Determination of the C_m-α curves for different positions of the axis of rotation for a 10 %, 5 % and 7.14 % cambered airfoil with an aspect ratio of 5.

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1 Introduction

In report KD 35 (ref. 1) a method is given for the design of a windmill rotor. The blade of the rotor must be provided with an aerodynamic airfoil. Kragten Design has developed some windmills for which the 7.14 % cambered airfoil is used. In report KD 398 (ref. 2), the aerodynamic characteristics are given for the 7.14 %, 10 % and 12.5 % cambered airfoil with an infinitive aspect ratio. An infinitive aspect ratio means that the blade has the same length as the width of the wind tunnel and so no wake is created at the blade ends. These characteristics are derived from measurements given in report R443D (ref. 3). The measurements for the 7.14 %, 10 % and 12.5 % airfoils have originally been performed by Buering of Imperial Collage but access to the original source is no longer possible. The 10 % cambered airfoil with an infinitive aspect ratio has also been measured by other researchers like Volkers and Bruining (see R443D). For all these measurements, the lift coefficient C_l and the drag coefficient C_d is given as a function of the angle of attack α. However, the moment coefficient C_m is not measured.

The moment coefficient C_m is only measured by Flachsbart but not for a blade with an infinitive aspect ratio but for a blade with an aspect ratio of 5 and for 5 %, 10 % and 15 % camber. An aspect ratio of 5 means that the blade length is five times the chord and that the width of the wind tunnel must have been larger than the blade length. So a wake is created at both sides of the blade and this results in reduction of the lift coefficient and increase of the drag coefficient.

At this moment I am investigating if it is possible to design a windmill rotor with a safety system based on pitch control activated by the aerodynamic moment acting on the blade. The blades will have a 7.14 % cambered airfoil. To investigate if this is possible, the C_m-α curve around a certain axis of rotation is needed. However, Flachsbart hasn't measured a 7.14 % cambered airfoil and therefore the measurements for 10 % and 5 % camber are investigated. The values for 7.14 % camber will lie somewhere in the middle.

2 Determination of the C_m-α curves for 10 % camber and different turning points

The measurements of Flachsbart for 10 % camber are given on page 3-5 of report R443D (ref. 3). Only a picture with the C_l-C_d and the C_m-C_d curves is given but fortunately the measuring points are also given in a table. It is not defined around which point the moment coefficient is taken. This can be the quart chord point or the airfoil nose. For α = 90° it is found that the moment coefficient C_m is just halve the drag coefficient C_d and this means that the moment coefficient is take around the airfoil nose. As C_d and C_m are both positive for α = 90°, it means that the left hand direction of the aerodynamic moment is taken positive. For most aerodynamic airfoils the right hand direction is taken positive but I will follow the definition of Flachsbart, so the left hand direction is taken positive. The table out of R443D has been copied as table 1. The measurements are performed for a Reynolds value $Re = 4 * 10^5$. The variation of C_m as a function of α is given in figure 2. It can be seen that C_m is increasing strongly for -20° < α < 12.5°. This is not nice for a pitch control system because the system will react very strongly for small variations of α. It would be better if C_m would be about constant because in this case the aerodynamic moment is only depending on the rotational speed of the rotor. It might be possible to realise this by taking another position of the axis of rotation. For normal airfoils, the moment coefficient is about constant for small angles of α if the moment is taken around the quart chord point. It will be investigated if an axis can be found for the 10 % cambered airfoil for which this is also the case.

α (°)	C_l (-)	C_d (-)	C_m (-)	$C_{m0.09}$ (-)	$C_{m0.17}$ (-)	$C_{m0.25}$ (-)	$C_{m0.33}$ (-)	$C_{m0.41}$ (-)	$C_{m0.5}$ (-)
-19.9	-0.340	0.237	-0.087	-0.051	-0.019	0.013	0.045	0.077	0.113
-10.0	-0.131	0.118	0.021	0.034	0.046	0.058	0.070	0.082	0.096
-5.0	0.111	0.0670	0.127	0.118	0.109	0.101	0.092	0.084	0.075
-0.2	0.580	0.0680	0.351	0.299	0.252	0.206	0.160	0.113	0.061
4.6	1.037	0.0965	0.510	0.416	0.333	0.250	0.166	0.083	-0.011
9.5	1.313	0.147	0.558	0.439	0.334	0.228	0.123	0.017	-0.102
12.5	1.459	0.194	0.592	0.460	0.343	0.225	0.108	-0.009	-0.141
14.4	1.520	0.239	0.595	0.457	0.335	0.212	0.090	-0.033	-0.171
16.5	1.462	0.308	0.605	0.471	0.352	0.233	0.114	-0.006	-0.140
19.6	1.181	0.368	0.547	0.436	0.337	0.238	0.139	0.040	-0.071
24.6	1.200	0.478	0.569	0.453	0.350	0.246	0.143	0.040	-0.076
29.6	1.156	0.588	0.575	0.458	0.355	0.251	0.147	0.044	-0.073
34.6	1.040	0.659	0.565	0.454	0.356	0.257	0.159	0.061	-0.050
39.7	0.945	0.705	0.540	0.434	0.340	0.246	0.151	0.057	-0.049
49.7	0.812	0.853	0.537	0.431	0.337	0.243	0.149	0.055	-0.051
59.8	0.671	0.984	0.549	0.442	0.347	0.252	0.157	0.062	-0.045
69.8	0.497	1.112	0.573	0.464	0.366	0.269	0.172	0.075	-0.035
79.9	0.299	1.185	0.585	0.475	0.378	0.280	0.183	0.085	-0.025
90.0	0	1.219	0.609	0.499	0.402	0.304	0.207	0.109	-0.001

table 1 C_l, C_d, C_m, $C_{m0.09}$, $C_{m0.17}$, $C_{m0.25}$, $C_{m0.33}$, $C_{m0.41}$ and $C_{m0.5}$ as a function of α for 10 % camber. C_m is taken around the nose. The left hand direction is taken positive. Re = 4 * 10^5

The 10 % cambered airfoil and the lift L, the drag D and the moment M acting on it, is given in figure 1.

