip. The blade is connected to the central strip and the connecting strip using five M10 bolts. Washers are mounted in between the central strip and the blade to prevent too strong deformation of the blade when the bolts are tightened. The bolts at the central strip are also used for connection of the balancing weights.

The hub is made of square bar 80 * 80 mm with a tapered hole in the centre for connection to the rotor shaft. The two central strips are clamped in between the hub and a square sheet by means of four M10 bolts and that is why the strip is not loaded by a bending moment at the position of the holes. The rotor shaft has a diameter of 35 mm. The hub is pulled on the tapered shaft end by one central M16 bolt. The mass of a central strip and two blades is about 34.5 kg so this mass can be handled by one man. The mass of the whole rotor including hub is about 73 kg which seems acceptable for a 3.6 m diameter steel rotor with a design tip speed ratio $\lambda_d = 2$. A sketch of the rotor is given in figure 1.

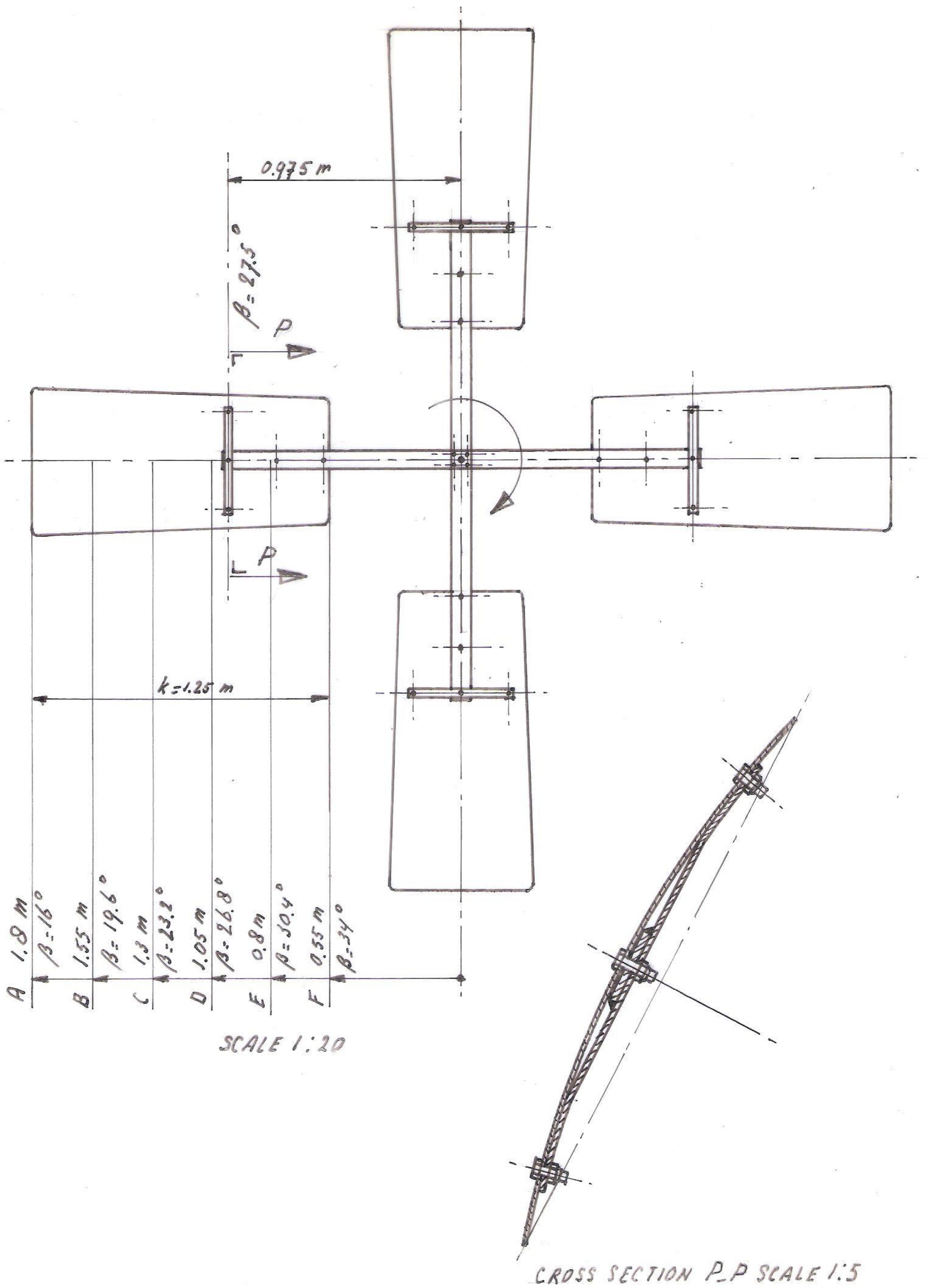

fig. 1 Sketch VIRYA-3.6L2 rotor, B = 4, λ_d = 2, blade angles β are given for increased twist

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 1). This report (KD 651) has its own formula numbering. Substitution of $\lambda_d = 2$ and R = 1.8 m in formula (5.1) of KD 35 gives:

$\lambda_{r\,d} = 1.1111 * r$ (-) (1)

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$\beta = \phi - \alpha$ (°) (2)

$\phi = 2/3 \arctan 1 / \lambda_{r\,d}$ (°) (3)

Substitution of B = 4 and c = 0.617 m in formula (5.4) of KD 35 gives:

$C_l = 10.183\ r\ (1 - \cos\phi)$ (-) (4)

Substitution of V = 4 m/s and c = 0.617 m in formula (5.5) of KD 35 gives:

$R_{e\,r} = 1.646 * 10^5 * \sqrt{(\lambda_{r\,d}^2 + 4/9)}$ (-) (5)

The blade is calculated for six stations A till F which have a distance of 0.25 m of one to another. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l, α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The aerodynamic characteristics of a 7.14 % cambered airfoil are given in report KD 398 (ref. 2). The Reynolds values for the stations are calculated for a wind speed of 4 m/s because this is a reasonable wind speed for a windmill with V_{rated} = 8 m/s. Those airfoil Reynolds numbers are used which are lying closest to the calculated values. The effect of the central strip on the aerodynamic characteristics of the inner part of the blade is neglected.

sta-tion	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 4 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	1.8	2	17.7	0.617	0.87	0.94	3.47	3.4	1.0	1.7	16.7	16.0	0.039
B	1.55	1.722	20.1	0.617	0.96	0.94	3.04	3.4	1.9	1.7	18.2	18.4	0.039
C	1.3	1.444	23.1	0.617	1.06	0.95	2.62	2.5	3.4	2.3	19.7	20.8	0.034
D	1.05	1.167	27.1	0.617	1.17	1.11	2.21	2.5	4.5	3.9	22.6	23.2	0.040
E	0.8	0.889	32.2	0.617	1.26	1.28	1.83	1.7	6.1	6.6	26.1	25.6	0.076
F	0.55	0.611	39.1	0.617	1.25	1.44	1.49	1.7	6.0	11.1	33.1	28.0	0.115

table 1 Calculation of the blade geometry of the original VIRYA-3.6L2 rotor

The theoretical blade angle β_{th} for stations A to F varies in between 16.7° and 33.1°. If the blade angle is linearised in between 16° at the blade tip and 28° at the blade root, the linearised angle of attack α_{lin} differs only a little from the theoretical value α_{th} for the most important outer side of the blade. So the blade is twisted 28° - 16° = 12°. The central strip is twisted 28° right hand in between the hub and the blade root. The blade angel at the heart of a connecting strip is about 23.7° so the outer part of the central strip is twisted back 4.3°.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.8 of KD 35 (for B = 4) and λ_{opt} = 2 and C_d/C_l = 0.04 gives $C_{p\ th}$ = 0.42.
Substitution of $C_{p\ th}$ = 0.42, R = 1.8 m and blade length k = 1.25 m in formula 6.3 of KD 35 gives $C_{p\ max}$ = 0.38.
$C_{q\ opt} = C_{p\ max} / \lambda_{opt}$ = 0.38 / 2 = 0.19. Substitution of $\lambda_{opt} = \lambda_d$ = 2 in formula 6.4 of KD 35 gives λ_{unl} = 3.2. The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

The average blade angle is 22°. For a non rotating rotor the average angle of attack α is therefore 90° - 22° = 68°. The estimated C_l-α curve for large values of α is given as figure 5 of KD 398. For α = 68° it can be read that C_l = 0.73.
Substitution of B = 4, R = 1.8 m, k = 1.25 m, C_l = 0.73 and c = 0.617 m in formula 6 gives that $C_{q\ start}$ = 0.11. For the ratio in between the starting torque and the optimum torque we find that it is 0.11 / 0.19 = 0.58. This is good for a rotor with a design tip speed ratio of 2. The ratio is expected to be high enough for combination of the VIRYA-3.6L2 windmill with a positive displacement pump which has about a constant torque. As the torque is proportional with V^2, it means that the starting wind speed will be a factor $(0.19 / 0.11)^{1/2}$ = 1.31 times the design wind speed. The design wind speed is the wind speed for which the rotor turns at its design tip speed ratio λ_d = 2.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \qquad (m/s) \qquad (7)$$

At this moment the starting torque Q_s of the positive displacement pump is not yet known so the starting wind speed can’t be calculated accurately. Assume Q_s = 20 Nm.
Substitution of Q_s = 20 Nm, $C_{q\ start}$ = 0.11, ρ = 1.2 kg/m^3 and R = 1.8 m in formula 7 gives that V_{start} = 4.1 m/s. This seems acceptable low for a 4-bladed rotor with a design tip speed ratio λ_d = 2 if the windmill is placed in a good wind regime.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for λ = 0.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 3). With this method, it can be determined that the C_q-λ curve is directly rising for low values of λ if a 7.14 % cambered sheet airfoil is used. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-3.6L2 rotor are given in figure 2 and 3.

