

**Investigation of the Sparta Ion front wheel hub motor as generator for
a small wind turbine**

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1 Introduction

On the wind forum of Otherpower website <https://fieldlines.com> there has been a discussion about the usability of a Sparta Ion front wheel hub motor. The discussion is started in the post of mbrouwer titled “axial generator with lamination core” at page 21, Replay #599. At this point, a photo is shown of a hub motor from which the bearing cover at the cable side is removed. The two printed circuit boards are removed too. In this photo it can be seen that the motor has an armature with 20 magnets and so 20 poles and a stator with 24 coils. It appears that the coils are connected such that the winding is a 3-phase winding. The three ends of the phases can be seen as three red isolated threaded holes in the bottom of the hub.

On YouTube there is a Dutch video of someone who has modified this motor by removing both internal printed circuit boards and who has connected a 3-phase cable directly to the 3-phase winding. He drives the motor with a standard 3-phase inverter. The video is called: “Sparta Ion GD Design”.

It might be possible to use this hub motor as generator for a small wind turbine which can charge a 12 V lead acid battery. However, this will only work acceptably if the generator has a sufficiently low peak on the cogging torque and if $P_{\text{mech}}-n$ curve of the generator matches with the optimum cubic line of the chosen wind turbine rotor. Matching in between rotor and generator is explained in chapter 8 of my public report KD 35 (ref. 1).

To get an impression of the qualities of this hub motor, I have bought a complete second hand front wheel (for only € 20). I have removed the rim and the spokes.

2 Description of the generator

The motor comes from the front wheel of an about ten years old Dutch E-bike Sparta Ion. This bike wheel has no brake at the motor but a brake on the wheel rim. There are E-bikes with an about identical motor at the back wheel but this motor has a longer shaft and a set of chain sprockets and sometimes also a brake connected to the motor. Therefore a front wheel motor was chosen.

The motor is direct drive and has an outside armature with 20 poles and a 3-phase stator with 24 coils. There are two printed circuit boards inside the hub which contain the inverter and the electronics needed for the torque regulation. There is a type plate in between the spoke flanges indicating: ION technology Hub motor 24-36 V, 250 W, Accell NL B.V.

The cable side of the motor is called the back side. The motor has two flanges to which the spokes are connected. Every flange has eighteen, 3 mm holes for the spokes at a pitch circle of 178 mm. The outside diameter of the flanges is 192 mm. The diameter at the heart of the hub is about 166 mm but it is a little hollow and there is a radius of about $r = 3$ mm with the flanges. The front bearing cover has a diameter of about 165 mm but there is a large radius $r = 5$ mm with the front flange.

The shaft has a diameter of 12 mm at the bearings but near the end, the diameter is reduced to 10 mm and both ends have a fine thread with an outside diameter of about 9 mm and a flat side at which the thickness is only 8 mm. In the 12 mm diameter part of the shaft there is a 6 mm wide and about 4 mm deep groove through which the cable is guided inside the bearing. In between the back bearing and the bicycle frame there is a bush with an outside diameter of 22 mm and the cable is guided through a groove in this bush. This bush increases the stiffness of the shaft if the nut is tightened. The shaft is made of high quality steel but the groove weakens the shaft strongly.

In a bicycle, the shaft is supported at both sides but if this motor is used as a generator for a wind turbine, only the back shaft end will be used to connect the generator to the head frame. I think that a rotor diameter of 1.5 m is the absolute maximum for this shaft and then this is only allowed if the wind turbine is provided with the hinged side vane safety system which limits the rotational speed and the yawing speed and so the gyroscopic moment.

3 Measuring of the cogging torque

The first thing which I did was measuring of the cogging torque. The back side shaft end was clamped in a vice. The hub has a diameter at the heart of the two flanges of 166 mm. A thin chord was wound around the hub and a hook was made at the end of the rope. Different ring spanners were hang at the hook until the hub just starts rotating. The spanners were weighed on a digital balance. The hub starts rotating at a weight of 850 gr = 0.85 kg. The radius of the string is $166 / 2 = 83 \text{ mm} = 0.083 \text{ m}$. So for the peak torque Q_{peak} it is valid that: $Q_{\text{peak}} = 9.81 * 0.083 * 0.85 = 0.692 \text{ Nm}$ which is very high. In report KD 78 (ref. 2), I give the measurements for a PM-generator made from an asynchronous motor frame size 90. This generator has a shaft diameter of 25 mm and can be used for a rotor with a diameter of 3 m. The cogging torque at $n = 0 \text{ rpm}$ is only 0.4 Nm.

It was also measured which weight is needed to keep a slowly rotating hub running, once it has started to rotate. This is the case for a weight of 387 gr = 0.387 kg. So for the average torque Q_{av} it is valid that $Q_{\text{av}} = 9.81 * 0.083 * 0.387 = 0.315 \text{ Nm}$. This is less than half the peak value. A photo of these measurements is given in figure 1.