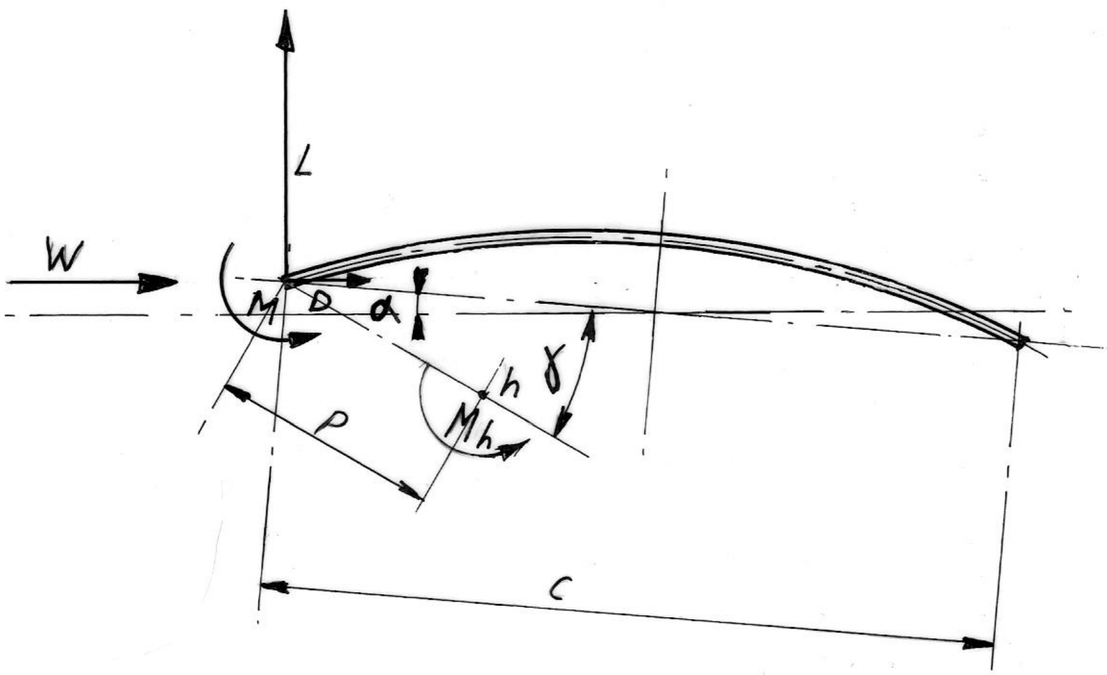

fig. 1 Lift L, drag D and moment M acting on the 10 % cambered airfoil

So the moment M and the moment coefficient C_m are taken around the airfoil nose by Flachsbart. If the moment is taken around another point h, lying at a distance p from the nose and if the line through the nose and point h makes a right hand angle γ with the zero line of the airfoil, it can be proven that C_{mh} is given by:

$C_{mh} = C_m - p/c \{C_l * \cos(\alpha + \gamma) + C_d * \sin(\alpha + \gamma)\}$ (1)

The value is C_{mh} is now determined for six values of p being: p = 0.09 c, p = 0.17 c, p = 0.25 c, p = 0.33 c, p = 0.41 c and p = 0.5 c. It is assumed that all points are lying on the zero line which means that $\gamma = 0°$.
Substitution of p = 0.09 c and $\gamma = 0°$ in formula 1 and taking $C_{mh} = C_{m0.09}$ gives:

$C_{m0.09} = C_m - 0.09 (C_l * \cos\alpha + C_d * \sin\alpha)$ (2)

Substitution of p = 0.17 c and $\gamma = 0°$ in formula 1 and taking $C_{mh} = C_{m0.17}$ gives:

$C_{m0.17} = C_m - 0.17 (C_l * \cos\alpha + C_d * \sin\alpha)$ (3)

Substitution of p = 0.25 c and $\gamma = 0°$ in formula 1 and taking $C_{mh} = C_{m0.25}$ gives:

$C_{m0.25} = C_m - 0.25 (C_l * \cos\alpha + C_d * \sin\alpha)$ (4)

Substitution of p = 0.33 c and $\gamma = 0°$ in formula 1 and taking $C_{mh} = C_{m0.33}$ gives:

$C_{m0.33} = C_m - 0.33 (C_l * \cos\alpha + C_d * \sin\alpha)$ (5)

Substitution of p = 0.41 c and $\gamma = 0°$ in formula 1 and taking $C_{mh} = C_{m0.41}$ gives:

$C_{m0.41} = C_m - 0.41 (C_l * \cos\alpha + C_d * \sin\alpha)$ (6)

Substitution of p = 0.5 c and $\gamma = 0°$ in formula 1 and taking $C_{mh} = C_{m0.5}$ gives:

$C_{m0.5} = C_m - 0.5 (C_l * \cos\alpha + C_d * \sin\alpha)$ (7)

$C_{m0.09}$, $C_{m0.17}$, $C_{m0.25}$, $C_{m0.33}$, $C_{m0.41}$ and $C_{m0.5}$, have been calculated for all values of α out of table 1 and the result is also mentioned in table 1. All curves are also given in figure 2.

Comparing the curves of figure 2 learns that, the $C_{m0.17}$-α curve and the $C_{m0.25}$-α curve are most constant for angles α in between 2° and 20°. The $C_{m0.333}$-α, the $C_{m0.41}$-α curve and the $C_{m0.5}$-α curve have a dip at $\alpha = 15°$. The $C_{m0.41}$-α curve is negative at $\alpha = 15°$. The $C_{m0.5}$-α curve is negative for $\alpha > 4°$. The C_m value of these two curves decreases strongly for $0 < \alpha < 15°$. Use of p/c = 0.41 and p/c = 0.5 is therefore not advised for a pitch control safety system for which the blade angle increases at increasing rotational speed. A certain wind gust will result in increase of α and this would result in strong decrease of the pitch moment.