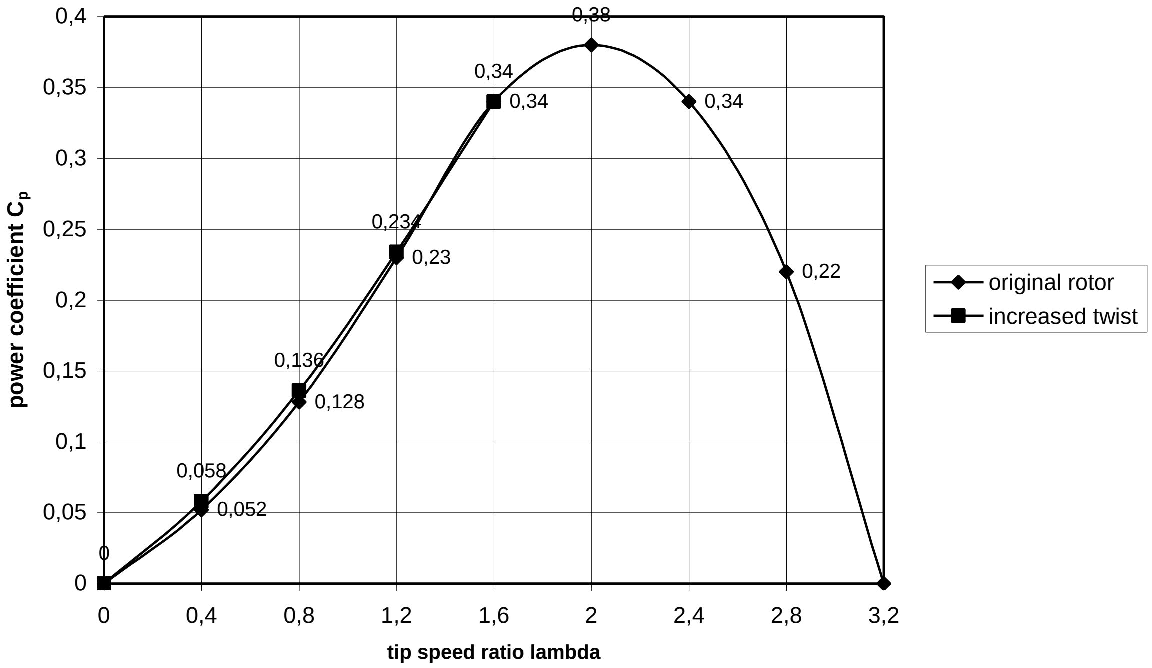

fig. 2 Estimated C_p-λ curve for the VIRYA-3.6L2 rotor for the wind direction perpendicular to the rotor for the original rotor and for increased twist

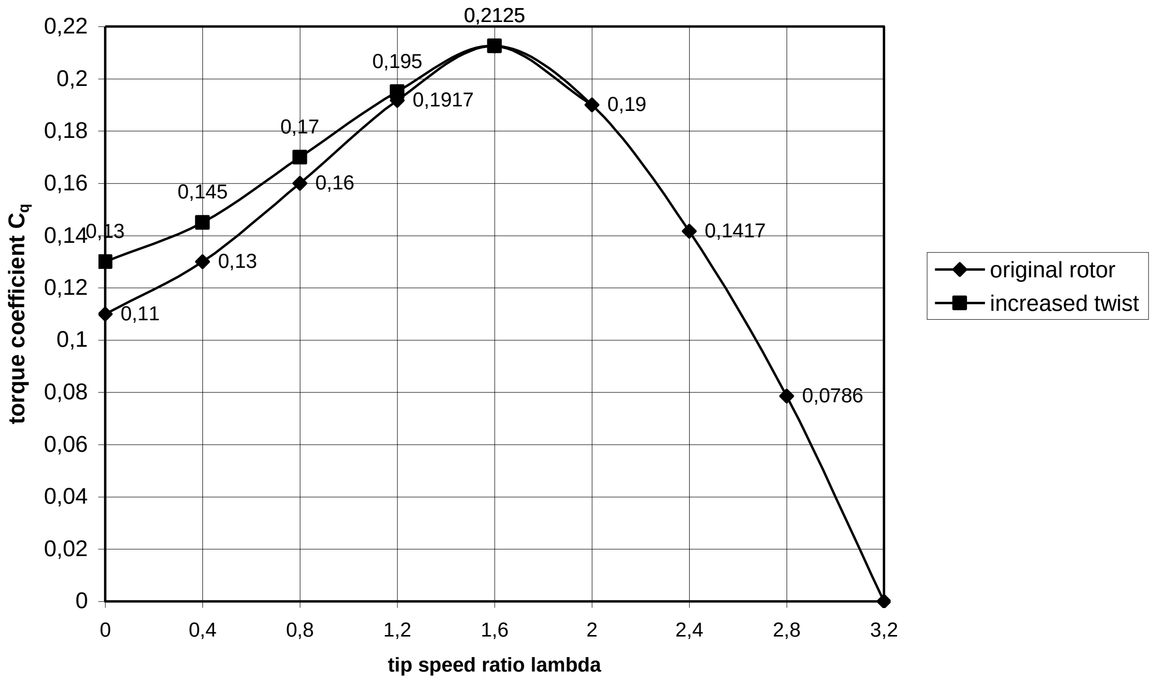

fig. 3 Estimated C_q-λ curve for the VIRYA-3.6L2 rotor for the wind direction perpendicular to the rotor for the original rotor and for increased twist

In figure 3 it can be seen that the starting torque coefficient is 0.11 and that the optimum torque coefficient for λ_d = 2 is 0.19. For a pump with a constant torque, this results in a starting wind speed which is a factor 1.31 higher than the design wind speed because the torque is proportional with V^2. So this means that if a design wind speed is chosen of 3 m/s, the rotor only starts at a wind speed of about 4 m/s. But wind is always fluctuating and if the average wind speed is 3 m/s, I expect that within a minute the wind speed will be 4 m/s for a short time and this might be enough to start the rotor. However, to increase the starting behaviour, it might be possible to increase the starting torque coefficient without changing the rotor solidity, so without changing the blade dimensions.

When I was working at the Wind Energy Group of the University of Technology Eindhoven we have done wind tunnel tests to check if the starting torque coefficient can be increased by increasing the blade twist without strong change of the design tip speed ratio and the maximum power coefficient. These measurements are given in report R 604 D (ref. 4). This report is no longer available but I have a copy of it and I have studied this report again. The main measurements were performed for a 6-bladed rotor with λ_d = 1.5 and a 3-bladed rotor with λ_d = 2.8 both having the same blade geometry. As the VIRYA-3.6L2 has a 4-bladed rotor with λ_d = 2, I only have looked at the measurements for the rotor with λ_d = 1.5. This rotor has been measured for six different blade angles. The blade angles β for measurements no 4 are lying closest to the theoretical values for β and the measured C_p for these blade angles is highest. For the measurements no 6, the blade angel at the tip is increased by 1° but the blade angle at the blade root is increased by 13°. The maximum C_p is only a little lower and belongs to a somewhat lower tip speed ratio but the starting torque coefficient is increased from about 0.13 up to about 0.18. So my conclusion from these measurements is that the blade twist can be increased a lot if the blade angle at the blade tip is maintained. The maximum C_p stays almost the same but there is a substantial increase of the starting torque coefficient.

Assume for the VIRYA-3.6L2 rotor, that the blade angle at the blade tip is maintained at 16° but that the blade angle at the blade root is increased by 6° and so it becomes 28° + 6° = 34°. So now the blade twist is 34° - 16° = 18° in stead of 12°. The central strip has now to be twisted 34° right hand in between the hub and the blade root. The blade angle at the heart of the connecting strip is now about 27.5°, so the central strip has to be twisted left hand 6.5° in between the blade root and the heart of the connecting strip.

The average blade angle is now (16° + 34°) / 2 = 20°. So the average blade angle is increased from 17° up to 20° and this means that the starting torque will increase by about a factor 20 / 17 = 1.176 resulting in a starting torque coefficient of about 0.13. This increase of the starting torque coefficient results in change of the C_p-λ and C_q-λ curves as given in figure 2 and figure 3. It is assumed that only the part in between $0 < \lambda < 1.6$ is changed. The changed part of the curves is labelled "increased twist". The curve for the original rotor is labelled "original rotor". The blade angles given in figure 1 are the blade angles for increased twist. The Q-n curves of the rotor as given in chapter 5 are also made using the values of C_q for increased twist.

The starting torque coefficient for increased twist is 0.13 and the optimum torque coefficient remains 0.19. So now the starting wind speed is a factor $(0.19 / 0.13)^{½}$ = 1.209 higher than the design wind speed. So the starting wind speed for a design wind speed of 3 m/s is now about 3.6 m/s in stead of 4 m/s for the original rotor. So increase of the blade twist is useful and costs no more material and labour.

5 Determination of the Q-n curves

The determination of the Q-n curves of a windmill rotor is described in chapter 8 of KD 35. One needs a C_q-λ curve of the rotor and a δ-V curve of the safety system together with the formulas for the torque Q and the rotational speed n. The C_q-λ curve is given in figure 3. The curve ”increased twist” will be used. The δ-V curve of the safety system depends on the safety system. As this safety system is not yet designed, the δ-V curve of it is estimated. The estimated δ-V curve is given in figure 4.