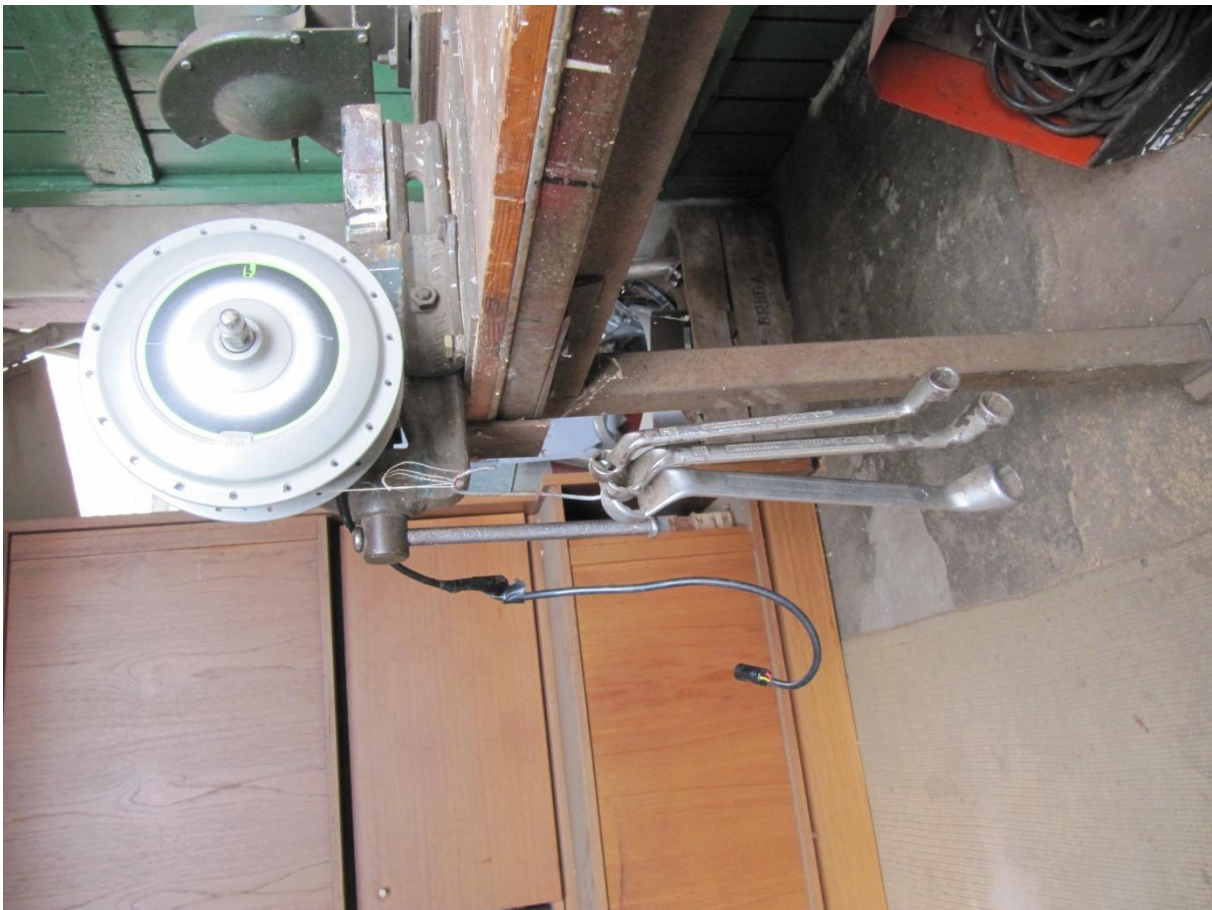


fig. 1 Measuring of the cogging torque

As the armature has 20 poles, the angle in between two adjacent poles is $360 / 20 = 18^\circ$. As the stator has 24 poles, the angle in between two adjacent poles is $360 / 24 = 15^\circ$. So the difference is 3° . Assume that the armature has a preference position if an armature pole is just opposite a stator pole. This happens every 3° for four armature poles at the same time and so the armature should have $360 / 3 = 120$ preference positions per revolution. However, in practice it appears that the armature has only 20 preference positions per revolution. I don't quite understand how this is possible.

It might be caused by the fact that the magnets are not exactly positioned under 18° . A slight deviation from the theoretical angle may make one of six preference positions stronger than the other five and only the strongest one is felt.

4 Measuring of the open voltage

The next thing was to check if a non modified motor can supply a DC voltage when it is used as a generator. Three cables with colours red, black and yellow are coming out of the back shaft. These three cables are not directly connected to the three phases of the winding. The cables with colours red and black are normally connected the battery. The inverter is built in the hub motor on the second printed circuit board. It is assumed that the 3-phase winding is connected in star and that the star point is hidden somewhere at the coils. The yellow cable is used to regulate the torque. A DC volt meter was connected to the red and black wires. A swing was given to the rope and the DC voltage became about 20 V. So the 3-phase inverter also works as a 3-phase rectifier. However, the voltage slows down only slowly for $n = 0$ rpm which is an indication that there are capacitors inside the generator.

To really measure the U-n curve, it is necessary to drive the motor with different constant rotational speeds. But this requires a test rig with which the generator can be driven. Some years ago I have built a simple test rig to measure a small Chinese axial flux PM-generator of Hefei Top Grand. This test rig is described in report KD 595 (ref. 3).

The test rig makes use of a permanent magnet DC motor which is driving a second shaft using a reducing chain transmission to increase the torque level. Unfortunately there is some non concentricity in the big chain wheel which makes that the tension in the chain varies and this causes vibrations which limits the maximum rotational speed. The motor can be driven at variable speed using a Variac and a 1-phase rectifier. The rotational speed is measured using a laser speed meter. The second shaft is made black with a white reflecting spot on it.

The second shaft has a hub with a hole pattern of the Chinese generator. A hexagonal 2 mm thick steel plate is bolted to this hub with three bolts M8. The Sparta hub motor is bolted to this sheet by six threaded rods M5 which are that long that the front shaft end of the hub motor isn't touching the hexagonal sheet.

So six of the eighteen spoke holes in the front flange are increased up to 5 mm. It was chosen to take that holes which are opposite the six screws which connect the back bearing cover. The hub was supported on three wooden blocks during drilling (see photo given in figure 2).

The flange has a rather large radius at both sides of the flange and some aluminium is filed away to create flat areas large enough for a nut M5 at both sides of the flange. Filing is also easy if the generator is lying on the three wooden blocks.

The sheet of the test rig is clamped in a vice. A symmetric lever is bolted to the shaft end and one end of the lever is connected by a rope to a weight lying on the ground. This prevents that the shaft is rotating. A photo of the test rig is given in figure 3.

First it was thought that the generator can't be coupled to a 12 V lead acid battery because then it might work as a motor. However, nothing happens if the red wire is connected to the plus and if the black wire is connected to the min of the battery. It appears that the generator works only as a motor if the correct signal is given to the electronics by means of the yellow wire. So if a battery is mounted, charging starts as soon as the open voltage is higher than the battery voltage. However, it isn't sure if the electronics on the printed circuit boards are strong enough for long use of a non modified motor as generator. It might still be required to remove both printed circuit boards, to lay a new 3-phase cable and to use an external 3-phase rectifier. The six diodes in a 3-phase rectifier make that there can only flow a current from the generator to the battery and not in the opposite direction.