The angle α depends on the blade design. For the blade with a constant chord, α is small at the blade tip and large at the blade root if the rotor runs at the design tip speed ratio. α increases suddenly at wind gusts. α is very large during starting of the rotor.

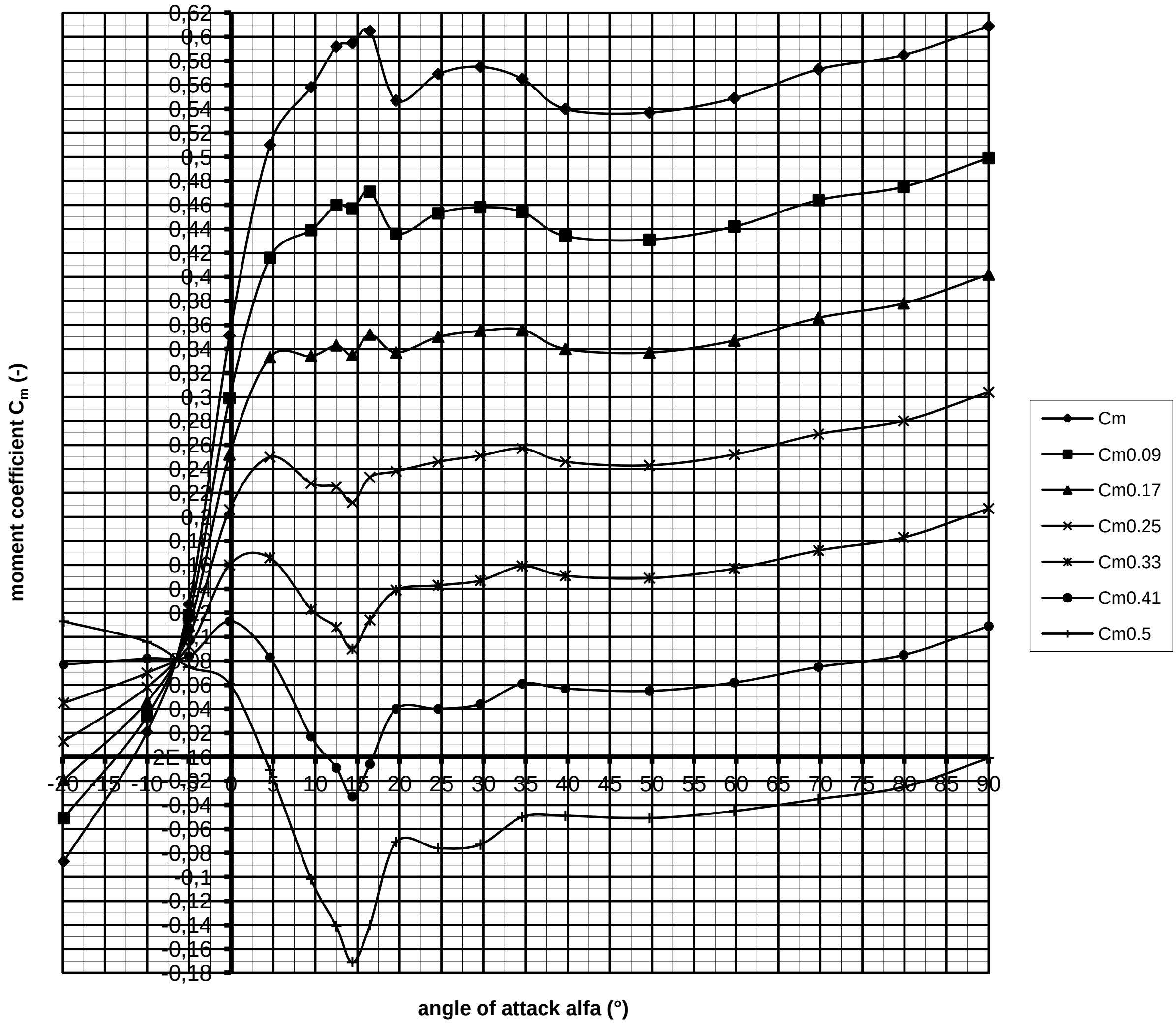

fig. 2 C_m-α curves for different positions of the axis of rotation for a 10 % cambered plate for Re = 4 * 10^5

3 Determination of the C_m-α curves for 5 % camber and different turning points

The measurements of Flachsbart for 5 % camber are also given on page 3-5 of report R443D (ref. 3). The table out of R443D has been copied as table 2. The measurements are performed for a Reynolds value Re = 4 * 10^5. The variation of C_m as a function of α is given in figure 3.

The $C_{m0.09}$, $C_{m0.17}$, $C_{m0.25}$, $C_{m0.33}$, $C_{m0.41}$ and $C_{m0.5}$ values are determined in the same way as done in chapter 2 for 10 % camber. The result of the calculations is given in table 2. The C_m-α curves for different turning points are given in figure 3.

