Calculations executed for the rotor of the VIRYA-5R windmill (λ_d = 6.5, Gö 711 airfoil) with a 2-bladed rotor made out of Roofmate, glass fibre and epoxy

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July 2023

KD 750

The rotor should not be used if the windmill has no safety system which turns the rotor fluently out of the wind at high wind speeds!

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1 Introduction

Already in 1977 I have designed and built a very light 2-bladed windmill rotor with a diameter D = 3.2 m, a design tip speed ratio λ_d = 8 and a NACA 4412 airfoil which was manufactured from the isolation material Roofmate covered with glass fibre and epoxy. A photo of this rotor is given in appendix 1. This rotor was damaged in a strong storm because the windmill for which it was used, was not provided with a safety system which limits the thrust and because the rotor was not strong enough. However, the manufacturing procedure was rather simple and a large rotor can be manufactured with the used procedure at a rather low price. So it is investigated how a rather large rotor looks like if it is manufactured from Roofmate material.

Roofmate is standard supplied in sheets with a length of 1250 mm and a width of 600 mm. It can be supplied in a large range of thicknesses. There is some tolerance on the length and so the sections out of which the rotor is built up can be a little shorter than 1250 mm or 625 mm but this effect is neglected for the rotor calculations.

A rotor is made of six 625 mm long blade sections and one 1250 mm long central section resulting in a rotor diameter of 5 m. The windmill is called the VIRYA-5R. The R (of Roofmate) is added to distinguish it from the VIRYA-5 with two wooden blades and the VIRYA-5B3 with three wooden blades.

It might be difficult in some countries to obtain wood of good quality and therefore this alternative rotor made out of Roofmate, glass fibre and epoxy is designed. An adjacent advantage is that this rotor is very light. The VIRYA-5R rotor is mounted to the same axial flux generator type TGET450-5KW-300R as used for the VIRYA-5B3 which is described in report KD 710 (ref. 1). It also makes use of the same head and tower.

2 Description of the rotor of the VIRYA-5R windmill

A blade is made of three 625 mm long segments which are glued together by epoxy. Two rotor blades are glued to a central part which is also made from Roofmate and which has a length of 1250 mm and a thickness of 102 mm. So the rotor diameter D = 6 * 625 + 1250 = 5000 mm = 5 m and the rotor radius R = 2.5 m. The whole rotor is covered with some layers of glass fibre imbedded in epoxy to make the rotor strong and stiff enough. The required number of layers have still to be determined. The original rotor had only one layer.

A blade segment is made of a 625 mm long part of Roofmate by connecting a jig with the required airfoil to both sides of the sheet. The blade segment is cut from the sheet by moving a hot wire simultaneously along both jigs. This procedure allows a small blade twist in between both sides of the blade segment.

The 2-bladed rotor of the VIRYA-5R windmill has a design tip speed ratio λ_d = 6.5. This is lower than the λ_d = 8 of the 3.2 m rotor made in 1977 and the chords are therefore relatively larger resulting in a stronger rotor. The most important advantages of a 2-bladed rotor above a 3-bladed rotor are that the connection of the blades to the hub is simple and that balancing of the rotor is rather simple too.

The rotor has blades with a linearised chord and blade angle and is provided with a Gö 711 airfoil. The geometry and characteristics of the Gö 711 airfoil are given in report KD 285 (ref. 2). The maximum thickness is 14.85 % of the chord. A disadvantage of this airfoil is that it has only been measured for a rather high Reynolds value Re = 4 * 10^5 but as the used chords are rather large, this is no problem.

The rather large thickness also makes that the rotor is relatively stronger and stiffer than the original rotor. The lower side of the Gö 711 airfoil is flat over 97.5 % of the chord which simplifies manufacture. The central part has a rectangular cross section in the middle with rounded edges but long transistion parts to realise a fluent transistion to the airfoil geometry. The chord is 0.28 m at the blade tip and 0.4 m at the blade root. The increase of the chord and so also of the blade thickness, results in a strong increase of the blade strength at the root.

A square 2 mm thick stainless steel sheet with sizes of 300 mm is glued by epoxy to both sides of the central part to guide the forces acting on the rotor to the generator. There are four M8 bolts at a pitch of 260 mm at the corners of the sheets. The rotor is connected to the generator by ten M12 bolts at a pitch circle of 130 mm. Inserts at the bolts prevent sqeezing of the central part. The blades will be rather flexible and therefore vibrations which are caused by the gyroscopic moment, by streaming under a certain yaw angle δ and by a non-uniform distribution of the wind speed over the rotor plane, are flattened. The rotor is balanced by adding some extra glass and epoxy to the lightest blade tip or by gluing small cylinders of lead in holes which are drilled in the lightest blade tip. Appendix 2 gives a sketch of the rotor.