The head starts to turn away at a wind speed of about 5 m/s. For wind speeds above 8 m/s it is supposed that the head turns out of the wind such that the component of the wind speed perpendicular to the rotor plane, is staying constant. The Q-n curve for 8 m/s will therefore also be valid for wind speeds higher than 8 m/s.

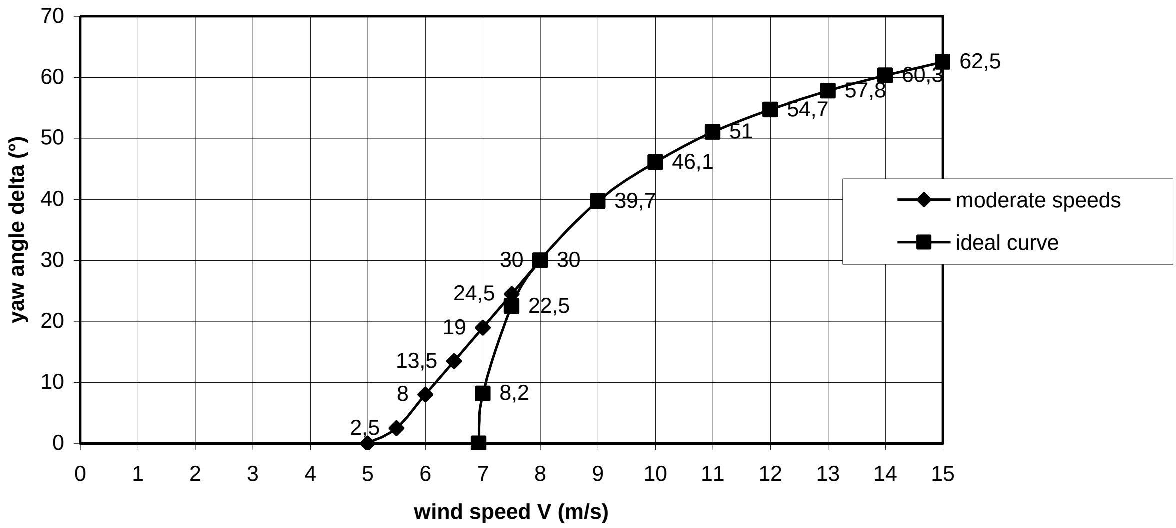

fig. 4 Estimated δ-V curve VIRYA-3.6L2 safety system with V_{rated} = 8 m/s

The Q-n curves are used to check the matching with the Q-n curve of the pump. The Q-n curves are determined for wind the speeds V = 2, 3, 4, 5, 6, 7 and 8 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for Q and n are used which are given in chapter 7 of KD 35.

Substitution of R = 1.8 m in formula 7.1 of KD 35 gives:

$$n_\delta = 5.3052 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of ρ = 1.2 kg / m^3 and R = 1.8 m in formula 7.7 of KD 35 gives:

$$Q_\delta = 10.9931 * C_q * \cos^2\delta * V^2 \qquad \text{(W)} \qquad (9)$$

The Q-n curves are determined for C_q values belonging to λ is 0, 0.4, 0.8, 1.2, 1.6, 2, 2.4, 2.8 and 3.2 (see figure 3). For a certain wind speed, for instance V = 2 m/s, related values of C_q and λ are substituted in formula 8 and 9 and this gives the Q-n curve for that wind speed. For the higher wind speeds the yaw angle δ as given by figure 4, is taken into account. The result of the calculations is given in table 2. The calculated values for n and Q are plotted in figure 5.

		V = 2 m/s δ = 0°		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 8°		V = 7 m/s δ = 19°		V = 8 m/s δ = 30°	
λ (-)	C_q (-)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	n (rpm)	Q (Nm)	$n_δ$ (rpm)	$Q_δ$ (Nm)	$n_δ$ (rpm)	$Q_δ$ (Nm)	$n_δ$ (rpm)	$Q_δ$ (Nm)
0	0.13	0	5.72	0	12.86	0	22.87	0	35.73	0	50.45	0	62.60	0	68.60
0.4	0.145	4.24	6.38	6.37	14.35	8.49	25.50	10.61	39.85	12.61	56.27	14.05	69.83	14.70	76.51
0.8	0.17	8.49	7.48	12.73	16.82	16.98	29.90	21.22	46.72	25.22	65.97	28.09	81.87	29.40	89.70
1.2	0.195	12.73	8.57	19.10	19.29	25.46	34.30	31.83	53.59	37.83	75.68	42.14	93.91	44.11	102.90
1.6	0.2125	16.98	9.34	25.46	21.02	33.95	37.38	42.44	58.40	50.43	82.47	56.18	102.33	58.81	112.13
2	0.19	21.22	8.35	31.83	18.80	42.44	33.42	53.05	52.22	63.04	73.74	70.23	91.50	73.51	100.26
2.4	0.1417	25.46	6.23	38.20	14.02	50.93	24.92	63.66	38.94	75.65	54.99	84.27	68.24	88.21	74.77
2.8	0.0786	29.71	3.46	44.56	7.78	59.42	13.82	74.27	21.60	88.26	30.50	98.32	37.85	102.92	41.47
3.2	0	33.95	0	50.93	0	67.91	0	84.88	0	100.87	0	112.36	0	117.62	0

table 2 Calculated values of n and Q as a function of λ and V for the VIRYA-3.6L2 rotor

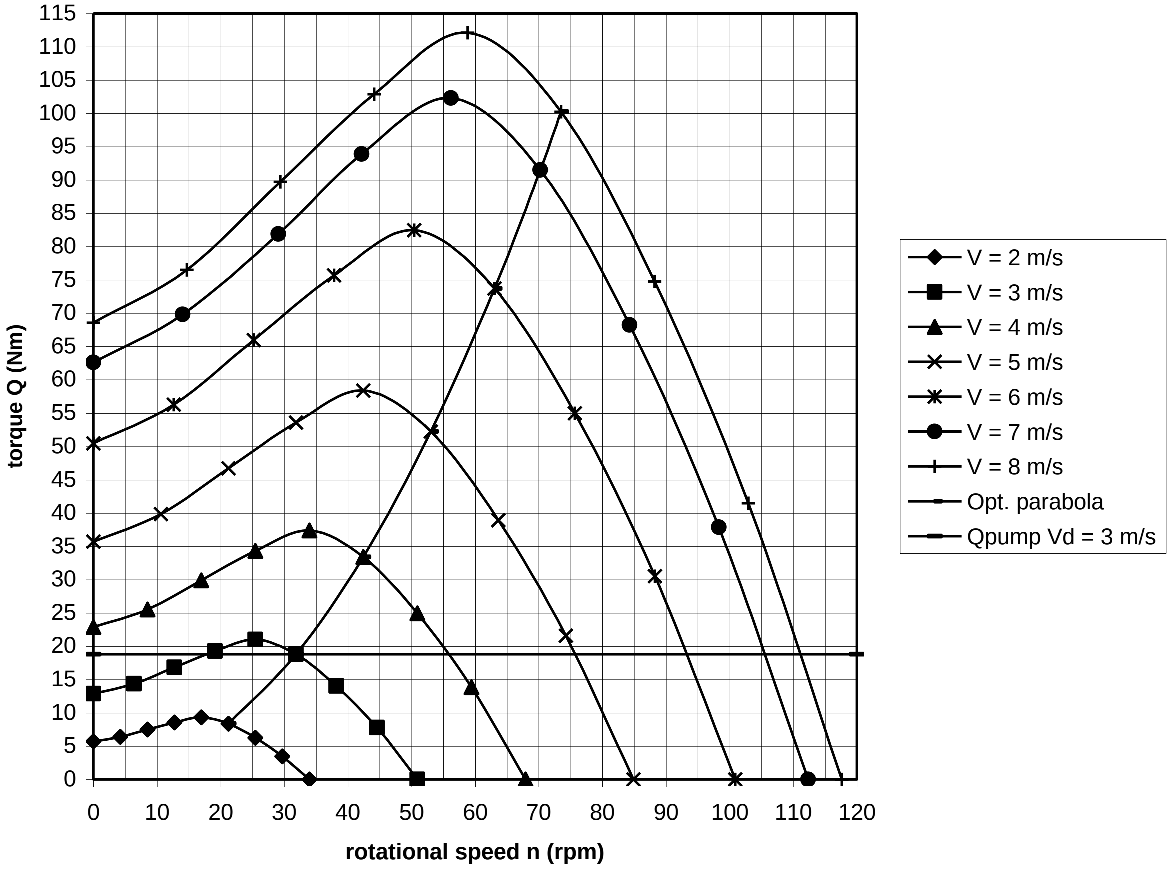

fig. 5 Q-n curves of the VIRYA-3.6L2 rotor for blades with increased twist

The optimum parabola which is going through the points with λ = 2, where C_p is maximum, is also drawn in figure 5. If the pump has a constant torque for all rotational speeds and if the pump geometry is chosen such that the design wind speed is 3 m/s, the pump characteristic is a horizontal line going through the point n = 32.82 rpm and Q = 18.8 Nm. This line is also given in figure 5. It was calculated earlier that the starting wind speed is 3.6 m/s but in figure 5 it can be seen that a wind speed of 3 m/s is enough to continue starting if a rotational speed of about 18 rpm is reached. The real tip speed ratio increases strongly for wind speeds above 3 m/s and at a wind speed of 8 m/s and higher, the rotor runs at a tip speed ratio of about 3 and at a rotational speed of about 111 rpm. Figure 5 gives the Q-n curves on the rotor shaft. The Q_v-n_v curves on the vertical shaft can be derived from figure 5 if the gear ratio i and the efficiency η of the accelerating Polycord transmission is taken into account. This is done in chapter 7.3.