α (°)	C_l (-)	C_d (-)	C_m (-)	$C_{m0.09}$ (-)	$C_{m0.17}$ (-)	$C_{m0.25}$ (-)	$C_{m0.33}$ (-)	$C_{m0.41}$ (-)	$C_{m0.5}$ (-)
-19.8	-0.580	0.262	-0.210	-0.153	-0.102	-0.051	-0.001	0.050	0.107
-9.8	-0.420	0.116	-0.118	-0.079	-0.044	-0.010	0.025	0.060	0.099
-4.9	-0.173	0.0600	0.009	0.025	0.039	0.053	0.068	0.082	0.098
-0.1	0.302	0.0248	0.186	0.159	0.135	0.111	0.086	0.062	0.035
4.8	0.660	0.037	0.282	0.223	0.170	0.117	0.064	0.011	-0.048
9.6	0.984	0.110	0.367	0.278	0.199	0.120	0.041	-0.038	-0.127
12.1	1.083	0.175	0.425	0.326	0.239	0.151	0.063	-0.024	-0.123
14.6	1.070	0.238	0.460	0.361	0.274	0.186	0.099	0.011	-0.088
22.1	1.050	0.355	0.463	0.363	0.275	0.186	0.098	0.009	-0.090
29.6	0.986	0.513	0.481	0.381	0.292	0.203	0.114	0.026	-0.074
39.7	0.867	0.665	0.465	0.367	0.279	0.192	0.105	0.017	-0.081
49.7	0.750	0.804	0.480	0.381	0.293	0.205	0.118	0.030	-0.069
59.8	0.614	0.964	0.515	0.412	0.321	0.229	0.138	0.047	-0.056
69.8	0.473	1.080	0.542	0.436	0.342	0.248	0.154	0.059	-0.046
79.9	0.290	1.160	0.573	0.466	0.370	0.275	0.179	0.084	-0.023
90.0	0	1.210	0.605	0.496	0.399	0.303	0.206	0.109	0.000

table 1 C_l, C_d, C_m, $C_{m0.09}$, $C_{m0.17}$, $C_{m0.25}$, $C_{m0.33}$, $C_{m0.41}$ and $C_{m0.5}$ as a function of α for 5 % camber. C_m is taken around the nose. The left hand direction is taken positive. Re = 4 * 10^5

Comparing the curves of figure 3, for angles α larger than 0°, learns that $C_{m0.25}$ is about constant for 0° < α < 10° for which $C_{m0.25}$ is about 0.12. $C_{m0.25}$ rises from 0.12 to 0.19 for 10° < α < 15°. $C_{m0.25}$ is again about constant and 0.19 for 15° < α < 25°. The $C_{m0.17}$-α curve rises for 0° < α < 15°. The $C_{m0.33}$-α, the $C_{m0.41}$-α and the $C_{m0.5}$-α curves all have a dip at α = 10°. The $C_{m0.41}$-α curve is negative at α = 10°. The $C_{m0.5}$-α curve is negative for α > 2°. Use of p/c = 0.41 and p/c = 0.5 is therefore also not advised for 5% camber for a pitch control safety system for which the blade angle increases at increasing rotational speed.

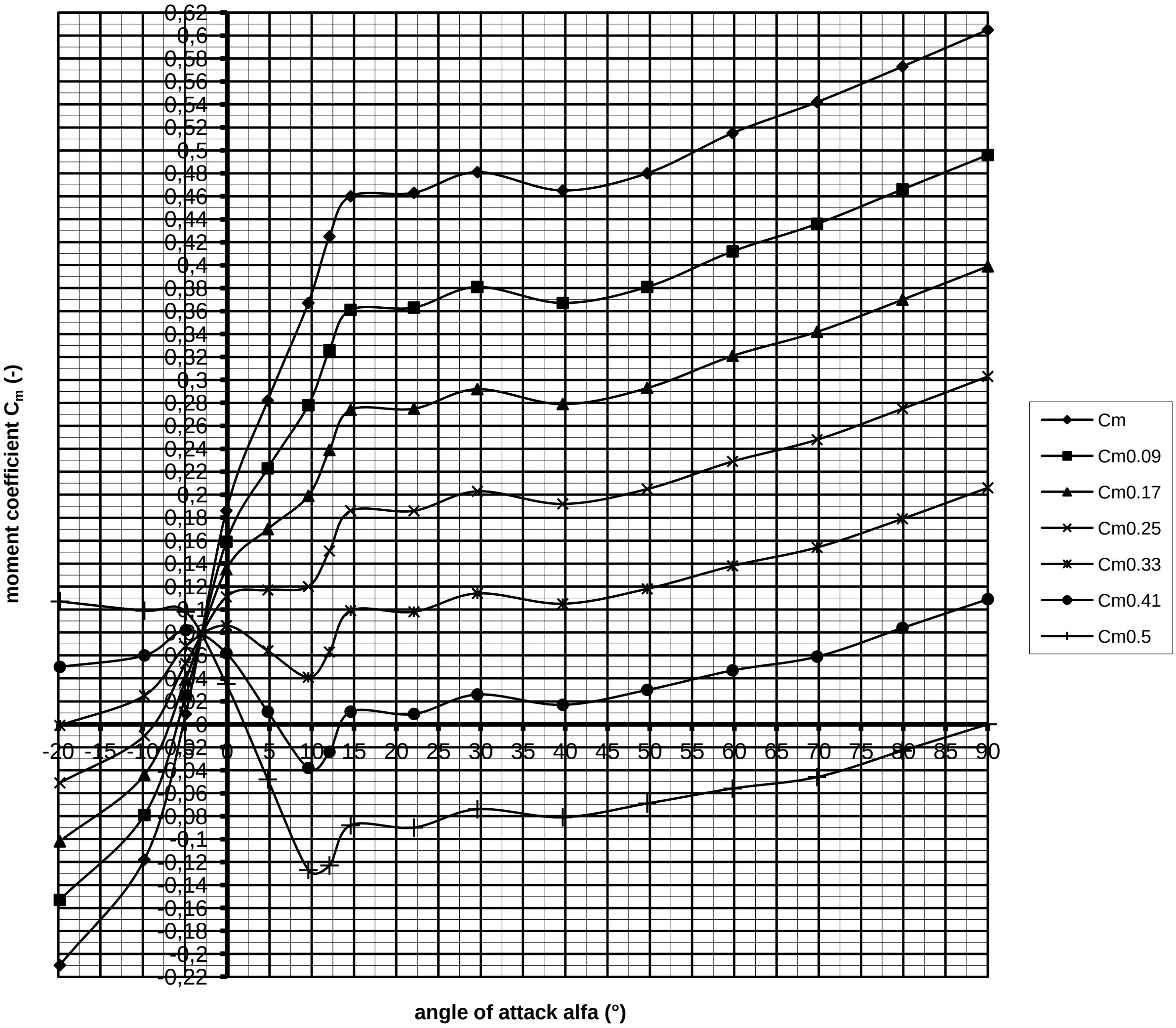

fig. 3 C_m-α curves for different positions of the axis of rotation for a 5 % cambered plate for Re = 4 * 10^5