3 Calculation of the rotor geometry

The rotor geometry is determined using the method and the formulas as given in report KD 35 (ref. 3). This report (KD 750) has its own formula numbering. Substitution of $\lambda_d = 6.5$ and $R = 2.5$ m in formula (5.1) of KD 35 gives:

$$\lambda_{r\,d} = 2.6 * r \qquad (-) \qquad (1)$$

Formula’s (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \qquad (°) \qquad (2)$$

$$\phi = 2/3 \arctan 1 / \lambda_{r\,d} \qquad (°) \qquad (3)$$

Substitution of B = 2 in formula (5.4) of KD 35 gives:

$$C_l = 12.566\, r\, (1 - \cos\phi) / c \qquad (-) \qquad (4)$$

Substitution of V = 5 m/s in formula (5.5) of KD 35 gives:

$$R_{e\,r} = 3.335 * 10^5 * c * \sqrt{(\lambda_{r\,d}^2 + 4/9)} \qquad (-) \qquad (5)$$

The blade is calculated for seven stations A till G which have a distance of 0.3125 m (so halve a blade section) of one to another. First the theoretical values are determined for C_l, α and β. Next β is linearised such that the twist is constant in between the ends of a blade section and that the linearised values correspond as good as possible with the theoretical values. The result of the calculations is given in table 1.

The Reynolds values for the stations are calculated for a wind speed of 5 m/s because for most working hours, the windmill will be used at rather low wind speeds.

sta-tion	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	$C_{l\,th}$ (-)	$C_{l\,lin}$ (-)	$R_{e\,r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ Gö 711	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	$C_d/C_{l\,lin}$ (-)
A	2.5	6.5	5.8	0.28	0.58	0.58	6.10	4	-1.0	-0.1	6.8	6.8	0.025
B	2.1875	5.688	6.6	0.3	0.62	0.62	5.73	4	-0.6	-0.6	7.2	7.2	0.022
C	1.875	4.875	7.7	0.32	0.67	0.68	5.25	4	0.0	0.1	7.7	7.6	0.020
D	1.5625	4.063	9.2	0.34	0.75	0.74	4.67	4	1.0	0.9	8.2	8.3	0.018
E	1.25	3.25	11.4	0.36	0.86	0.87	3.98	4	2.3	2.4	9.1	9.0	0.015
F	0.9375	2.438	14.9	0.38	1.04	0.99	3.20	4	4.6	3.9	10.3	11.0	0.015
G	0.625	1.625	21.1	0.4	1.31	1.31	2.34	4	8.1	8.1	13.0	13.0	0.021

table 1 Calculation of the blade geometry of the VIRYA-5R rotor

β is linearised in between station A and C, in between station C and E and in between station E and G. The twist per station is 0.4° in between stations A up to C, 0.7° in between stations C and E and 2° in between stations E up to G. For these values, the linearised angles are lying very close to the theoretical angles. A sketch of the rotor is given in appendix 2.

4 Determination of the C_p-λ and the C_q-λ curves

The determination of the C_p-λ and C_q-λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the four most important outer stations of the blade is 0.021. Figure 4.6 of KD 35 (for B = 2) and $\lambda_{opt} = 6.5$ and $C_d/C_l = 0.021$ gives $C_{p\ th} = 0.46$. The blade isn't stalling, even not at station G and so the whole part of the blade which has an airfoil is effective for the calculation of the C_p. This gives k = 2.5 – 0.625 = 1.875 m.
Substitution of $C_{p\ th} = 0.46$, R = 2.5 m and blade length k = 1.875 m in formula 6.3 of KD 35 gives $C_{p\ max} = 0.43$. $C_{q\ opt} = C_{p\ max} / \lambda_{opt} = 0.43 / 6.5 = 0.0662$.
Substitution of $\lambda_{opt} = \lambda_d = 6.5$ in formula 6.4 of KD 35 gives $\lambda_{unl} = 10.4$.
The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - ½k) * C_l * c * k / \pi R^3 \qquad (-) \qquad (6)$$

Formula 6 is only valid for a blade with a constant chord and a constant blade angle but it gives a good approximation for a tapered blade if the values at half the blade length are used. Section D is lying at halve the blade length. The chord at section D is 0.34 m and the blade angle $\beta = 8.3°$. If the rotor is not rotating, the angle of attack $\alpha = 90° - \beta$. So the average angle of attack is 90° – 8.3° = 81.7°.