fig. 2 Drilling of six 5 mm holes

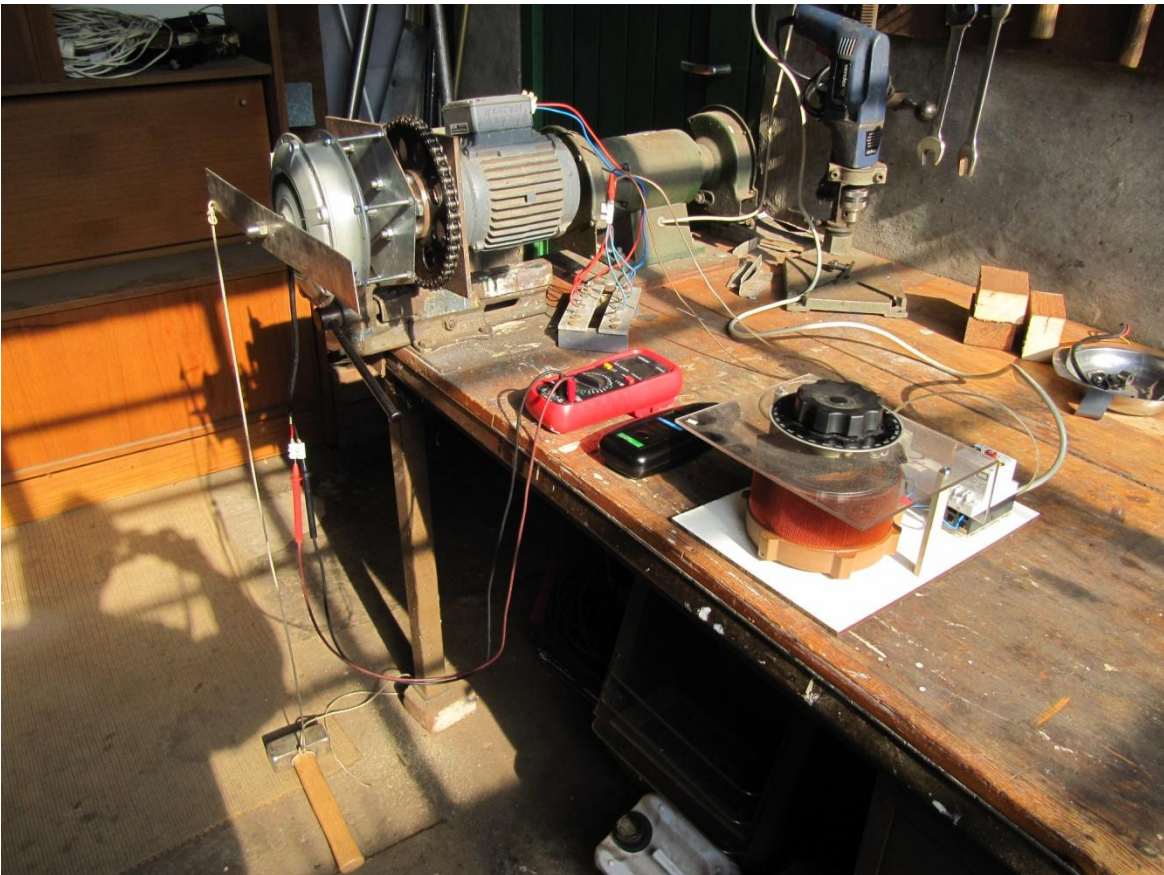


fig. 3 Test rig for measuring of the open voltage

The measured open voltage U_{open} (V) as a function of the rotational speed n (rpm) is given in figure 4.

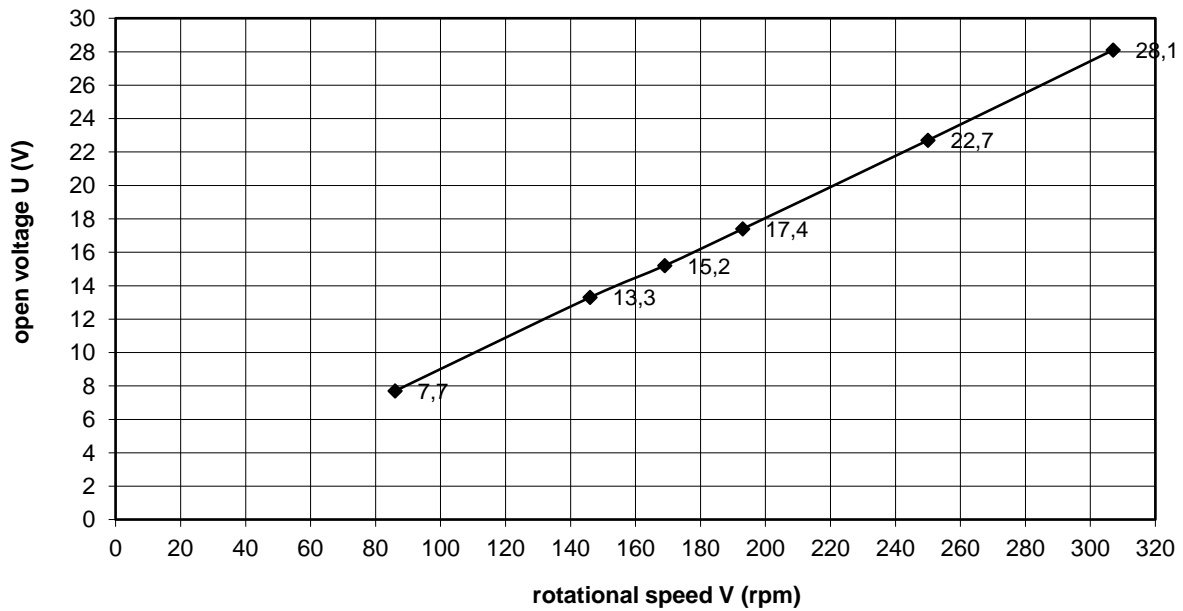


fig. 4 Open voltage U (V) as a function of rotational speed n (rpm) for a standard motor

Charging of a 12 V lead acid battery starts about at an open voltage of 12.5 V. In figure 4 it can be seen that this voltage is reached at a rotational speed of about 135 rpm. So at this rotational speed the $P_{\text{mech}}-n$ curve of the generator suddenly rises. To find the $P_{\text{mech}}-n$ curve for a 12 V battery load, the torque Q has to be measured. This is possible for the described test rig if the weight at the end of the rope is laid on a balance.