6 Calculation of the strength of the strip which connects the blades

Two opposite blades are connected to each other by a strip with a length of 2000 mm, a width b = 80 mm and a height h = 8 mm. The strip is loaded by a bending moment with axial direction which is caused by the rotor thrust and by the gyroscopic moment. The strip is also loaded by a centrifugal force and by a bending moment with tangential direction caused by the torque and by the weight of the blade but the stresses which are caused by these loads can be neglected.

Because the strip is thin and long it makes the blade connection elastic and therefore the blade will bend backwards already at a low load. As a result of this bending, a moment with direction forwards is created by a component of the centrifugal force in the blade. The bending is substantially decreased by this moment and this has a favourable influence on the bending stress.

It is started with the determination of the bending stress which is caused by the rotor thrust. There are two critical situations:

1[e] The load which appears for a rotating rotor at V_{rated} = 8 m/s. For this situation the bending stress is decreased by the centrifugal moment. The yaw angle is 30° for V_{rated} = 8 m/s.

2[e] The load which appears for a locked rotor. The rotor is locked by connecting a blade strip to the head (never connect a blade strip to the tower).

6.1 Bending stress in the strip for a rotating rotor and V = 8 m/s

The rotor thrust is given by formula 7.4 of KD 35. The rotor thrust is the axial load of all blades together and exerts in the hart of the rotor. The thrust per blade $F_{t\,\delta\,bl}$ is the rotor thrust $F_{t\,\delta}$ divided by the number of blades B. This gives:

$$F_{t\,\delta\,bl} = C_t * \cos^2\delta * ½\rho V^2 * \pi R^2 / B \qquad \text{(N)} \qquad (10)$$

For the rotor theory it is assumed that every small area dA which is swept by the rotor, supplies the same amount of energy and that the generated energy is maximised. For this situation the wind speed in the rotor plane has to be slowed down till 2/3 of the undisturbed wind speed V. This results in a pressure drop over the rotor plane which is the same for every value of r. It can be proven that this results in a triangular axial load which forms the thrust and in a constant radial load which supplies the torque. The theoretical thrust coefficient C_t for the whole rotor is 8/9 = 0.889 for the optimal tip speed ratio. In practice C_t is lower because of the tip losses and because the blade is not effective up to the rotor centre. The blade length k of the VIRYA-3.6L2 rotor is 1.25 m but the rotor radius R = 1.8 m. Therefore there is a disk in the centre with an area of about 0.093 of the rotor area on which no thrust is working. This results in a theoretical thrust coefficient C_t = 8/9 * 0.907 = 0.806. Because of the tip losses, the real C_t value is substantially lower. Assume this results in a real practical value of C_t = 0.75.

Substitution of C_t = 0.75, δ = 30°, ρ = 1.2 kg/m³, V = 8 m/s, R = 1.8 m and B = 4 in formula 10 gives $F_{t\,\delta\,bl}$ = 55 N.

For a pure triangular load, the same moment is exerted in the hart of the rotor as for a point load which exerts in the centre of gravity of the triangle. The centre of gravity is lying at 2/3 R = 1.2 m. Because the blade length is only k, there is no triangular load working on the blade but a load with the shape of a trapezium as the triangular load over the part R – k falls off. The centre of gravity of the trapezium has been determined graphically and is lying at about r_1 = 1.25 m.

The maximum bending stress is not caused at the hart of the rotor but at the edge of the hub because the strip bends backwards from this edge. This edge is lying at r_2 = 0.04 m.

At this edge we find a bending moment $M_{b\,t}$ caused by the thrust which is given by:

$M_{b\,t} = F_{t\,\delta\,bl} * (r_1 - r_2)$ (Nm) (11)

Substitution of $F_{t\,\delta\,bl} = 55$ N, $r_1 = 1.25$ m and $r_2 = 0.04$ m gives $M_{b\,t} = 66.6$ Nm = 66600 Nmm.

For the stress we use the unit N/mm^2 so the bending moment has to be given in Nmm. The bending stress σ_b is given by:

$\sigma_b = M / W$ (N/mm^2) (12)

The moment of resistance W of a strip is given by:

$W = 1/6\ bh^2$ (mm^3) (13)

(12) + (13) gives:

$\sigma_b = 6\ M / bh^2$ (N/mm^2) (M in Nmm) (14)

Substitution of M = 66600 Nmm, b = 80 mm and h = 8 mm in formula 14 gives $\sigma_b = 78$ N/mm^2. For this stress the effect of the stress reduction by bending forwards of the blade caused by the centrifugal force in the blade has not yet been taken into account. The gyroscopic moment has also not yet been taken into account.

Next it is investigated how far the blade bends backwards as a result of the thrust load and what influence this bending has on the centrifugal moment. Hereby it is assumed that the strip is bending only in between the hub and the inner connection bolt of blade and central strip. So it is assumed that the blade itself is not bending. The inner connection bolt is lying at $r_3 = 0.575$ m = 575 mm. So the length of the strip l which is loaded by bending is given by:

$l = r_3 - r_2$ (mm) (15)

The load from the blade on the strip at r_3 can be replaced by a moment M and a point load F. F is equal to $F_{t\,\delta\,bl}$. M is given by:

$M = F * (r_1 - r_3)$ (Nmm) (16)

The bending angle ϕ (in radians) at r_3 for a strip with a length l is given by (combination of the standard formulas for a moment plus a point load):

$\phi = l * (M + ½\ Fl) / EI$ (rad) (17)

The bending moment of inertia I of a strip is given by:

$I = 1/12\ bh^3$ (mm^4) (18)

(15) + (16) + (17) + (18) gives:

$\phi = 12 * F * (r_3 - r_2) * \{(r_1 - r_3) + ½\ (r_3 - r_2)\} / (E * bh^3)$ (rad) (19)

Substitution of F = 55 N, $r_3 = 575$ mm, $r_2 = 40$ mm, $r_1 = 1250$ mm, $E = 2.1 * 10^5$ N/mm^2, b = 80 mm and h = 8 mm in formula 19 gives: $\phi = 0.03869$ rad = 2.22°.

This is an angle which can not be neglected. In report R409D (ref. 5) a formula is derived for the angle ε with which the blade moves backwards if it is connected to the hub by a hinge. This formula is valid if both the axial load and the centrifugal load are triangular. For the VIRYA-3.6L2 this is not exactly the case but the formula gives a good approximation. The formula is given by:

$$\varepsilon = \arcsin\left(\frac{C_t * \rho * R^2 * \pi}{B * A_{pr} * \rho_{pr} * \lambda^2}\right) \quad (°) \qquad (20)$$

In this formula A_{pr} is the cross sectional area of the airfoil (in m^2) and ρ_{pr} is the density of the used airfoil material (in kg/m^3). For a plate width of 625 mm and a plate thickness of 2 mm it is found that $A_{pr} = 0.00125\ m^2$. The blade is made of steel sheet with a density ρ_{pr} of about $\rho_{pr} = 7.8 * 10^3\ kg/m^3$. If the rotor is coupled to a positive displacement pump it will run with a rather high tip speed ratio at high wind speeds. It is supposed that the tip speed ratio is 3 for V = 8 m/s. Substitution of $C_t = 0.75$, $\rho = 1.2\ kg/m^3$, R = 1.8 m, B = 4, $A_{pr} = 0.00125\ m^2$, $\rho_{pr} = 7.8 * 10^3\ kg/m^3$ and $\lambda = 3$ in formula 20 gives: $\varepsilon = 1.50°$. This angle is smaller than the calculated angle of 2.22° with which the blade would bend backwards if the compensating effect of the centrifugal moment is not taken into account. This means that the real bending angle will be less than 1.72°.

The real bending angle ε is determined a follows. A thrust moment M_t = 66.6 Nm is working backwards and M_t is independent of ε for small values of ε. A bending moment M_b is working forwards and M_b is proportional with ε. M_b = 66.6 Nm for $\varepsilon = 2.22°$. A centrifugal moment M_c is working forwards and M_c is also proportional with ε. M_c = 66.6 Nm for $\varepsilon = 1.50°$. The path of these three moments is given in figure 6. The sum total of $M_b + M_c$ is determined and the line $M_b + M_c$ is also given in figure 6.

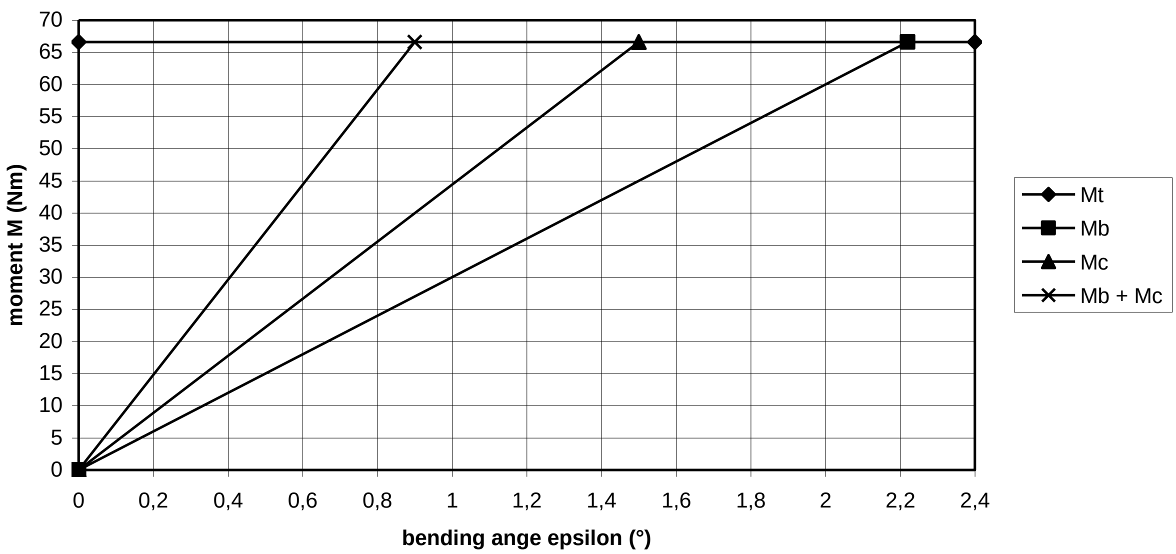

fig. 6 Path of M_t, M_b, M_c, and $M_b + M_c$ as a function of ε

The point of intersection of the line of M_t with the line of $M_b + M_c$ gives the final angle ε. In figure 5 it can be seen that $\varepsilon = 0.9°$. This is a factor 0.405 of the calculated angle of 2.22°. Because the bending stress is proportional to the bending angle it will also be a factor 0.405 of the calculated stress of 78 N/mm^2 resulting in a stress of about 32 N/mm^2. This is a low stress but up to now the gyroscopic moment, which can be rather large, has not yet been taken into account.