The C_l-α curve for the Gö 711 airfoil arn't given for large angles of α in KD 285. However, the C_l-α curve for Gö 623 airfoil for large angles α is given in figure 5.10 of report KD 35 (ref. 3) and it is assumed that this curve can also be used for the Gö 711 arfoil if the airfoil is stalling. For $\alpha = 81.7°$ it can be read in this figure that $C_l = 0.3$. During starting the whole blade length is stalling and so the blade length k = 1.875 is used for the calculation of $C_{q\ start}$.

Substitution of B = 2, R = 2.5 m, k = 1.875 m, $C_l = 0.3$ and c = 0.34 m in formula 6 gives that $C_{q\ start} = 0.0091$. The real starting torque coefficient will be somewhat lower than the calculated value because we have used the average chord and the average blade angle. It is assumed that $C_{q\ start} = 0.0085$. For the ratio between the starting torque and the optimum torque we find that it is 0.0085 / 0.0662 = 0.128. This is acceptable for a rotor met a design tip speed ratio of 6.5. The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * ½\rho * \pi R^3} \right)} \qquad (m/s) \qquad (7)$$

For the sticking torque Q_s of the generator it is specified that it is smaller than 0.5 Nm. However, this is the case if no oil seal is mounted. An extra oil seal is needed for horizontal positioning of the generator shaft. It is assumed that the sticking torque with an oil seal is 2 Nm. Substitution of $Q_s = 2$ Nm, $C_{q\ start} = 0.0085$, $\rho = 1.2$ kg/m^3 and R = 2.5 m in formula 7 gives that $V_{start} = 2.8$ m/s. This is acceptable low for a 2-bladed rotor with a design tip speed ratio of 6.5.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p-λ and C_q-λ curves can be determined if only two points of the C_p-λ curve and one point of the C_q-λ curve are known. The first part of the C_q-λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for $\lambda = 0$.

Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 4). With this method, it can be determined that the C_q-λ curve is about straight and horizontal for low values of λ if a Gö 711 airfoil is used. A scale model of a three bladed rotor with constant chord and blade angle and with a design tip speed ratio $\lambda_d = 6$ has been measured in the open wind tunnel of the University of Technology Delft already on 20-11-1980. It has been found that the maximum C_p was more than 0.4 and that the C_q-λ curve for low values of λ was not horizontal but somewhat rising. This effect has been taken into account and the estimated C_p-λ and C_q-λ curves for the VIRYA-5R rotor are given in figure 1 and 2. The low C_q and C_p values at low values of λ are caused by stalling of the airfoil.

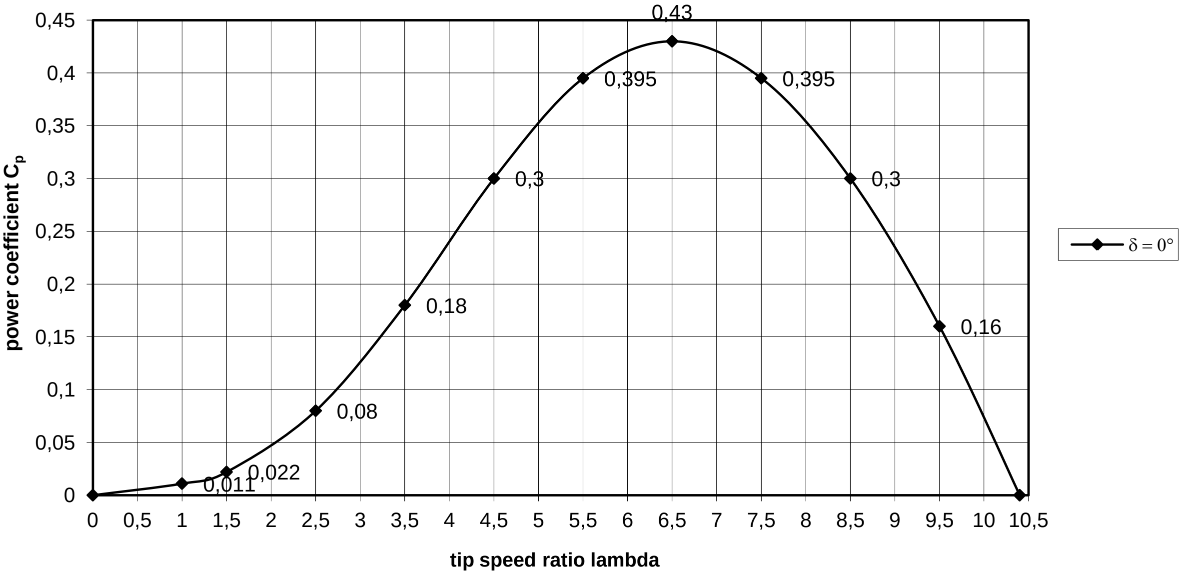

fig. 1 Estimated C_p-λ curve for the VIRYA-5R rotor for the wind direction perpendicular to the rotor ($\delta = 0$