However, I stopped the measurements at this point because the very large peak on the cogging torque seems a big problem. The weak shaft may also break if the moment in the shaft is too high and the weak shaft strongly limits the maximum rotor diameter. So my first conclusion was that it is difficult to use this hub motor as a generator for a small wind turbine. Therefore I didn't take the effort to measure the $P_{\text{mech}}-n$ and $P_{\text{el}}-n$ curves for a 12 V battery load.

5 Design of the 4-bladed VIRYA-1.4 rotor with $\lambda_d = 2.5$

My conclusion in chapter 4 was that because of the very high peak on the cogging torque, it is difficult to design a rotor for this Sparta Ion hub motor which has an acceptably low starting wind speed. So in the first instance, use of this motor as generator for a wind turbine, wasn't advised. However, as this used second hand motor is rather cheap, it seems worthwhile to investigate if a proper rotor can be designed. An important limitation is the rather weak generator shaft. So it is assumed that the rotor diameter is only 1.4 m. The windmill with this rotor is therefore called the VIRYA-1.4.

A high starting torque coefficient is only gained if the design tip speed ratio λ_d is chosen rather low. So it is chosen that $\lambda_d = 2.5$. An attendant advantage of this low design tip speed ratio is that the noise level is low. It is chosen that the number of blades $B = 4$. The rotor hub has to be connected to the six 5 mm holes in the front generator flange at a pitch circle of 178 mm. It is chosen that the hub plate is made out of 2 mm thick galvanised or stainless steel sheet size 500 * 500 mm. It is chosen that a blade is made out of 1 mm thick galvanised or stainless steel sheet size 500 * 250 mm and that the blade length is 500 mm. The overlap in between the blade and the hub plate is 50 mm and so the free blade length is 450 mm which is expected to be short enough to prevent flutter in the blades at high wind speeds.

A good airfoil is gained if the chord is 7.14 % cambered. However, this requires a complicated blade press. So it is assumed that the airfoil is formed by a central 150 mm wide flat part and that the both 50 mm wide edges are bent 15° forwards. Manufacture of such an airfoil is rather simple. It is assumed that the aerodynamic characteristics of a 7.14 % cambered airfoil can still be used. The chord c for a 250 mm wide sheet with 15° bent edges is about 246 mm = 0.246 m.

The hub plate has a central square part size 200 * 200 mm which is flat. The six 5 mm holes for the connecting bolts to the generator are made in this flat part. This flat part has a 40 mm central hole for the bulge on the front bearing cover. Six 13 mm long bushes with an inside diameter of 5 mm and an outside diameter of 8 mm are placed in between the hub plate and the generator flange. The hub plate is connected to the generator flange by six stainless steel bolts M5 * 25 and six stainless steel self locking nuts M5.

The hub plate has four 150 mm wide ears. The outer parts are widened to 250 mm for the outer 50 mm. The 150 mm wide part of the ear is twisted 24° right hand to give the blade the correct blade angle at the blade root. The 250 mm wide part has a flat part in the centre but both 50 mm wide outer parts are bent 15° to the front just as it is also done for the blades. A blade is connected to the hub plate by three stainless steel bolts M6 * 10 and three stainless steel self locking nuts M6. The mass of the whole rotor is about 6.5 kg which seems acceptable for a steel rotor with $D = 1.4$ m. The mass of the generator is about 5.5 kg and so the total mass is about 12 kg. A sketch of the rotor is given in figure 5. All parts are drawn flat in this sketch and the measures are given for flat parts.