The gyroscopic moment is caused by simultaneously rotation of rotor and head. One can distinguish the gyroscopic moment in a blade and the gyroscopic moment which is exerted by the whole rotor on the rotor shaft and so on the head. On a rotating mass element dm at a radius r, a gyroscopic force dF is working which is maximum if the blade is vertical and zero if the blade is horizontal and which varies with $\sin\alpha$ with respect to a rotating axis frame. α is the angle with the blade axis and the horizon. So it is valid that $dF = dF_{max} * \sin\alpha$. The direction of dF depends on the direction of rotation of both axis and dF is working forwards or backwards. The moment dF * r which is exerted by this force with respect to the blade is therefore varying sinusoidal too.

However, if the moment is determined with respect to a fixed axis frame it can be proven that it varies with $dF_{max} * r \sin^2\alpha$ with respect to the horizontal x-axis and with $dF_{max} * \sin\alpha * \cos\alpha$ with respect to the vertical y-axis. For two and more bladed rotors it can be proven that the resulting moment of all mass elements around the y-axis is zero.

For a single blade and for two bladed rotors, the resulting moment of all mass elements with respect to the x-axis is varying with $\sin^2\alpha$, so just the same as for a single mass element. However, for three and more bladed rotors, the resulting moment of all mass elements with respect to the x-axis is constant. The resulting moment with respect to the x-axis for a three (or more) bladed rotor is given by the formula:

$$M_{gyr\ x\text{-}as} = I_{rot} * \Omega_{rot} * \Omega_{head} \quad \text{(Nm)} \qquad (21)$$

In this formula I_{rot} is the mass moment of inertia of the whole rotor, Ω_{rot} is the angular velocity of the rotor and Ω_{head} is the angular velocity of the head. The resulting moment is constant for a four bladed rotor because adding four $\sin^2\alpha$ functions which make an angle of 90° which each other, appear to result in a constant value. The four functions are given in figure 7. It can be proven that the sum moment for all four blades is two times the peak value of one blade.

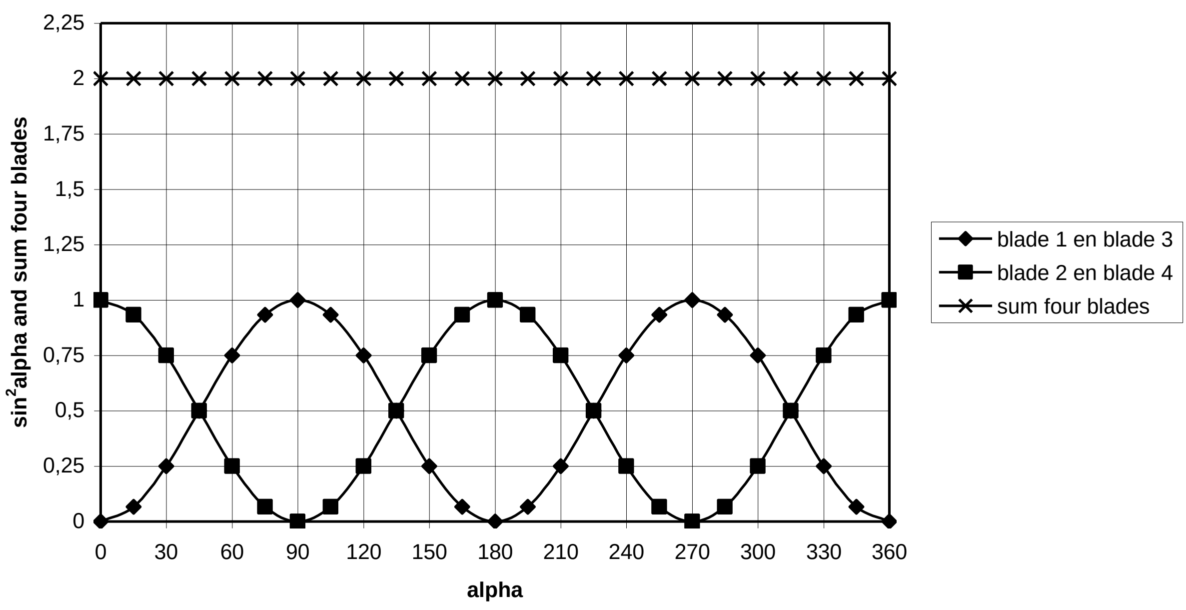

fig. 7 Variation of $\sin^2\alpha$ and the sum of four moments

We are not interested in the variation of the gyroscopic moment in a blade with respect to the x-axis but in the variation of the moment with respect to an axis frame with rotates with the blade. If the blade is vertical both axis coincide and the moment is the same for both axis frames. The peak moment in one blade is therefore half the value of the moment given by formula 21, so:

$M_{gyr\ bl\ max} = 0.5 * I_{rot} * \Omega_{rot} * \Omega_{head}$ (Nm) (22)

For a four bladed rotor, the moment of inertia of the whole rotor I_{rot} is four times the moment of inertia of one blade I_{bl}. Therefore it is valid that:

$M_{gyr\ bl\ max} = 2\ I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (23)

Up to now it is assumed that the blades have an infinitive stiffness. However, in reality the blades are flexible and will bend by the fluctuations of the gyroscopic moment. Therefore the blade will not follow the curve for which formula 21 and 23 are valid. I am not able to describe this effect physically but the practical result of it is that the strong fluctuation on the $\sin^2\alpha$ function is rather flattened. However, the average moment is assumed to stay the same as given by formula 21. I estimate that the flattened peak value of $M_{gyr\ bl\ max}$ is given by:

$M_{gyr\ bl\ max\ flattened} = 1.2\ I_{bl} * \Omega_{rot} * \Omega_{head}$ (Nm) (24)

For the chosen blade geometry it is calculated that I_{bl} = 21 kgm^2. The maximum loaded rotational speed of the rotor can be read in figure 5 for λ = 3 and it is found that n_{max} = 111 rpm. This gives $\Omega_{rot\ max}$ = 11.6 rad/s (because $\Omega = \pi * n / 30$).

It is not easy to determine the maximum yawing speed. The VIRYA-2.B2 will be with a safety system which has a large moment of inertia of the head around the tower axis and therefore sudden variations in wind speed and wind direction will be followed only slowly. It is assumed that the maximum angular velocity of the head can be 0.2 rad/s at very high wind speeds.

Substitution of I_{bl} = 21 kgm^2, $\Omega_{rot\ max}$ = 11.6 rad/s and $\Omega_{head\ max}$ = 0.2 rad/s in formula 24 gives: $M_{gyr\ bl\ max}$ = 58.5 Nm = 58500 Nmm. Substitution of M = 58500 Nmm, b = 80 mm and h = 8 mm in formula 14 gives $\sigma_{b\ max}$ = 69 N/mm^2. This value has to be added to the bending stress of 32 N/mm^2 which was the result of the thrust because there is always a position were both moments are strengthening each other. This gives $\sigma_{b\ tot\ max}$ = 101 N/mm^2. The minimum stress is 32 - 69 = - 37 N/mm^2. So the stress is becoming negative and therefore it is necessary to take the load as a fatigue load.

For the strip material hot rolled strip Fe360 is chosen. For hot rolled strip the allowable stress for a load in between zero and maximum is about 190 N/mm^2 and for a fatigue load it is about 140 N/mm^2. However, these are tensile stresses and the allowable bending stress is a lot higher. It is assumed that the allowable bending stress is about 240 N/mm^2 for a load in between zero and maximum and about 175 N/mm^2 for a fatigue load. The calculated stress is much lower than the allowable fatigue stress so the strip is strong enough. In reality the blade is not extremely stiff and will also bend somewhat. This reduces the bending of the strip and therefore the stress is the strip will be somewhat lower.

6.2 Bending stress in the strip for a locked rotor

The rotational speed for a rotor which is stopped by locking of the rotor is zero. Therefore there is no compensating effect of the centrifugal moment on the moment of the thrust. However, there is also no gyroscopic moment. The safety system is also working if the rotor is slowed down but a much larger wind speed will be required to generate the same thrust as for a rotating rotor.