4 Estimation of the $C_{m0.17}$-α, the $C_{m0.25}$-α and the $C_{m0.33}$-α curves for 7.14 % camber

The $C_{m0.17}$-α, the $C_{m0.25}$-α and the $C_{m0.33}$-α curves seem useful for a positive pitch control system for which the blade turns to larger blade angles. It is assumed that the curve for 7.14 % camber is the average of the curves of 10 % and 5 % camber.

The $C_{m0.17}$-α curve of figure 2 for 10 % camber and the $C_{m0.17}$-α curve of figure 3 for 5 % camber are put together in figure 4. The estimated $C_{m0.17}$-α curve for 7.14 % camber is also given in figure 4.

The $C_{m0.25}$-α curve of figure 2 for 10 % camber and the $C_{m0.25}$-α curve of figure 3 for 5 % camber are put together in figure 5. The estimated $C_{m0.25}$-α curve for 7.14 % camber is also given in figure 5.

The $C_{m0.33}$-α curve of figure 2 for 10 % camber and the $C_{m0.33}$-α curve of figure 3 for 5 % camber are put together in figure 6. The estimated $C_{m0.33}$-α curve for 7.14 % camber is also given in figure 6.

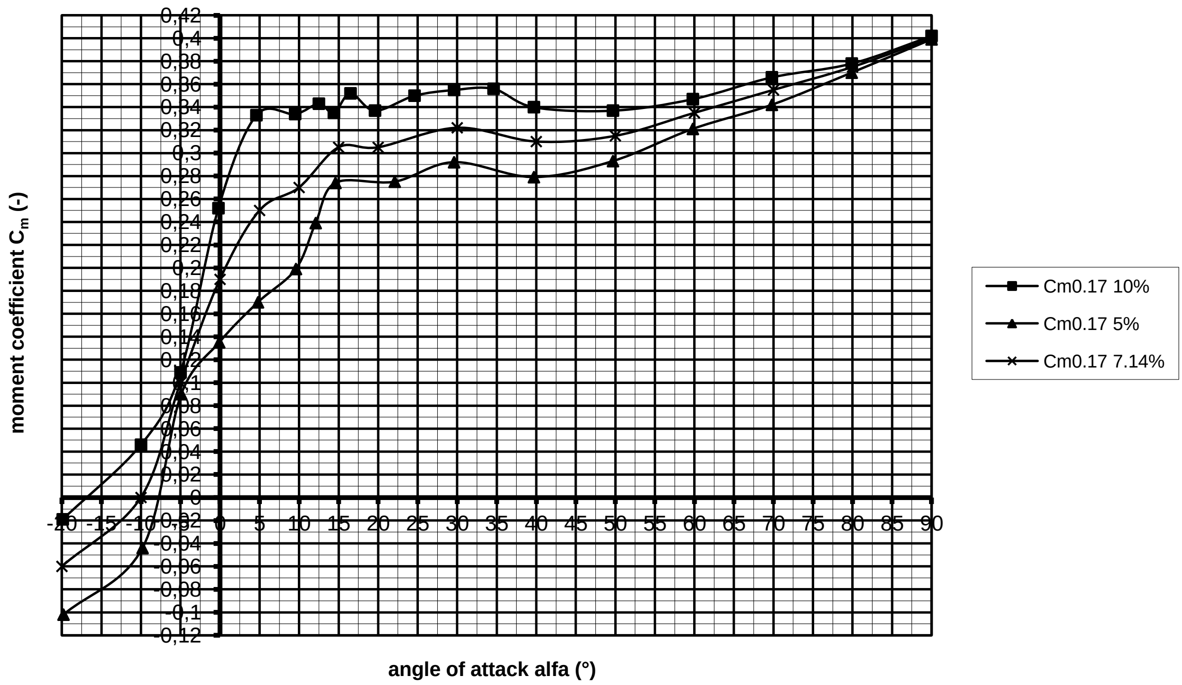

fig. 4 Measured $C_{m0.17}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.17}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

In figure 4 it can be seen that the estimated $C_{m0.17}$-α curve for 7.14 % camber is rising strongly from 0.19 at α = 0° up to 0.305 for α = 15° which are rather high values.