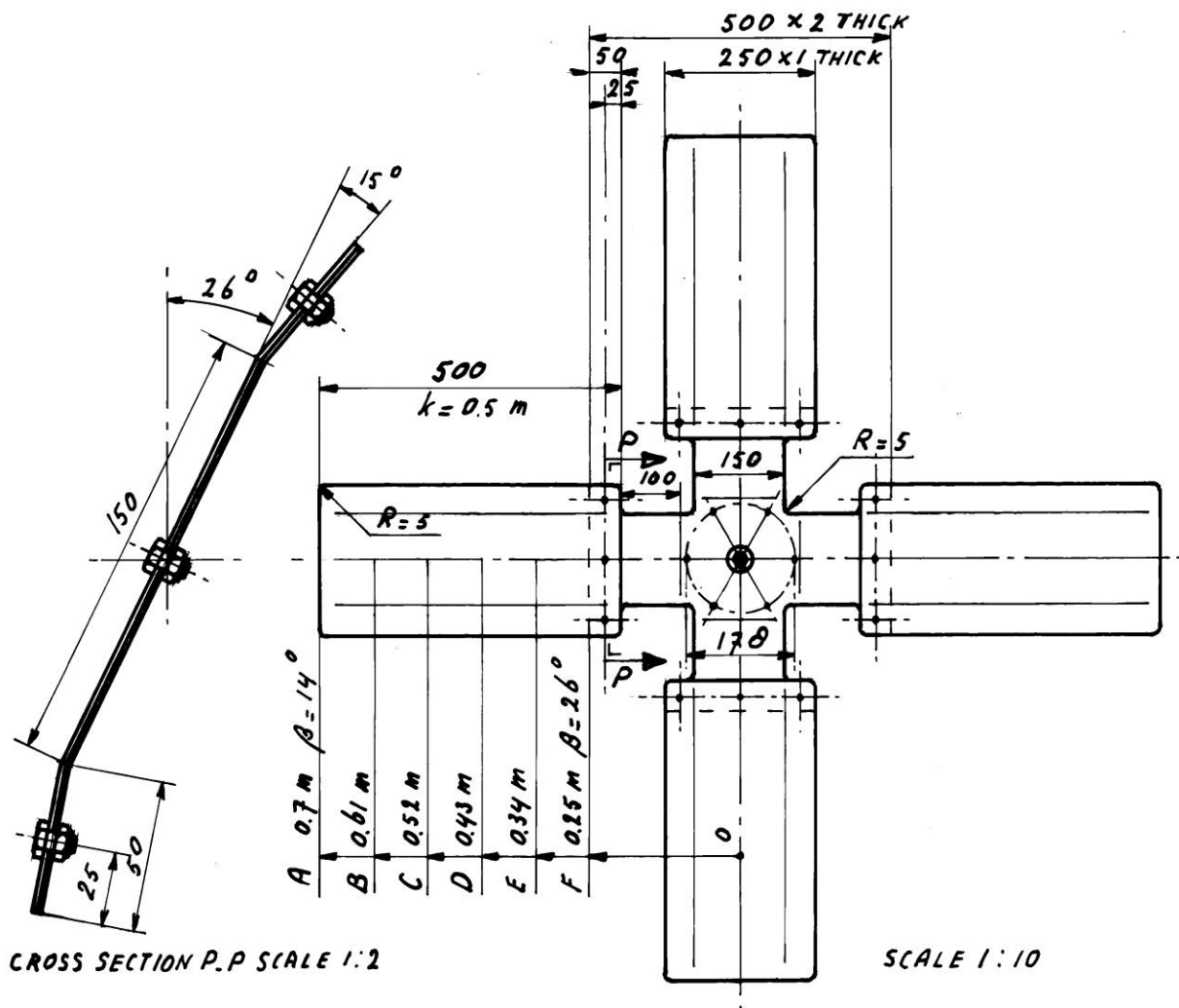


fig. 5 Sketch of the VIRYA-1.4 rotor

The required formulas for the rotor calculations are given in chapter 5 of report KD 35 (ref. 1). Substitution of $\lambda_d = 2.5$ and $R = 0.7$ m in formula (5.1) of KD 35 gives:

$$\lambda_{rd} = 3.5714 * r \quad (-) \quad (1)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \quad (^\circ) \quad (2)$$

$$\phi = 2/3 \arctan 1 / \lambda_{rd} \quad (^\circ) \quad (3)$$

Substitution of $B = 4$ and $c = 0.246$ m in formula (5.4) of KD 35 gives:

$$C_l = 25.541 r (1 - \cos\phi) \quad (-) \quad (4)$$

Substitution of $V = 5$ m/s and $c = 0.246$ m in formula (5.5) of KD 35 gives:

$$R_{e_r} = 0.82 * 10^5 * \sqrt{(\lambda_{rd}^2 + 4/9)} \quad (-) \quad (5)$$

The blade is calculated for six stations A till F which have a distance of 0.09 m of one to another. Station F corresponds to the end of the ear of the hub plate. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l , α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 3.

The aerodynamic characteristics of 7.14 % cambered airfoil are given in figure 1 and 2 of report KD 398 (ref. 4). The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for a good wind regime. Those airfoil Reynolds numbers are used which are lying closest to the calculated values.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	C_{lth} (-)	C_{lin} (-)	$R_{e_r} * 10^{-5}$ V = 5 m/s	$R_e * 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	C_d/C_{lin} (-)
A	0.7	2.5	14.5	0.246	0.57	0.79	2.12	2.5	-0.9	0.5	15.4	14.0	0.038
B	0.61	2.179	16.4	0.246	0.64	0.62	1.87	1.7	0.2	0.0	16.2	16.4	0.061
C	0.52	1.857	18.9	0.246	0.71	0.63	1.62	1.7	0.7	0.1	18.2	18.8	0.060
D	0.43	1.536	22.0	0.246	0.80	0.67	1.37	1.2	1.7	0.8	20.3	21.2	0.038
E	0.34	1.214	26.3	0.246	0.90	0.92	1.14	1.2	2.4	2.7	23.9	23.6	0.030
F	0.25	0.893	32.2	0.246	0.98	1.27	0.91	1.2	3.0	6.2	29.2	26.0	0.049

table 1 Calculation of the blade geometry of the VIRYA-1.4 rotor

The theoretical blade angle β_{th} for stations A up to F varies in between 15.4° and 29.2° . If the blade angle is chosen 14° at the blade tip and 26° at the blade root, the linearised blade angles are lying close to the theoretical values for the most important outer part of the blade. The ears of the hub plate are twisted 26° right hand to get the correct blade angle at the blade root. The blade itself is twisted 12° left hand to get the correct blade angle at the blade tip. A sketch of the rotor and the PM-generator is given in figure 5.

The determination of the C_p - λ and C_q - λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.8 of KD 35 (for $B = 4$) and $\lambda_{opt} = 2.5$ and $C_d/C_l = 0.04$ gives $C_{p_{th}} = 0.43$ (interpolation in between the lines for $C_d/C_l = 0.03$ and $C_d/C_l = 0.05$).

Substitution of $C_{p\ th} = 0.43$, $R = 0.7$ m and blade length $k = 0.5$ m in formula 6.3 of KD 35 gives $C_{p\ max} = 0.395$. However, a blade isn't provided with a real 7.14 % cambered airfoil and this will reduce the maximum C_p . Assume $C_{p\ max} = 0.38$.

$C_{q\ opt} = C_{p\ max} / \lambda_{opt} = 0.38 / 2.5 = 0.152$. Substitution of $\lambda_{opt} = \lambda_d = 2.5$ in formula 6.4 of KD 35 gives $\lambda_{unl} = 4$.

The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{q\ start} = 0.75 * B * (R - \frac{1}{2}k) * C_1 * c * k / \pi R^3 \quad (-) \quad (6)$$

The average blade angle is 20° . For a non rotating rotor, the average angle of attack α is therefore $90^\circ - 20^\circ = 70^\circ$. The C_1 - α curve for large values of α is given as figure 5 of KD 398 for 10 % camber. It is assumed that this figure can also be used for 7.14 % camber if the angle of attack is very large. For $\alpha = 70^\circ$ it can be read that $C_1 = 0.67$.

Substitution of $B = 4$, $R = 0.7$ m, $k = 0.5$ m, $C_1 = 0.67$ and $c = 0.246$ m in formula 6 gives that $C_{q\ start} = 0.103$. For the ratio in between the starting torque and the optimum torque we find that it is $0.103 / 0.152 = 0.68$. This is rather high for a rotor with a design tip speed ratio of 2.5. The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{start} = \sqrt{\left(\frac{Q_s}{C_{q\ start} * \frac{1}{2}\rho * \pi R^3} \right)} \quad (m/s) \quad (7)$$

For the peak on the sticking or cogging torque Q_s it has been measured that $Q_s = 0.692$ Nm. Substitution of $Q_s = 0.692$ Nm, $C_{q\ start} = 0.103$, $\rho = 1.2$ kg/m³ and $R = 0.7$ m in formula 7 gives that $V_{start} = 3.2$ m/s. This seems acceptable for regions with a moderate wind regime.