In chapter 6.1 it has been calculated that the maximum thrust on one blade for a rotating rotor is 55 N for $V = V_{rated} = 8$ m/s and $\delta = 30°$. The head turns out of the wind such at higher wind speeds, that the thrust stays almost constant above V_{rated}. A slowed down rotor will therefore also turn out of the wind by 30° if the force on one blade is 55 N. Also for a slowed down rotor the force is staying constant for higher yaw angles. However, for a slowed down rotor, the resulting force of the blade load is exerting in the middle of the blade at $r_4 = 1.175$ m because the relative wind speed is constant along the whole blade. The bending moment around the edge of the hub is therefore somewhat smaller. Formula 11 changes into:

$$M_{b\ t} = F_{t\ \delta\ bl} * (r_4 - r_2) \quad \text{(Nm)} \qquad (25)$$

Substitution of $F_{t\ \delta\ bl} = 55$ N, $r_4 = 1.175$ m and $r_2 = 0.04$ m in formula 25 gives $M_{b\ t} = 62.4$ Nm = 62400 Nmm. Substitution of M = 62400 Nmm, b = 80 mm and h = 8 mm in formula 14 gives $\sigma_b = 73$ N/mm^2. This is lower than the calculated stress for a rotating rotor. The load is not fluctuating and therefore it is surly not necessary to use the allowable fatigue stress. The allowable bending stress is about 240 N/mm^2 for no fatigue load and for hot rolled strip, so the strip is strong enough.

7 Description of the transmission and the pump

7.1 General

The main disadvantage of a single acting piston pump which is used in traditional water pumping windmills is that the torque and the flow vary sinusoidal during the upwards stroke and that they are zero during the downwards stroke. The peak torque is a factor π times the average torque. This requires a windmill rotor with a very high starting torque coefficient and only very high solidity rotors with a design tip speed ratio of about 1 have a starting torque coefficient which is large enough to realise an acceptable low starting wind speed.

A rotating positive displacement pump has an almost constant flow and torque and it is therefore expected that a rotor with a design tip speed ratio $\lambda_d = 2$ has a starting torque coefficient which is high enough to get an acceptable low starting wind speed. But the real pump characteristics can only be known if a certain pump is chosen. There are many different rotating positive displacement pumps available on the market and choosing a certain pump is without the scope of this report. As the pump is coupled directly to the vertical shaft, it has a rather low rotational speed and therefore it must be rather big to give an acceptable flow. One has to chose a pump principle which has only limited internal friction. In my report KD 651 (ref. 6), I have described a vane pump which has only two vanes and a larger version of this pump might be suitable. As the suction and the pressure height are limited for irrigation, it might also be possible to use a pump with a flexible rotor like an impeller pump.

A problem with a rotating positive displacement pump is that it also works as a hydraulic motor. So this means that the pump wants to turn the windmill rotor backwards if the wind speed becomes zero. This would result in the loss of all water in the suction and the pressure pipes and it will take a certain time to fill these pipes if the wind is blowing again.

This can be prevented in two ways. One way is to mount a one-direction clutch in the vertical shaft. Another way is to mount a foot valve in the suction pipe. I prefer this second option. The foot valve is mounted at the entrance of the suction pipe and is combined with a filter which prevents that lumber is sucked into the pump.

Another problem of using a pump which is driven by a vertical shaft in the tower is that the vertical shaft gives a reaction torque on the head and this reaction torque influences the safety system. The reaction torque is smaller as the gear ratio of the transmission is chosen larger. Several transmissions haven been researched and a Polycord transmission with a round string and an accelerating gear ratio i = 2.5 seems the most promising option. An advantage of this transmission is that it is also able the bridge the eccentricity in between the rotor shaft and the vertical shaft. This transmission is described recently in report KD 693 (ref. 7) for the VIRYA-3.5 wind driving a centrifugal pump. It is expected that the same transmission with a 15 mm round string can be used for the VIRYA-3.6L2. The peak torque of the rotor of the VIRYA-3.6L2 is larger than that of the VIRYA-3.5 because the design tip speed ratio is lower. However, the real loaded torque depends on the design wind speed and on the pump characteristics. The pump torque for a positive displacement pump is almost constant but for a centrifugal pump it is increasing at increasing rotational speed.

For the centrifugal pump of the VIRYA-3.5, a design wind speed of 4 m/s was chosen resulting in a design torque of 19.7 Nm. For the positive displacement pump of the VIRYA-3.6L2, a design wind speed of 3 m/s is chosen resulting in a design torque of 18.8 Nm. So the Polycord transmission of the VIRYA-3.5 is certainly strong enough for the VIRYA-3.6L2.

7.2 Description of the transmission

The Polycord transmission is described in chapter 6 and 7 of KD 693 (ref. 7) and this description will not be repeated for the VIRYA-3.6L2 in this report KD 651. The vertical shaft calculations of the VIRYA-3.5 are given in chapter 5 of KD 653. However, the design tip speed ratio of the VIRYA-3.6L2 is much lower than for the VIRYA-3.5 and this results in a much lower maximum rotational speed of the vertical shaft which may allow a larger distance in between the bearings. So the stability calculations for the vertical shaft have to be made again for the VIRYA-3.6L2.

In figure 5 it can be seen that the maximum unloaded rotational speed of the rotor is about 118 rpm for V = 8 m/s or higher. The rotational speed of the vertical shaft n_v is given by:

$$n_v = n * i \qquad \text{(rpm)} \qquad (26)$$

In practice the gear ratio may vary a little depending on the load because there will be some slip of the string around the wheels but this effect is neglected. Substitution of $n = n_{max}$ = 118 rpm and i = 2.5 in formula 21 gives $n_{v\ max}$ = 295 rpm. This is rather high and the vertical shaft therefore has to be checked for “instability” (the Dutch word is zwiep, the correct English word could not be found). Instability for a rotating shaft can be compared to buckling for a compressed rod. In practice the maximum pump speed will normally be lower than the calculated value because of the pump load. In figure 5 it can be seen that the maximum loaded rotational speed is about 111 rpm at V = 8 m/s (for V_d = 3 m/s). However, it is possible that the pump load disappears, for instance if there is no water. Therefore the worst case of an unloaded rotor has to be taken for instability calculations.

Instability of a rotating shaft can be understood best for a long horizontal shaft turning in two bearings. The shaft will bend downwards in the middle because of the weight of the shaft itself. Therefore a certain distance will be created between the centre of gravity and the real rotational axis. Because of this distance a centrifugal force exists which has a tendency to increase the distance.

At a certain critical rotational speed n_c, the stiffness of the shaft is no longer large enough to prevent rapid growth of the distance and this results in a sudden break out of the shaft which is very dangerous. If the shaft is not horizontal but vertical, the problem is not initiated by the weight of the shaft but by sudden vibrations or if the shaft is bent somewhat. The shaft is normally calculated for horizontal position even if it is vertical in practice. However, I think that there is more reserve if the shaft is vertical. The instability is influenced by the stiffness of the bearings. It is assumed that the bearings have no stiffness and the given formulas are valid for this situation. The critical rotational speed n_c is given by:

$n_c = 30/\pi * \sqrt{(g / f)}$ (rpm) (27)

In this formula is g is the acceleration of gravity (m/s^2) and f is the deflection (m) because of the weight of the shaft. For a shaft with length l (m) and an equal spread load q (caused by the weight), the deflection f is given by:

$f = 5 q * l^4 / 384 EI$ (m) (28)

For a shaft with diameter d (m) and a density ρ_s (kg/m^3), the weight per length q is given by:

$q = \pi/4 d^2 * \rho_s * g$ (N/m) (29)

The moment of inertia I (m^4) of a round shaft with diameter d (m) is given by:

$I = \pi/64 d^4$ (m^4) (30)

(28) + (29) + (30) gives:

$f = 5/24 \rho_s * g * l^4 / (E * d^2)$ (m) (31)

(27) + (31) gives:

$n_c = 20.92 * d/l^2 * \sqrt{(E / \rho_s)}$ (rpm) (32)

Assume we take a steel shaft. This gives $\rho_s = 7.8 * 10^3$ kg/m^3 and $E = 2.1 * 10^{11}$ N/m^2. The 6 metre high tapered tower has four legs made out of pipe which are connected to each other by horizontals. Assume that the horizontals are welded every 0.5 m so it is easy for connection of the bearings if l is a whole number times this value. The value for d must be taken as small as possible because the vertical shaft must pass through a hole in the head pin. Assume d = 12 mm = 0.012 m and l = 2 m. Substitution of these values in formula 32 gives n_c = 326 rpm. This is larger than the calculated maximum rotational speed of 295 rpm so the chosen distance of 2 m in between the bearings is acceptable for a 12 mm shaft.

The shaft will have a length of 6 m, so the same as for the tower height. The Polycord wheel at the top of the shaft and the centrifugal pump at the bottom will be connected to the shaft by clamping couplings so the shaft isn't weakened by thread.

The torsion stress for a 12 mm shaft will be calculated for the maximum moment which is allowed for the Polycord transmission which is 48.6 Nm (see KD 653). Substitution of $Q = Q_{max}$ = 48.6 Nm, η_{tr} = 0.95 and i = 2.5 in formula 4 of KD 653 gives Q_{react} = 18.47 Nm = 18470 Nmm.

The torsion stress τ is calculated by formula:

$\tau = Q / (\pi/16 * d^3)$ (N/mm^2) (33)

Substitution of Q = Q_{max} = 18470 N/mm^2 and d = 12 mm in formula 28 gives τ = 54 N/mm^2. This is a rather low stress which is certainly acceptable, even if stainless steel is used as material for the vertical shaft.

The vane dimensions for the VIRYA-3.5 are given in appendix 1 of KD 693. As the torque level in the rotor shaft is about the same for the design wind speed, the torque level in the vertical shaft will also be about the same. So the influence of the reaction torque will be small with respect to the rotor torque. The vane geometry of the VIRYA-3.5 can therefore also be used for the VIRYA-3.6L2.