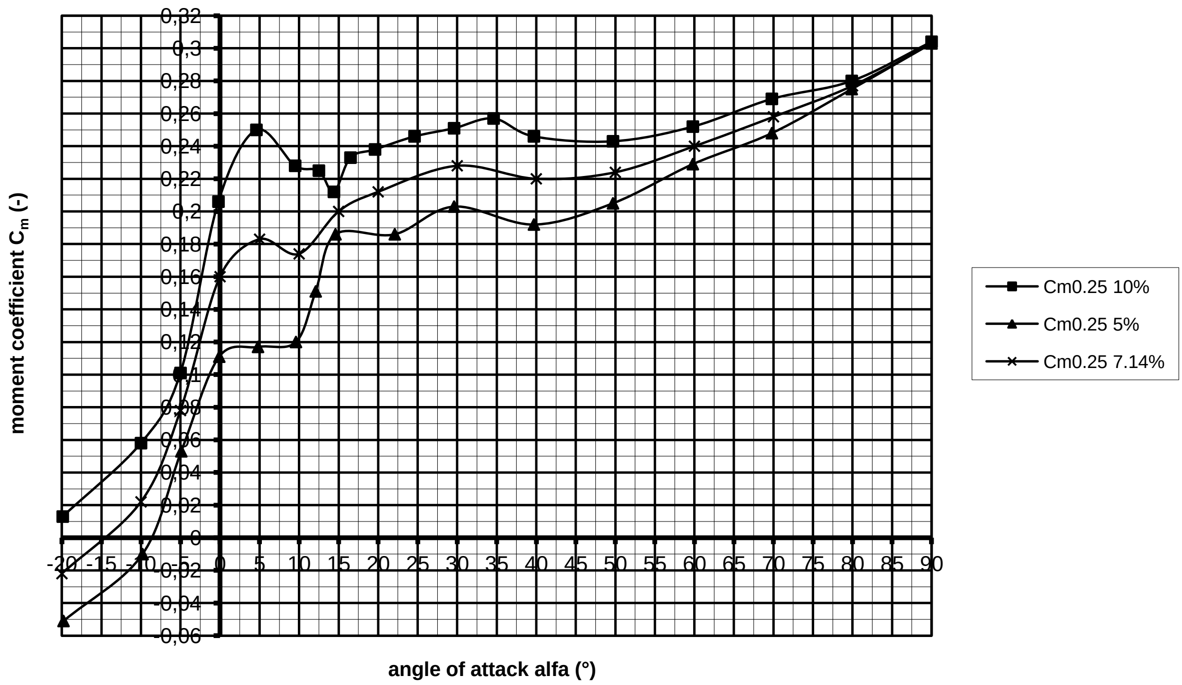

fig. 5 Measured $C_{m0.25}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.25}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

In figure 5 it can be seen that the estimated $C_{m0.25}$-α curve for 7.14 % camber is about constant for 2° < α < 12°. The average $C_{m0.25}$ value is about 0.18 which is rather high.

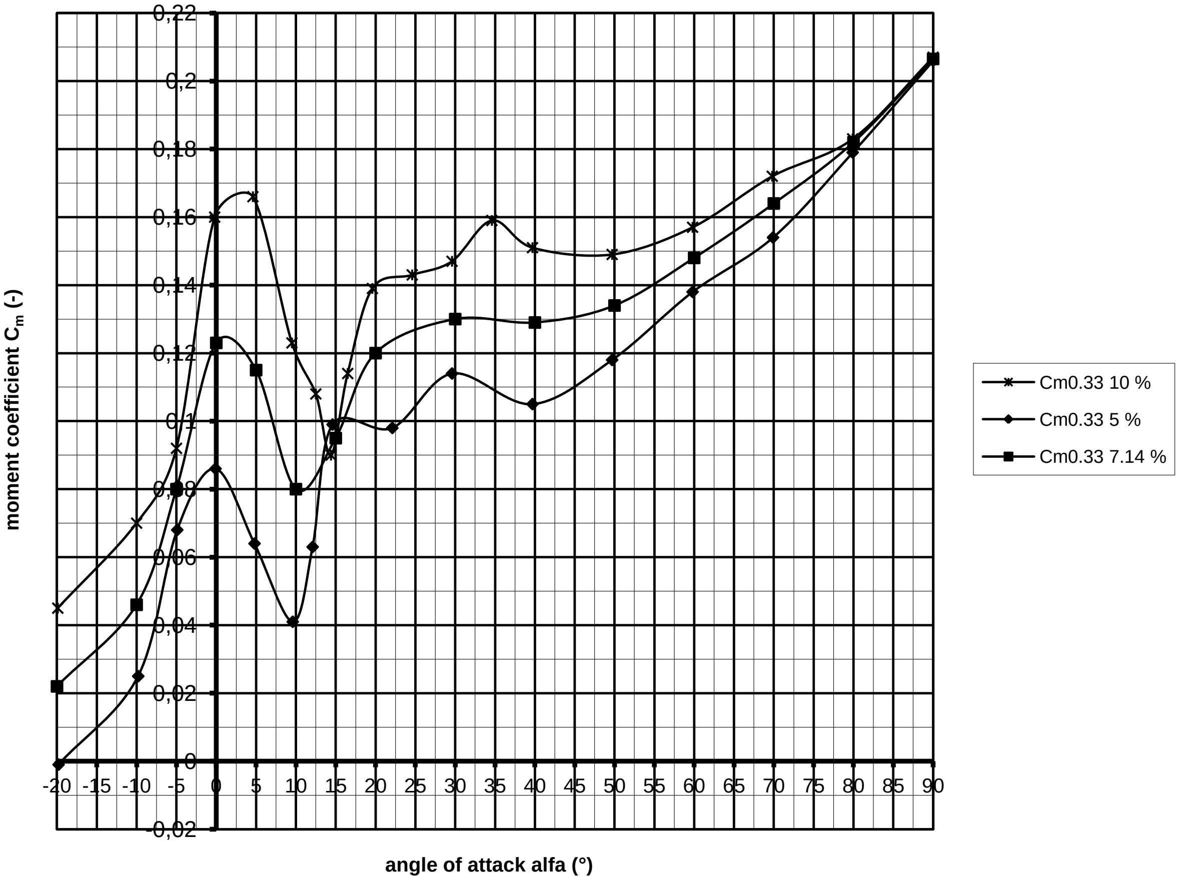

fig. 6 Measured $C_{m0.33}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.33}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

In figure 6 it can be seen that the estimated $C_{m0.33}$-α curve for 7.14 % camber is decreasing for about 0° < α < 10°. It is 0.123 for α = 0° and 0.08 for α = 10°. So this results in a small negative angle g (see figure 1). It is

5 Choosing of the turning point at the airfoil nose

In chapter 2 it is said that choosing the turning point at the airfoil nose is not advised because the system will react very strongly for small variations of α. In figure 2 and figure 3 it can be seen that the value of C_m rises strongly in between about α = - 10° and α = 15°. So once the blade has started to rotate around the blade axis, the pitch moment increases and therefore the blade angle β will increase even more. However, if this would really happen depends on the stiffness of the spring which counteracts the increase of the pitching moment. If the spring is very stiff, the system can still be stable for an increasing value of C_m.