Once the rotor is started, it will stop at a much lower wind speed than the starting wind speed of 3.2 m/s. This has two reasons. The first reason is that now the rotor is working as a flywheel and therefore only the average cogging torque has to be supplied. It has been measured that this average torque is 0.315 Nm for low rotational speeds. However, during these measurements only the generator housing was working as a flywheel. The moment of inertia is much larger if the windmill rotor is connected to the generator and the real average cogging torque will therefore be lower than 0.315 Nm. Assume $Q_s = 0.25$ Nm. The second reason is that the rotor stops only if the peak torque of the rotor is lower than the average generator torque. The peak torque is gained for the peak value of the torque coefficient $C_{q\ peak}$. In figure 8 it can be seen that $C_{q\ peak} = 0.1675$ for $\lambda = 2$. Formula 7 can also be used to calculate the stopping wind speed V_{stop} but only V_{start} has to be changed by V_{stop} and $C_{q\ start}$ has to be changed by $C_{q\ peak}$. Substitution of $Q_s = 0.25$ Nm, $C_{q\ peak} = 0.1675$, $\rho = 1.2$ kg/m³ and $R = 0.7$ m in formula 7 gives that $V_{stop} = 1.5$ m/s. So the starting behaviour seems acceptable for this 4-bladed rotor with $D = 1.4$ m and $\lambda_d = 2.5$ in combination with the tested Sparta Ion wheel motor. But to be sure, it must be tested and I won't do that.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p - λ and C_q - λ curves can be determined if only two points of the C_p - λ curve and one point of the C_q - λ curve are known. The first part of the C_q - λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for $\lambda = 0$. Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 5). With this method, it can be determined that the C_q - λ curve is rising slowly for low values of λ if a 7.14 % cambered airfoil is used. This effect has been taken into account and the estimated C_p - λ and C_q - λ curves for the VIRYA-1.4 rotor are given in figure 6 and 7.

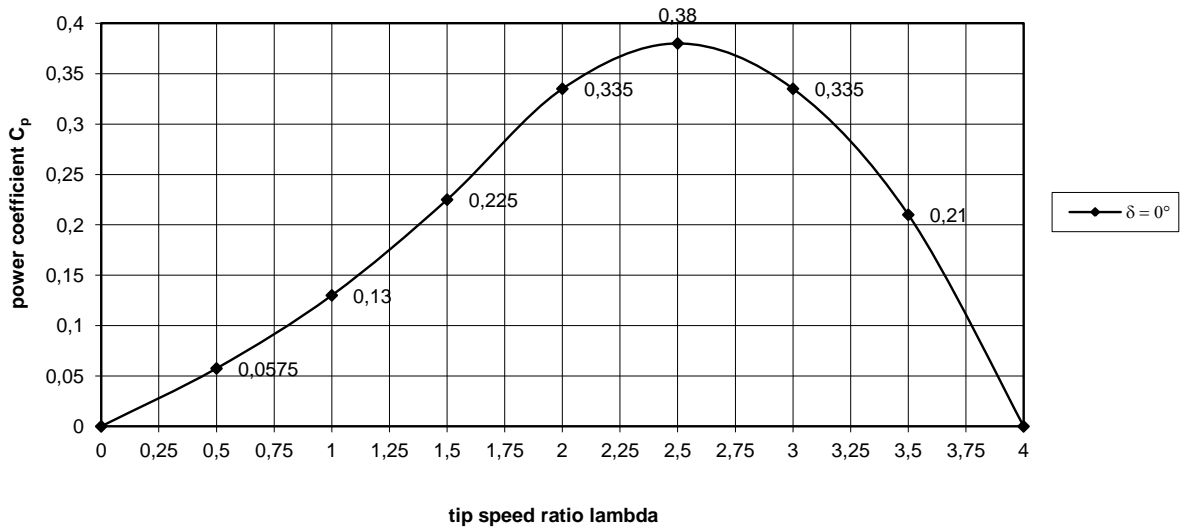


fig. 6 Estimated C_p - λ curve for the VIRYA-1.4 rotor for the wind direction perpendicular to the rotor ($\delta = 0^\circ$)

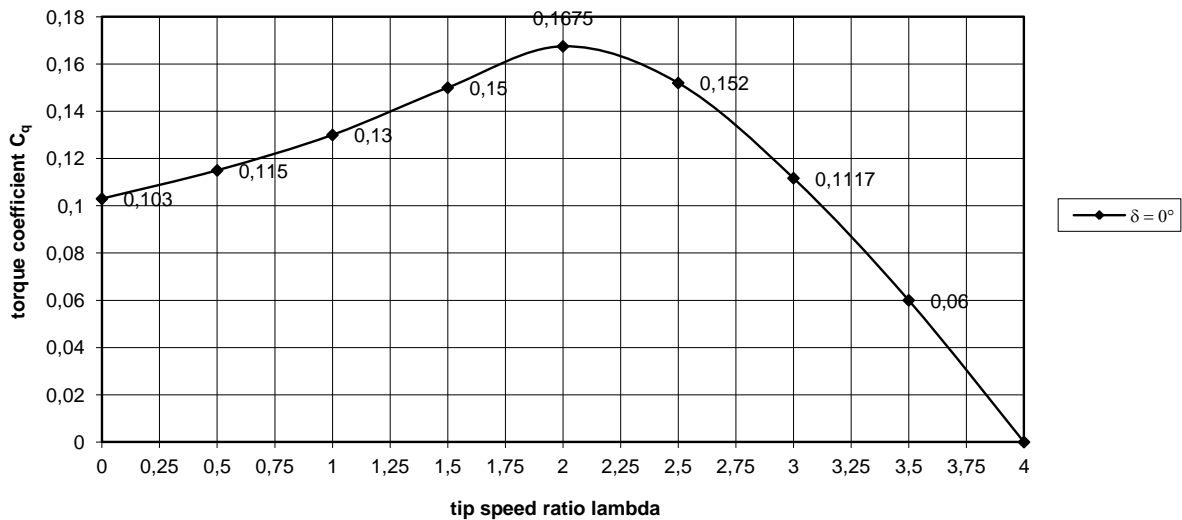


fig. 7 Estimated C_q - λ curve for the VIRYA-1.4 rotor for the wind direction perpendicular to the rotor ($\delta = 0^\circ$)