The positive displacement pump will be driven directly by the vertical shaft. So no rectangular gear box is needed like it is the case for a standard centrifugal pump. It is expected that the positive displacement pump can be connected to the foundation of the tower or to the lowest horizontals of the tower.

7.3 Transformation of the Q-n curves to the Q_v-n_v curves

The Q-n curves as given in figure 5 are valid for the rotor shaft. Similar curves can also be derived for the vertical shaft if the gear ratio i (-) and the transmission efficiency η_{tr} (-) is taken into account. The rotational speed of the vertical shaft is called n_v (rpm). The torque in the vertical shaft is called Q_v (Nm). So the Q-n curves have to be transferred to the Q_v-n_v curves. The Q_v-n_v curves are needed for the determination of the pump geometry. n_v is given by formula 26.

Q_v = given by:

$$Q_v = Q * \eta_{tr} / i \qquad \text{(Nm)} \qquad (34)$$

The transmission efficiency is caused by friction of the string and by friction of the bearings of the vertical shaft. Elasticity of the spring may also cause a small variation of the gear ratio but for the calculations it is assumed that i is constant and that i = 2.5. It is assumed that η_{tr} = 0.95. Using formula 26 and 34, table 2 with the values for n and Q is now transformed into table 3 with the values for n_v and Q_v.

		V = 2 m/s δ = 0°		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 8°		V = 7 m/s δ = 19°		V = 8 m/s δ = 30°	
λ (-)	C_q (-)	n_v (rpm)	Q_v (Nm)	n_v (rpm)	Q_v (Nm)	n_v (rpm)	Q_v (Nm)	n_v (rpm)	Q_v (Nm)	nv_δ (rpm)	Qv_δ (Nm)	nv_δ (rpm)	Qv_δ (Nm)	nv_δ (rpm)	Qv_δ (Nm)
0	0.13	0	2.17	0	4.89	0	8.69	0	13.58	0	19.17	0	23.79	0	26.07
0.4	0.145	10.63	2.42	15.93	5.45	21.23	9.69	26.53	15.14	31.53	21.38	35.13	26.54	36.75	29.07
0.8	0.17	21.23	2.84	31.83	6.39	42.45	11.36	53.05	17.75	63.05	25.07	70.23	31.11	73.50	34.09
1.2	0.195	31.83	3.26	47.75	7.33	63.65	13.03	79.58	20.36	94.58	28.76	105.35	35.69	110.28	39.10
1.6	0.2125	42.45	3.55	63.65	7.99	84.88	14.20	106.10	22.19	126.08	31.34	140.45	38.89	147.03	42.61
2	0.19	53.05	3.17	79.58	7.14	106.10	12.70	132.63	19.84	157.60	28.02	175.58	34.77	183.78	38.10
2.4	0.1417	63.65	2.37	95.50	5.33	127.33	9.47	159.15	14.80	189.13	20.90	210.68	25.93	220.53	28.41
2.8	0.0786	74.28	1.31	111.40	2.96	148.55	5.25	185.68	8.21	220.65	11.59	245.80	14.38	257.30	15.76
3.2	0	84.88	0	127.33	0	169.78	0	212.20	0	252.18	0	280.90	0	294.05	0

table 3 Calculated values of n_v and Q_v as a function of λ and V for the VIRYA-3.6L2 rotor

The calculated values for n_v and Q_v are plotted in figure 8.

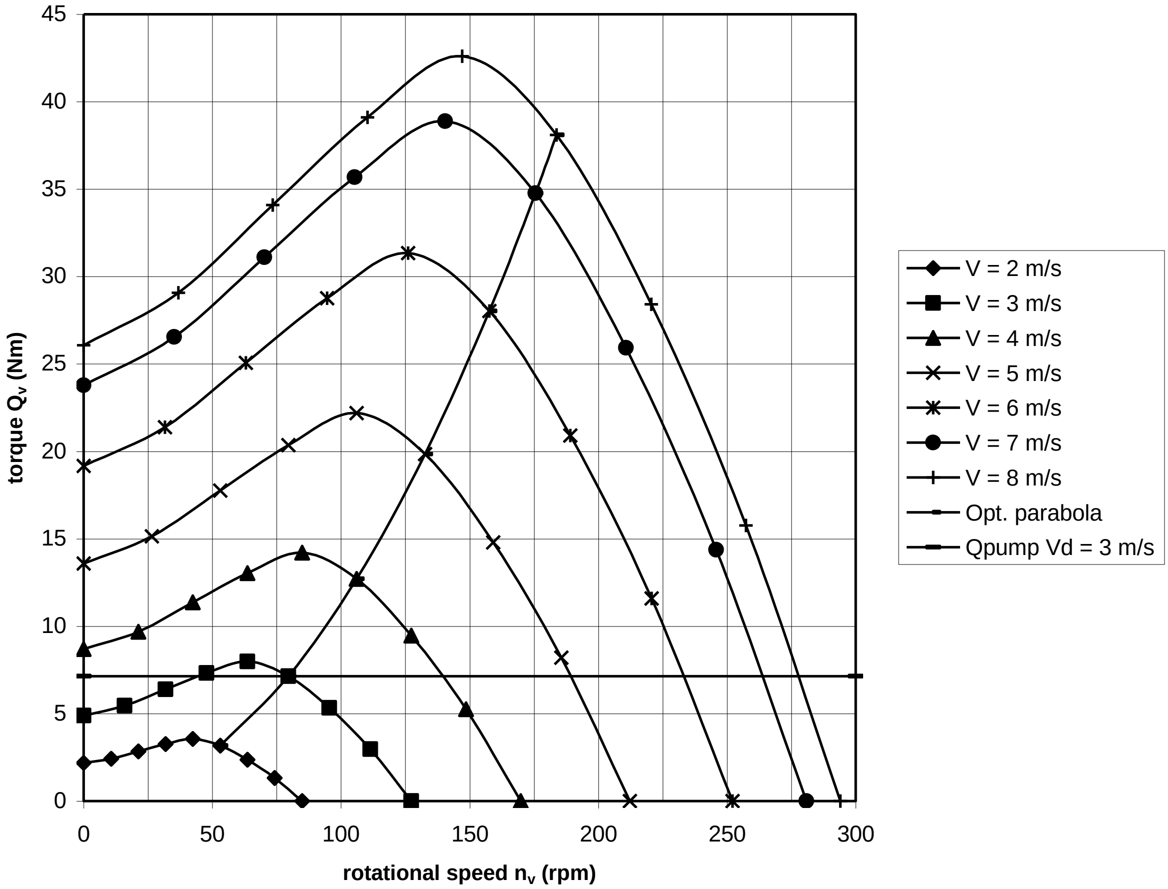

fig. 8 Q_v-n_v curves of the VIRYA-3.6L2 rotor for blades with increased twist

The optimum parabola and the Q-n curve of the pump for a design wind speed V_d = 3 m/s are also given in figure 8. In figure 8 and table 8 it can be seen that for V_d = 3 m/s, the design torque Q_{dv} = 7.14 Nm for a design rotational speed n_{dv} = 79.58 rpm. For the determination of the stroke volume of the pump, the design power at the vertical shaft P_{dv} is needed. The relation in between the power P (W), the torque Q (Nm) and the rotational speed n (rpm) is given by:

$$P = \pi * Q * n / 30 \quad \text{(W)} \qquad (35)$$

Substitution of Q = Q_{dv} = 7.14 Nm and n = n_{dv} = 79.58 rpm in formula 35 gives that P_{dv} = 59.5 W.

7.4 Determination of the stroke volume of the pump and the flow

The stroke volume ∇ (m^3) of a rotating positive displacement pump is the volume of water which is pumped for one revolution of the pump shaft. The stroke volume may change for different pressures because water is leaked away through internal gaps in between the pressure side and the suction side of the pump or because of elastic deformation of certain pump components. The pressure drop over a pump used for irrigation is only little if the height H is limited and therefore it is assumed that the stroke volume is constant for every height. So it is also assumed that the volumetric efficiency is 1. The energetic pump efficiency of the pump η_p will certainly be lower than 1 because of internal friction. At this moment it is assumed that η_p = 0.9 for all pump conditions.

The hydraulic power P_{hyd} (W) supplied by the pump is given by:

$$P_{hyd} = \rho_w * g * H * q \quad (W) \tag{36}$$

In this formula ρ_w is the density of water (1000 kg/m^3), g is the acceleration of gravity (9.81 m/s^2), H is the total height (m) so the height in between the water level of the river and the opening of the pressure pipe at the reservoir. q is the flow (m^3/s). The mechanical power supplied to the pump shaft is P_{mech}. P_{mech} is given by:

$$P_{mech} = P_{hyd} / \eta_p \quad (W) \tag{37}$$

For the design wind speed V_d = 3 m/s, P_{mech} is equal to P_{dv} and the flow q is equal to the design flow q_d. So for the design wind speed it is valid that:

$$P_{dv} = \rho_w * g * H * q_d / \eta_p \quad (W) \tag{38}$$

Formula 38 can be written as:

$$q_d = P_{dv} * \eta_p / (\rho_w * g * H) \quad (m^3/s) \tag{39}$$

So q_d depends on H and the larger H is chosen, the smaller q_d will be.