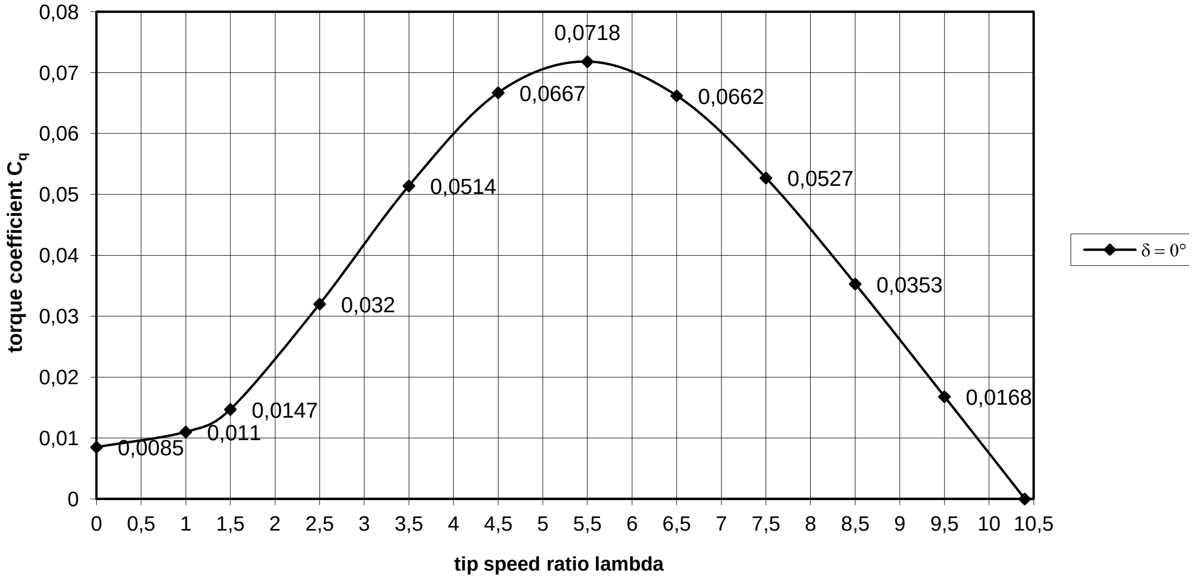

fig. 2 Estimated C_q-λ curve for the VIRYA-5R rotor for the wind direction perpendicular to the rotor ($\delta = 0°$)

5 Determination of the P-n curves and the optimum cubic line

The determination of the P-n curves of a windmill rotor is described in chapter 8 of KD 35 (ref. 3). One needs a C_p-λ curve of the rotor and the δ-V curve of the safety system together with the formulas for the power P and the rotational speed n. The VIRYA-5R will be executed with the hinged side vane safety system which is described in report KD 213 (ref. 5) for the VIRYA-4.2 windmill. The VIRYA-5R makes use of the same head and so of the same vane blade as used for the VIRYA-5B3. This vane blade is made out of 12 mm meranti plywood resulting in a rated wind speed of about 11 m/s. The estimated δ-V curve is given in figure 4 of KD 710. This δ-V curve is copied as figure 3. The rotor starts turning out of the wind at a wind speed of about 7 m/s. The rotor is turned out of the wind 30° at a wind speed of 11 m/s. It is assumed that the ideal curve is followed for wind speeds above 11 m/s.