The C_p - λ curve can be used to derive the P-n curves for different wind speeds but for higher wind speeds one needs the δ -V curve of the chosen safety system. Assume that the head of the VIRYA-1.4 is derived from the head of the former VIRYA-1.8. Drawings of the head and tower of the VIRYA-1.8 are given in the manual of the VIRYA-1.81. This head is strong enough for a total mass of rotor and generator of 12 kg. The VIRYA-1.8 is provided with a 1 mm stainless steel vane blade resulting in a rated wind speed of about 11 m/s. This is too high for the VIRYA-1.4 and so this vane blade is replaced by a 2 mm aluminium vane blade. This results in a rated wind speed of about 9 m/s. The VIRYA-1.8 has an eccentricity $e = 0.15$ m. The rotor area of the VIRYA-1.8 is a factor $(1.8 / 1.4)^2 = 1.65$ larger than of that of the VIRYA-1.4. So the rotor thrust is also about a factor 1.65 larger. To make that the rotor thrust of the VIRYA-1.4 gives about the same moment around the tower axis, the eccentricity e therefore has to be increased by a factor 1.65 and becomes $1.65 * 0.15 = 0.25$ m.

So the generator bracket has to be modified such that the eccentricity is now 0.25 m and such that the shaft of the generator can be mounted. This means that the strip of the generator bracket must now be connected to the head pipe such that it is in parallel to the rotor plane and that it is provided with a hole for the generator shaft at the correct place. The estimated δ -V curve is given in figure 8.

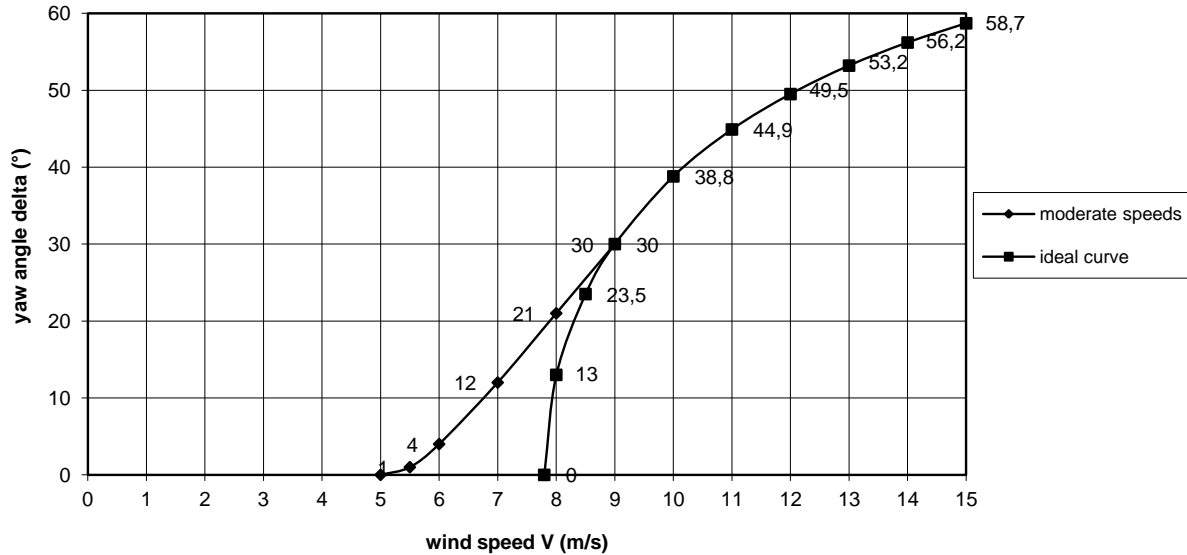


fig. 8 Estimated δ -V curve VIRYA-1.4 for a 2 mm aluminium vane blade

The rotor starts turning out of the wind at a wind speed of about 5 m/s. The ideal curve is followed for wind speeds higher than 9 m/s which means that the P-n curve of the rotor for $V = 9$ m/s is also valid for higher wind speeds than 9 m/s.

The P-n curves are used to check the matching with the P_{mech} -n curve of the generator for a certain gear ratio i (the VIRYA-1.4 has no gearing so $i = 1$). The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8 and 9 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of $R = 0.7$ m in formula 7.1 of KD 35 gives:

$$n_{\delta} = 13.642 * \lambda * \cos\delta * V \quad (\text{rpm}) \quad (8)$$

Substitution of $\rho = 1.2 \text{ kg} / \text{m}^3$ and $R = 0.7$ m in formula 7.10 of KD 35 gives:

$$P_{\delta} = 0.9236 * C_p * \cos^3\delta * V^3 \quad (\text{W}) \quad (9)$$

The P-n curves are determined for C_p values belonging to $\lambda = 1, 1.5, 2, 2.5, 3, 3.5$ and 4 (see figure 6). For a certain wind speed, for instance $V = 3$ m/s, related values of C_p and λ are substituted in formula 8 and 9 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 8, is taken into account. The result of the calculations is given in table 2.

λ	C_p	V = 3 m/s $\delta = 0^\circ$		V = 4 m/s $\delta = 0^\circ$		V = 5 m/s $\delta = 0^\circ$		V = 6 m/s $\delta = 4^\circ$		V = 7 m/s $\delta = 12^\circ$		V = 8 m/s $\delta = 21^\circ$		V = 9 m/s $\delta = 30^\circ$	
		n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_s (rpm)	P_s (W)	n_s (rpm)	P_s (W)	n_s (rpm)	P_s (W)	n_s (rpm)	P_s (W)
1	0.13	40.9	3.24	54.6	7.68	68.2	15.01	81.7	25.75	93.4	38.54	101.9	50.02	106.3	56.85
1.5	0.225	61.4	5.61	81.9	13.30	102.3	25.98	122.5	44.56	140.1	66.71	152.8	86.57	159.5	98.40
2	0.335	81.9	8.35	109.1	19.80	136.4	38.68	163.3	66.34	186.8	99.32	203.8	128.90	212.7	146.50
2.5	0.38	102.3	9.48	136.4	22.46	170.5	43.87	204.1	75.26	233.5	112.66	254.7	146.22	265.8	166.18
3	0.335	122.8	8.35	163.7	19.80	204.6	38.68	245.0	66.34	280.2	99.32	305.7	128.90	319.0	146.50
3.5	0.21	143.2	5.24	191.0	12.41	238.7	24.24	285.8	41.59	326.9	62.26	356.6	80.80	372.2	91.84
4	0	163.7	0	218.3	0	272.8	0	326.6	0	373.6	0	407.5	0	425.3	0

table 2 Calculated values of n and P as a function of λ and V for the VIRYA-1.4 rotor

The calculated values for n and P are plotted in figure 9. The optimum cubic line which can be drawn through the maximum of all P-n curves is also given in figure 9.