The relation in between the design flow q_d (m^3/s), the stroke volume ∇ (m^3) and the design rotational speed of the vertical shaft n_{dv} (rpm) is given by:

$$q_d = \nabla * n_{dv} / 60 \quad (m^3/s) \tag{40}$$

(39) + (40) gives that:

$$\nabla = 60 * P_{dv} * \eta_p / (n_{dv} * \rho_w * g * H) \quad (m^3) \tag{41}$$

So also ∇ depends on H and the larger H is chosen, the smaller ∇ will be. This means that one needs different pumps for different heights H if one wants to have a design wind speed V_d = 3 m/s. It also means that one will get different design wind speeds if one pump is used for different heights H. To make a choice for a certain pump, a certain design wind speed V_d and a certain height H have to be chosen. Assume V_d = 3 m/s and H = 6 m. Substitution of P_{dv} = 59.5 W, η_p = 0.9, n_{dv} = 79.58 rpm, ρ_w = 1000 kg/m^3, g = 9.81 m/s^2 and H = 6 m in formula 41 gives that ∇ = 0.000686 m^3 = 0.686 litre.

So this is a rather big pump and I don't know if a positive displacement pump with about this stroke volume can be found on the market. If such a big pump can't be found, an alternative is to use a smaller pump but to drive it by a second accelerating gearing. This second gearing can also be used to match a certain pump to a certain height H by variation of the gear ratio.

Substitution of ∇ = 0.000686 m^3 and n_{dv} = 79.58 rpm in formula 40 gives that q_d = 0.00091 m^3/s = 0.91 litre/s = 3.276 m^3/hour. For a design wind speed of only 3 m/s this is an acceptable amount of water for irrigation.

In reality the wind speed will not have a constant value of 3 m/s but it will vary and the real flow depends on the real rotational speed of the pump. The real rotational speed n can be read in figure 8 for each wind speed if it is assumed that the pump torque is constant and equal to the design torque Q_d = 7.14 Nm. The read values are given in table 4.

For the real flow q (m^3/s) at a certain wind speed V it is valid that:

$$q = q_d * n / n_d \qquad (m^3/s) \qquad (42)$$

Using formula 42, q has been calculated in m^3/s and in $m^3/hour$ for every value of n and the result is also given in table 4. The variation of q as a function of V is given in figure 9.

V (m/s)	n (rpm)	q (m^3/s)	q ($m^3/hour$)
3	79.58	0.00091	3.276
4	137	0.00157	5.640
5	190	0.00217	7.822
6	232	0.00265	9.551
7	262	0.00300	10.786
8	277	0.00300	11.403

table 4 Variation of n and q as a function of V

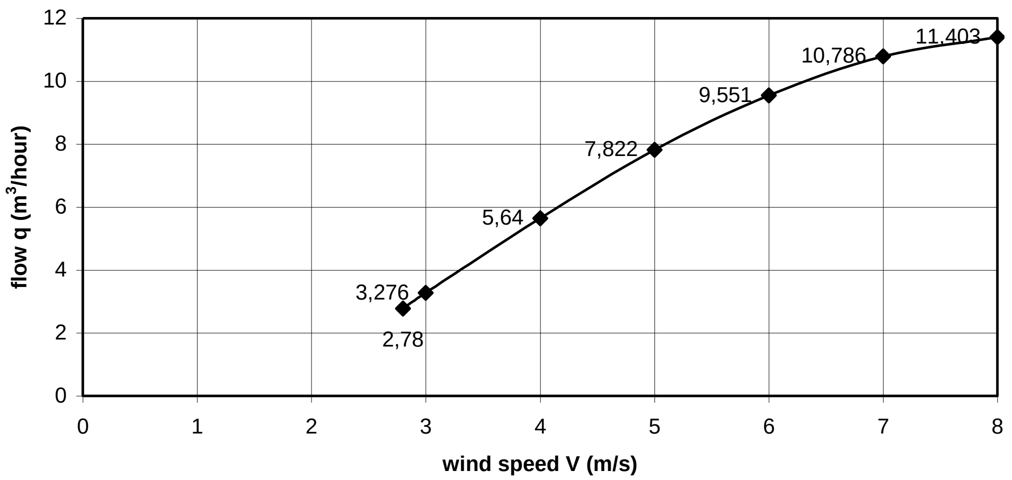

fig. 9 Variation of q ($m^3/hour$) as a function of V for V_d = 3 m/s and H = 6 m.

So the flow at a wind speed of 8 m/s is a factor 11.403 / 3.276 = 3.48 larger than at the design wind speed V_d = 3 m/s. If the rotor is running and if the wind speed is decreasing, the flow stops for the wind speed for which the peak of the Q_v-n_v curve of the rotor touches the horizontal pump curve for Q = 7.14 Nm. This is the case for a wind speed of about 2.8 m/s. The rotor starts only at a wind speed of 3.6 m/s, so there is hysteresis in the q-V curve in for wind speeds in between 2.8 m/s and 3.6 m/s.

Even if the wind speed would be about constant and only 3 m/s, the flow is 3.276 $m^3/hour$ = 83.5 m^3/day. This is an acceptable amount of water if the windmill is used for irrigation. For higher wind speeds, the flow is even considerably higher. So the VIRYA-3.6L2 in combination with a positive displacement pump seems a realistic concept.

If the total height is 6 m, one still has a choice to take a certain suction height and a certain pressure height. The absolute maximum suction height is determined by the under pressure for which cavitation starts. This will be about 9 m if the flow is almost constant. But the maximum suction height for a pump with a flexible rotor is probably much smaller as the rotor must be able to maintain its shape if there is under pressure at the suction side. I think that the maximum suction height for a flexible rotor is about 3 m.

The maximum pressure height of a pump with a flexible rotor depends on the strength of the rotor and I think that the maximum pressure height is about 10 m for a well designed pump. So the maximum total height for such a pump is about 13 m. If the height H is substantially higher or lower than 6 m, a pump with a smaller or larger stroke volume has to be used and the flow q increases proportional to the stroke volume. To really develop the VIRYA-3.6L2, it is required to select a rotating positive displacement pump and to make at least a composite drawing of the windmill. Detailed drawings of all single parts are needed for manufacture but I won't make them.

7.5 Ideas about the pump

The pump can be a standard pump which is bought on the market or it can be manufactured. I did some research on the Internet to standard positive displacement pumps but most of them have a much too small stroke volume and must normally run at a too high rotational speed to give an acceptable flow. A disadvantage of pumps which have a flexible element like flexible impeller pumps or hose pumps, is that they have a lot of mechanical friction resulting in a low efficiency. Also for vane pumps there is a lot of friction if the vanes are pressed outwards by springs. So it is investigated if a low friction vane pump can be manufactured.

Recently, I have designed a vane pump which is directly driven by a 24 V permanent magnet DC motor. This vane pump is described in report KD 661 (ref. 6). This vane pump is very simple as it has only four vanes. Two opposite vanes are made out of one piece and no springs are used. Therefore the internal friction is very low. However, for minimal clearance in between the vanes and the pump housing, the inside bore of the pump housing can't be a circle. In KD 661 is was chosen to curve the inside of the pump according to a cosine function but manufacture of this bore requires a programmable milling machine.

The vane pump described in KD 661 runs at a nominal rotational speed of about 1500 rpm and at a maximum rotational speed of about 1750 rpm (for a voltage of 28 V). It is meant for a static height of maximal 10 m. The outside diameter of the housing is 140 mm. The rotor diameter is 75 mm. The height of the rotor is 40 mm. The vane thickness is 12 mm and the vane stroke is 14 mm. The stroke volume is 0.0001076 m^3 so it is about a factor 6.38 too small for the VIRYA-3.6L2. This means that all dimensions of the vane pump of KD 661 would have to be scaled up with about a factor $6.38^{1/3} = 1.85$ to get the correct stroke volume if it is directly driven by the vertical shaft. This results in a rotor diameter of about 139 mm a vane height of about 74 mm and an outside diameter of about 259 mm. I expect that manufacture of a pump with these large dimensions will be very difficult in developing countries and that the required large diameter stainless steel bar is also not available..

So it is investigated if the pump of KD 661 can be used with an accelerating gearing in between the vertical shaft and the pump shaft. To use a pump with a stroke volume of 0.0001076 m^3, an accelerating gearing with a gear ratio of about 6.4 would be necessary to get the wanted flow. This is possible in one step with a poly-V-belt transmission. This transmission makes use of a belt which has many small V grooves. It will have a big wheel on the vertical shaft without V-grooves and a small wheel on the pump shaft with the same number of grooves as the number of grooves in the belt. This transmission is generally used in washing machines. This transmission seems possible but further specification of this transmission is out the scope of this report.

Assume an accelerating transmission with a gear ratio i = 6.5 is chosen. At V_d = 3 m/s, the vertical shaft rotates at a rotational speed of about 80 rpm (see figure 8). So the design rotational speed of the pump is 6.5 * 80 = 520 rpm. This is much lower than the nominal rotational speed of the pump if it is driven by a PM-motor and the volumetric efficiency will therefore be lower. I expect that it is about 0.85 if it is 0.95 at n = 1500 rpm but this is acceptable. The maximum loaded rotational speed of the vertical shaft is about 280 rpm so a factor 3.5 higher than at V_d. This means that the maximum rotational speed of the pump is also a factor 3.5 larger and it becomes 3.5 * 520 = 1820 rpm. This seems acceptable too.

Another option for the pump is to use the diaphragm pump with three in line diaphragms which is described in report KD 542 (ref. 8). The gear box and the electric motor have to be cancelled. This pump has a torque and flow which is fluctuating only a little. The stroke volume of this pump is about 0.000119 m^3 so about a factor 0.000686 / 0.000119 = 5.76 too small. This means that this pump can be used if a second accelerating gearing with a gear ratio of about 5.5 is used in between the vertical shaft and the pump shaft. In this case the bearing of the motor can't be used and the pump must have bearings at both sides. The pump shaft is vertical but this seems no problem if the pump is supplied with a foot at the bottom and with right angle hose pillars making that the suction and the pressure lines are horizontal.

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