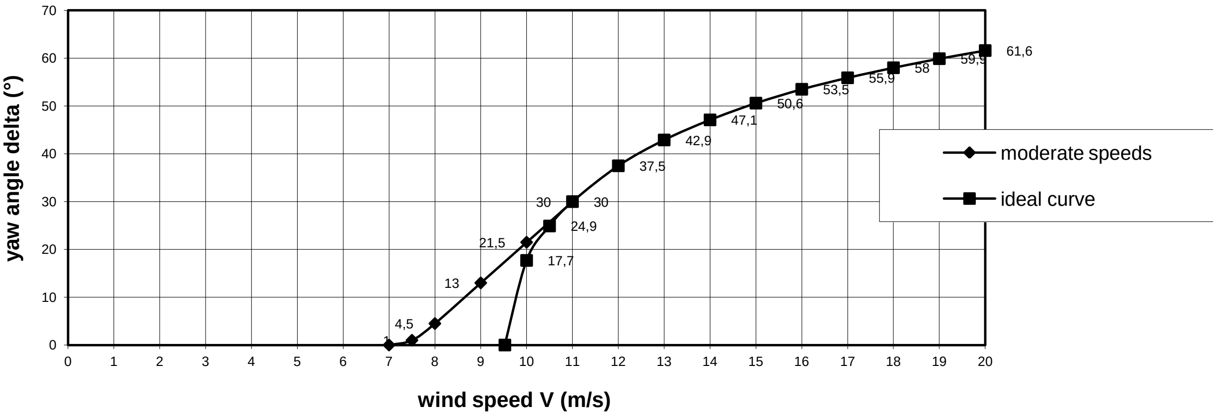

fig. 3 Estimated δ-V curve for a 12 mm meranti plywood vane blade

The P-n curves are used to check the matching with the P_{mech}-n curve of the generator and to determine the P_{el}-V curve. Because the P-n curve for low values of λ appears to lie very close to each other, the P-n curves are not determined for very low values of λ. The P-n curves are determined for C_p values belonging to λ is 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5 and 10.4 (see figure 1). The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8, 9, 10 and 11 m/s.

Substitution of R = 2.5 m in formula 7.1 of KD 35 gives:

$$n = 3.8197 * \lambda * \cos\delta * V \qquad \text{(rpm)} \qquad (8)$$

Substitution of $\rho = 1.2$ kg / m^3 and R = 2.5 m in formula 7.10 of KD 35 gives:

$$P = 11.781 * C_p * \cos^3\delta * V^3 \qquad \text{(W)} \qquad (9)$$

For a certain wind speed, for instance V = 3 m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For wind speeds higher than 7 m/s, the yaw angle δ is taken into account. The result of the calculations is given in table 2.

		V = 3 m/s δ = 0°		V = 4 m/s δ = 0°		V = 5 m/s δ = 0°		V = 6 m/s δ = 0°		V = 7 m/s δ = 0°		V = 8 m/s δ = 4.5°		V = 9 m/s δ = 13°		V = 10 m/s δ = 21.5°		V = 11 m/s δ = 30°	
λ (-)	C_p (-)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)	n_δ (rpm)	P_δ (W)
3.5	0.18	40.1	57	53.5	136	66.8	265	80.2	458	93.6	727	106.6	1076	117.2	1430	124.4	1708	127.4	1833
4.5	0.3	51.6	95	68.8	226	85.9	442	103.1	763	120.3	1212	137.1	1793	150.7	2383	159.9	2847	163.7	3055
5.5	0.395	63.0	126	84.0	298	105.0	582	126.1	1005	147.1	1596	167.5	2361	184.2	3138	195.5	3748	200.1	4023
6.5	0.43	74.5	137	99.3	324	124.1	633	149.0	1094	173.8	1738	198.0	2570	217.7	3416	231.0	4080	236.5	4379
7.5	0.395	85.9	126	114.6	298	143.2	582	171.9	1005	200.5	1596	228.5	2361	251.2	3138	266.5	3748	272.9	4023
8.5	0.3	97.4	95	129.9	226	162.3	442	194.8	763	227.3	1212	258.9	1793	284.7	2383	302.1	2847	309.3	3055
9.5	0.16	108.9	51	145.1	121	181.4	236	217.7	407	254.0	647	289.4	956	318.2	1271	337.6	1518	345.7	1630
10.4	0	119.2	0	158.9	0	198.6	0	238.3	0	278.1	0	316.8	0	348.4	0	369.6	0	378.4	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-5R rotor

The calculated values for n and P are plotted in figure 4. The optimum cubic line which is going through the tops of the P_{mech}-n curves is also given in figure 4.