It might even be acceptable that the blade angle is increased suddenly because this results in a strong decrease of the tip speed ratio and the rotational speed will therefore decrease very effectively. If the rotational speed has decreased enough, the pitch moment is reduced that much that the blade turns back to its starting position. So this behaviour is safe and reduction of the rotational speed at high wind speeds will have no negative influence on the output at moderate wind speeds.

Choosing the turning point at the airfoil nose might also have an advantage in how the blade is connected to the shaft which turns in the blade shaft bearings. Assume that there is an overlap of the shaft and the blade with a length of 1/6 of the blade length. If the blade would be welded to the shaft over this length, the connection would not be strong enough. However if an auxiliary plate with a length of 1/6 of the blade length and a width equal to the blade chord c is welded at the other side of the shaft and at the tailing edge of the airfoil, the connection is very strong.

So it seems worthwhile to determine the C_m-α curve for the turning point at the airfoil nose for the 7.14 % cambered airfoil. It is assumed that the curve is the average of the curves for 5 % and 10 % camber. The C_m-α curves for 10 % and 5 % camber as given in figure 2 and 3, are copied in figure 7. The estimated C_m-α curve for 7.14 % camber is also given in figure 7.

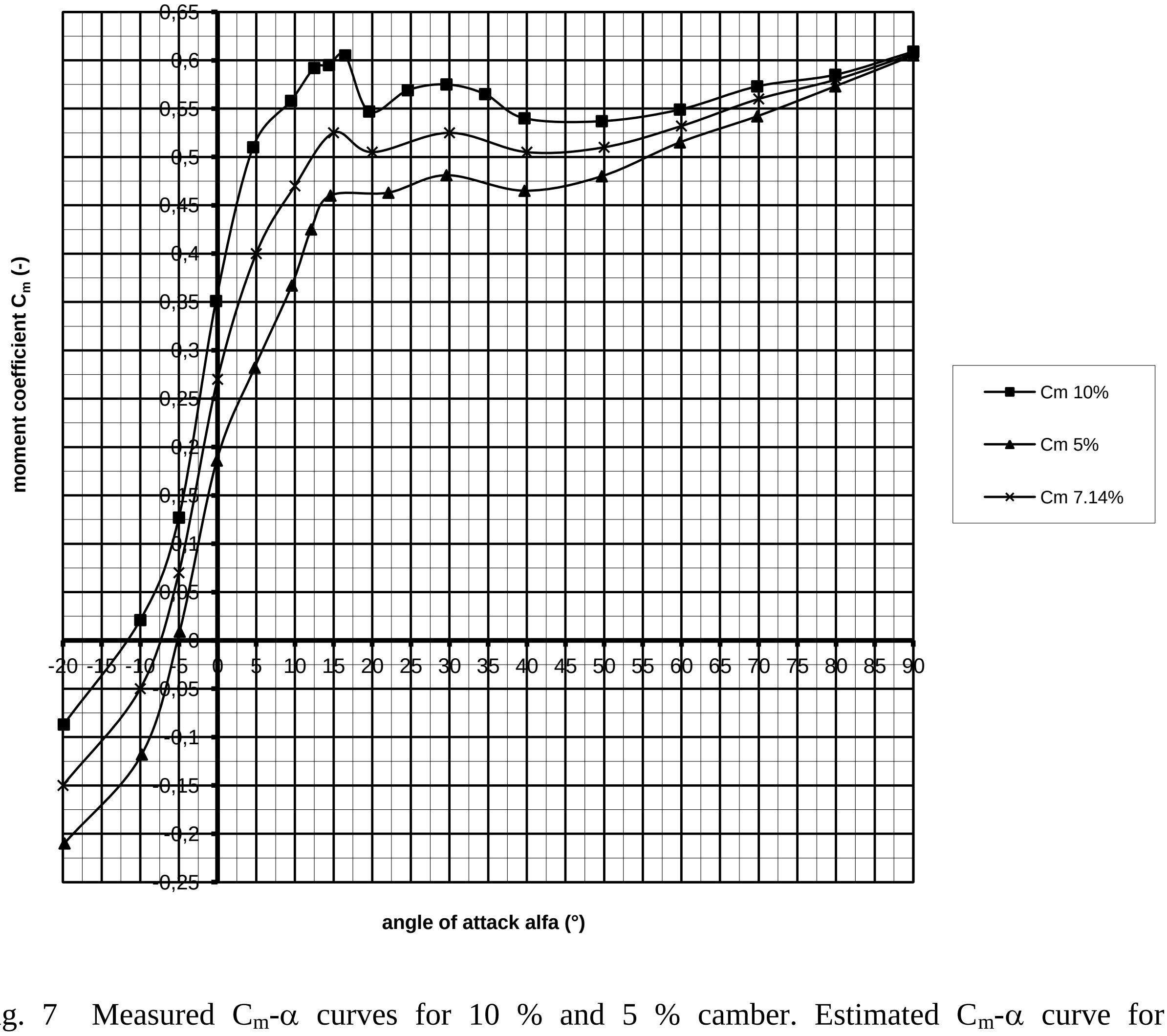

fig. 7 Measured C_m-α curves for 10 % and 5 % camber. Estimated C_m-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

6 Choosing of the turning point at the heart of the blade

Sometimes it is wanted that the pitch moment is minimal and then it might be good to choose the turning point at the heart of the airfoil. So in this case one needs to know the $C_{m0.5}$-α curve for the 7.14 % cambered airfoil. In figure 2 and figure 3 it can be seen that the value of $C_{m0.5}$ decreases strongly in between about α = - 5° and α = 10° for the 10 % and 5 % cambered airfoils. But for small blade angles α, the moment coefficient will be close to zero.