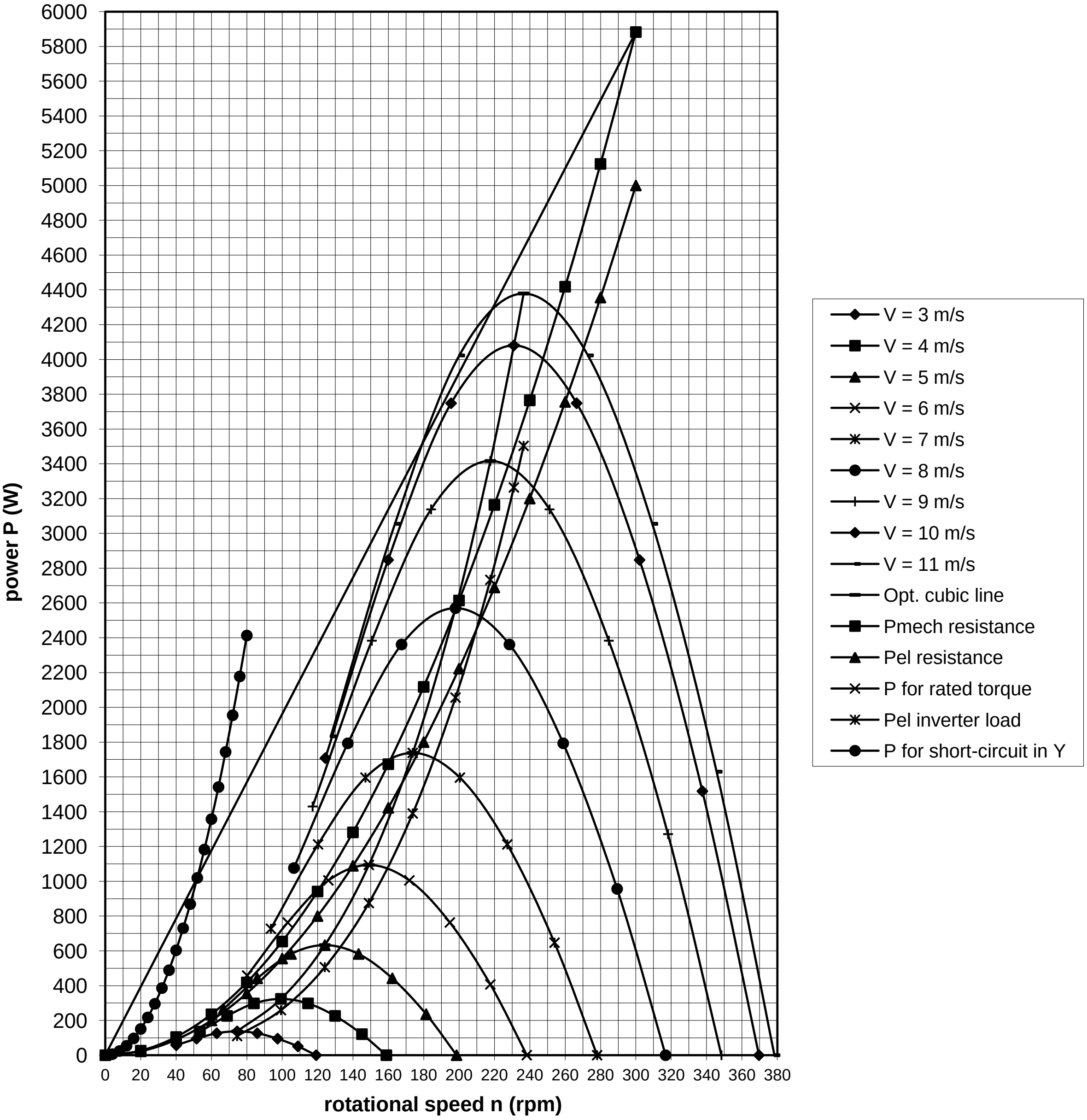

fig. 4 P-n curves of the VIRYA-5R rotor, optimum cubic line, P_{mech}-n and P_{el}-n curves for the generator with a resistance load such that P_{el} = 5000 W at n = 300 rpm, P-n curve for the rated torque, P_{el}-n curve for an inverter load, P-n curve for short-circuit in star

6 Use of the VIRYA-5R rotor with the axial flux generator TGET450-5KW-300R

The characteristics of the generator of Hefei Top Grand type TGET450-5KW-300R are derived in chapter 6 of KD 710 for a resistance load with a value of the resistance such that the electrical power is 5 KW at a rotational speed of 300 rpm. The P_{mech}-n and P_{el}-n curve for this condition are copied in figure 4. It can be seen that the P_{mech}-n curve is intersecting with the optimum cubic line at a wind speed of about 8 m/s.

It is assumed that the inverter can be adjusted such that the optimum cubic line is followed and that the total efficiency of generator, rectifier and inverter is 0.8. The P_{el}-n curve for an inverter load is also given in figure 4. So if the optimum cubic line is followed, the load is lighter than a resistance load for wind speeds below 8 m/s and higher than a resistance load for wind speed above 8 m/s. As the VIRYA-5R has the the same rotor diameter and the same maximum C_p as the VIRYA-5B3, the maximum power at a certain wind speed will also be the same. The VIRYA-5R will therefore have the same P_{el}-V curve as given in figure 8 of KD 710. This curve is copied as figure 5.