Choosing the turning point at the centre of the blade might also have as an advantage that a welded connection of a blade shaft and a blade is very strong if the shaft is provided with a groove in which the blade is shifted.

So it seems worthwhile to determine the $C_{m0.5}$-α curve for the turning point at the heart of the blade. The $C_{m0.5}$-α curves for 10 % and 5 % camber as given in figure 2 and 3, are copied in figure 8. The estimated $C_{m0.5}$-α curve for 7.14 % camber is also given in figure 8.

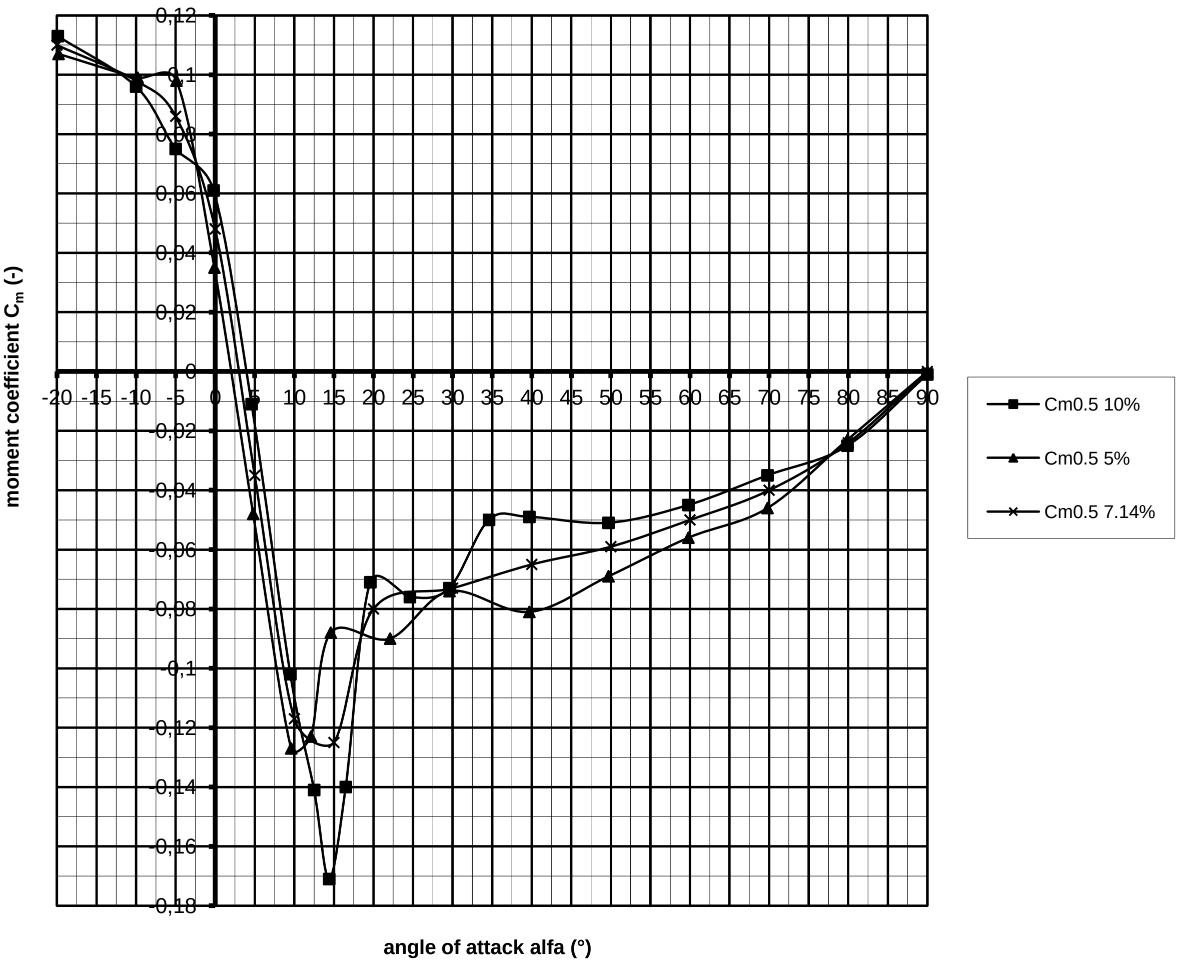

fig. 8 Measured $C_{m0.5}$-α curves for 10 % and 5 % camber. Estimated $C_{m0.5}$-α curve for 7.14 % camber for cambered sheet with aspect ratio of 5

If figure 8 it can be seen that $C_{m0.5} = 0$ for an angle α of about 3°. The real blade angles of a constant chord blade with a 7.14 % cambered airfoil varies in between about 0° at the blade tip and about 10° at the blade root. This means that the pitch moment of a blade for which the axis of rotation lays at the heart of the blade will be rather small.

The estimated $C_{m0.5}$-α curve for 7.14 % camber as given in figure 8 is determined for the turning point lying at the zero line of the airfoil. If the blade shaft is provided with a groove in which the blade is shifted, the turning point isn't lying at the zero line but at the camber line of the airfoil. This makes that there is a small negative angle γ (see figure 1). This angle γ is mainly effective for the influence of the drag coefficient on $C_{m0.5}$ but as the C_d value is rather small, this effect on $C_{m0.5}$–α curve can be neglected for small angles α.

7 References

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3 Hageman A. Catalogue of Aerodynamic Characteristics of Airfoils in the Reynolds number range $10^4 – 10^6$, July 1980, report R443D, (no longer available) University of Technology Eindhoven, Department of Physics, Laboratory of Fluid Dynamics and Heat Transfer, (former) Wind Energy Group. A scan of this report is given on my website at the bottom of the menu KD-reports.