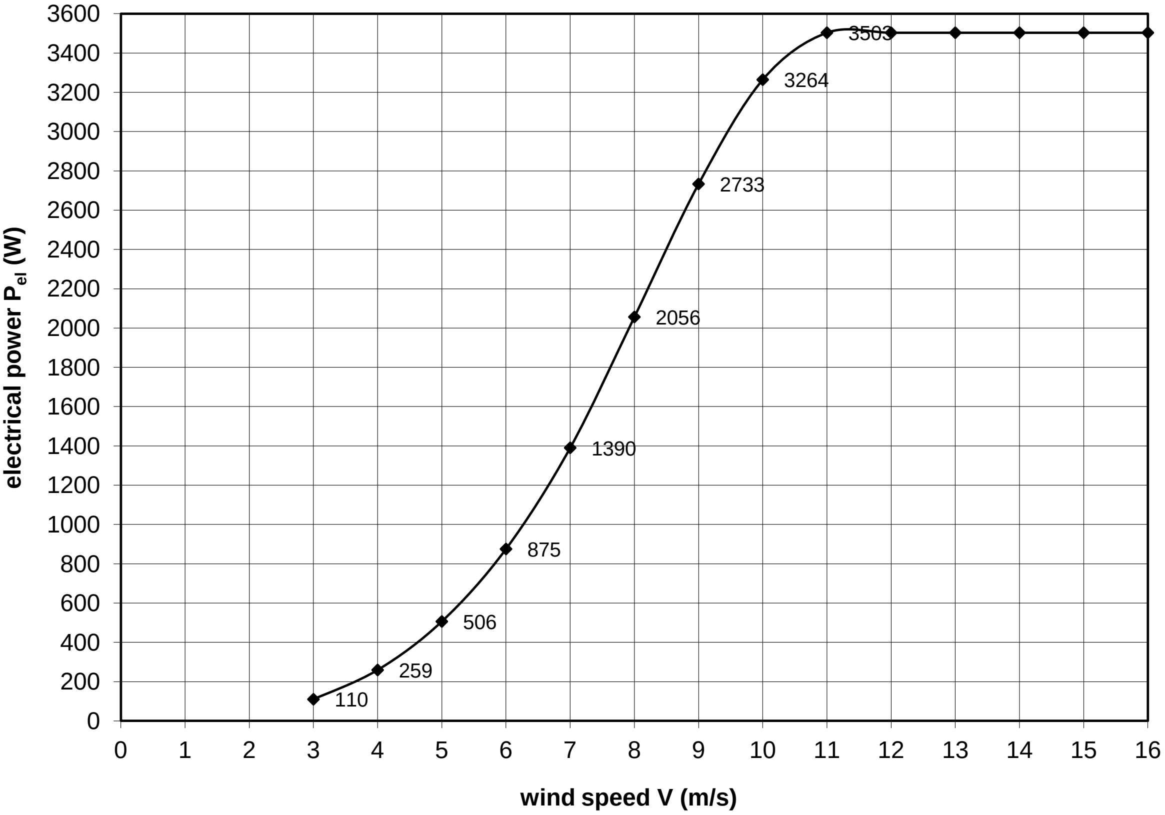

fig. 5 P_{el}-V curve VIRYA-5R for a load such that the optimum cubioc line is followed.

The inverter needs a minium voltage to start functioning. It is assumed that this voltage is generated at V = 3 m/s. This is the reason why the P_{el}-V curve starts suddenly with P_{el} = 110 W at V = 3 m/s. In chapter 4 it was calculated that the starting wind speed is 2.8 m/s and so there is no hysteresis in the P_{el}-V curve. The maximum power is about 3.5 kW at a wind speed of 11 m/s or higher which is very good for a wind turbine with a rotor diameter of 5 m and a rated wind speed of 11 m/s.

The torque for a resistance load is maximal for n = 300 rpm. This torque can also be supplied at lower rotational speeds. A constant torque results in a power curve which increases linear. The P-n line for the rated torque and the line for short-circuit in star are also given in figure 4.

Just as it is the case for the VIRYA-5B3, it is also possible to use the VIRYA-5R for 120 V battery charging or for heating by a resistance load (see KD 710 chapter 7 and 8).

7 References

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Appendix 1

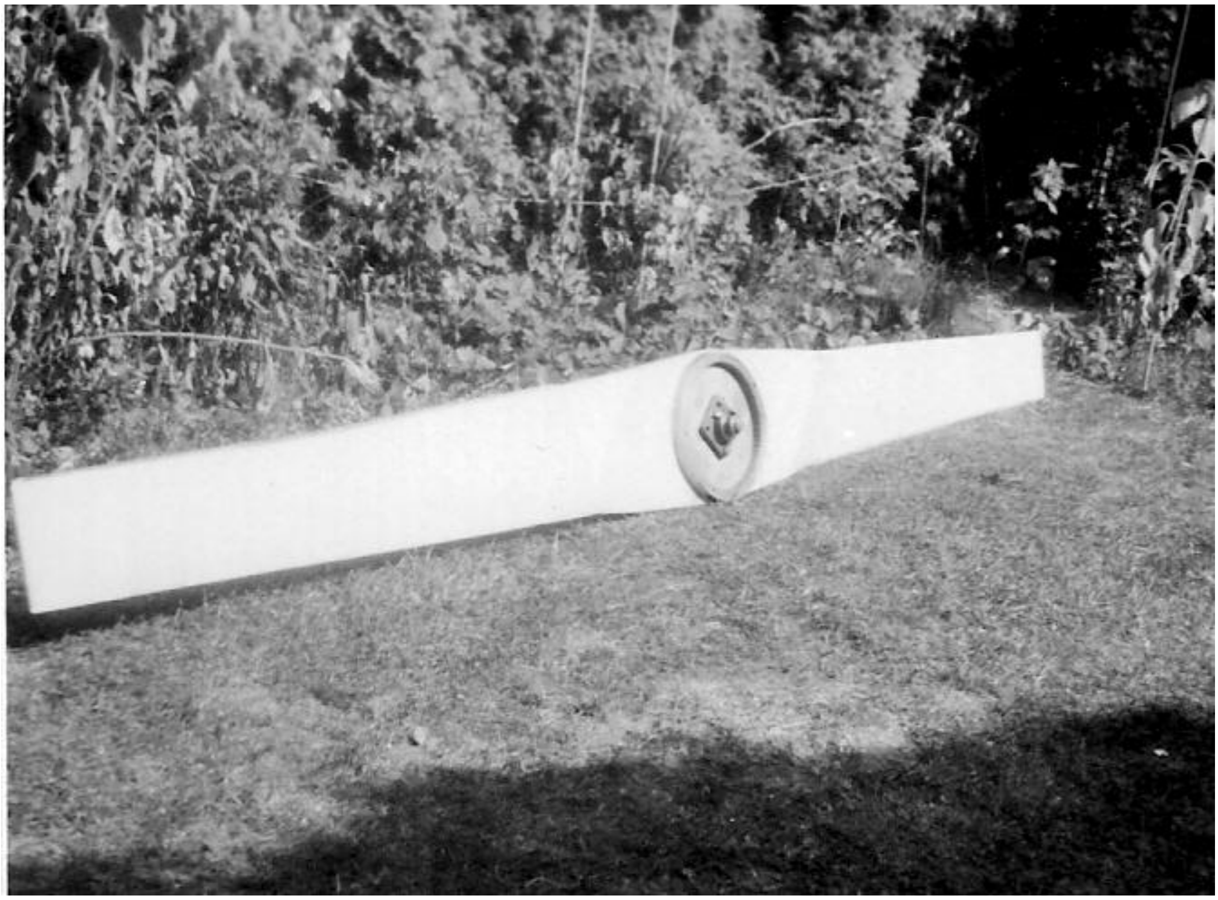
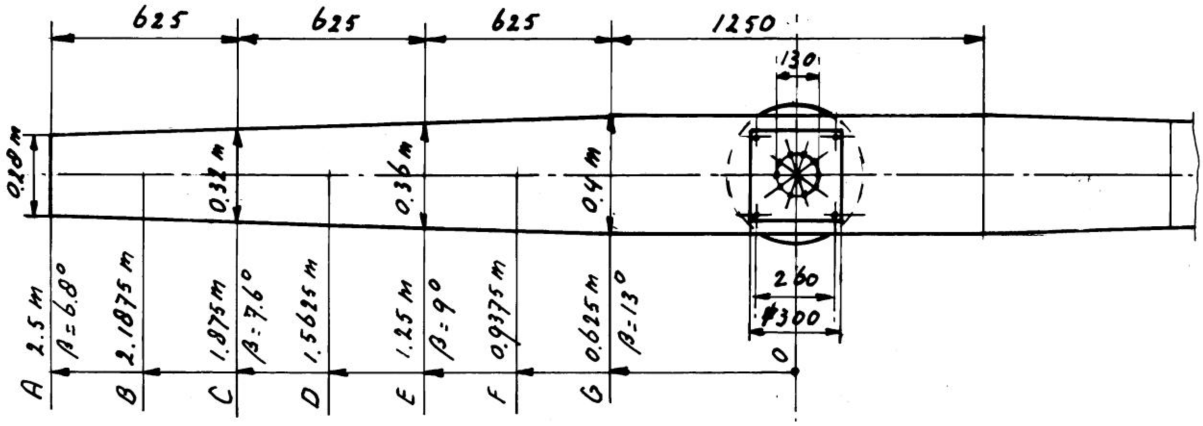


Photo Roofmate rotor with D = 3.2 m and λ_d = 8 made in 1977

Appendix 2



Sketch of the VIRYA-5R rotor