Unfortunately no measurements have been performed for a 12 V battery load. So the P_{mech} -n and the P_{el} -n curves have not been measured and therefore it isn't possible to check the matching in between rotor and generator and to determine the P_{el} -V curve. However, the unloaded voltage has been measured as a function of the rotational speed and is given in figure 4. Assume that charging of a 12 V lead acid battery starts if the open voltage is 12.5 V. In figure 4 it can be seen that this voltage is gained at a rotational speed of about 135 rpm. In figure 9 it can be seen that this rotational speed is gained for an unloaded rotor, so for $\lambda = 4$, at a wind speed below 3 m/s. It can be calculated that this unloaded rotational speed is already gained at a cut-in wind speed of 2.5 m/s (if the rotor has started). So the P_{mech} -n curve for a 12 V battery load will increase suddenly above $n = 135$ rpm and therefore it can be expected that the matching with the optimum cubic line of the rotor will be rather good.

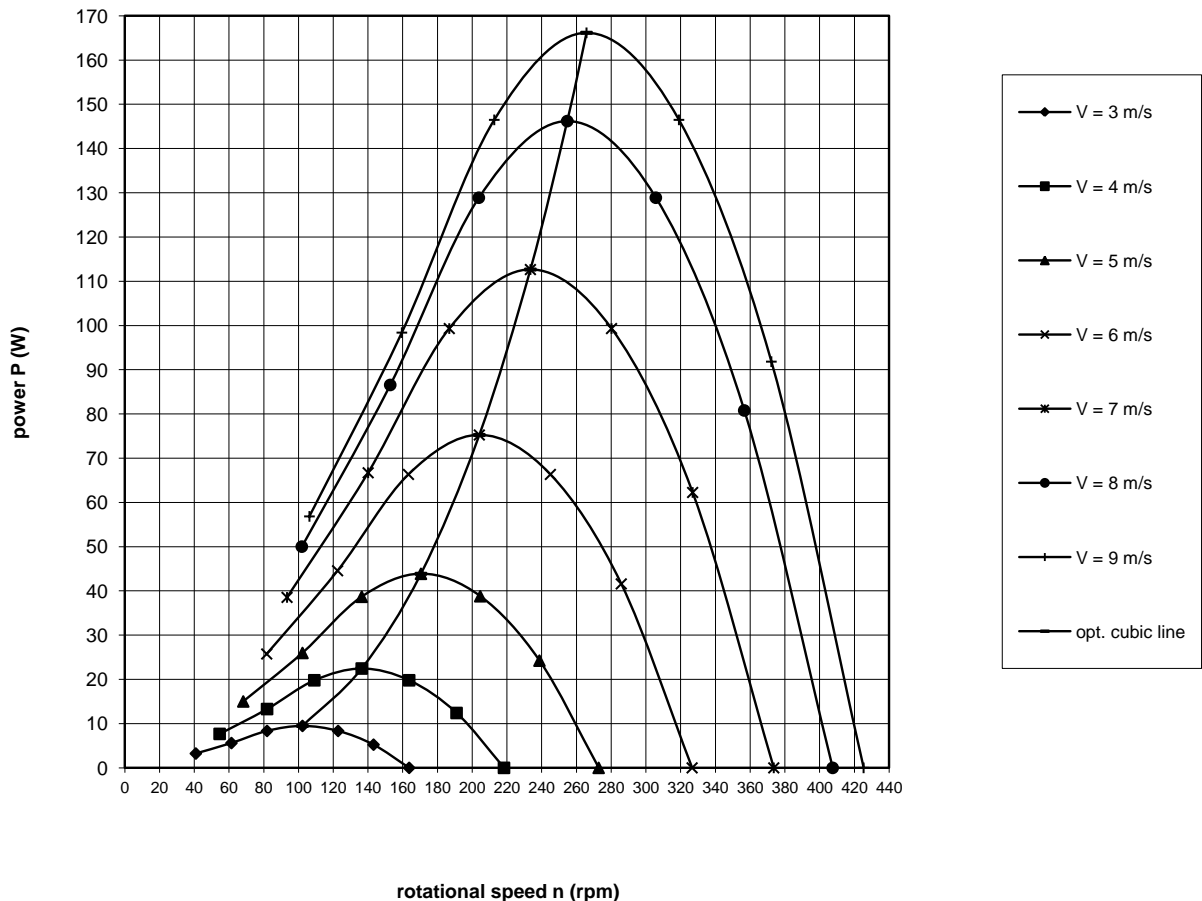


fig. 9 P-n curves of the VIRYA-1.4 rotor and optimum cubic line

If the matching is good at a wind speed of 9 m/s and if the generator efficiency is about 50 % at maximum power, the maximum electrical power will be about 75 W. This is certainly acceptable and relatively much higher than the maximum power of 6 W of the VIRYA-1.04 or the VIRYA-0.98 which make use of a Nexus hub dynamo from a bicycle wheel. So it seems worthwhile to build a rotor according to the sketch of figure 5 and to test this rotor with the generator. However, I won't do that.

6 Design of the 3-bladed VIRYA-1.38 rotor with $\lambda_d = 2.5$

The 4-bladed VIRYA-1.4 rotor as designed in chapter 5 has as disadvantages that manufacture of the hub plate is rather complicated and that a substantial part of the used square stainless steel sheet is wasted. It is possible to reduce the mass by connecting each blade to the generator by a separate spoke. However, as the generator has six, 5 mm holes at a pitch circle of 178 mm, it is logic to use a 3-bladed rotor.

A blade is made out of a 1 mm thick stainless steel sheet size 500 * 333.3 * 1 mm and 12 sheets can be made from a standard sheet size 1 * 2 m. The corners are rounded with $r = 5$ mm. A spoke is made out of a 2 mm thick stainless steel sheet size 200 * 200 * 2 mm and 50 sheets can be made from a standard sheet size 1 * 2 m. The overlap in between a blade and a spoke is 40 mm. The distance in between the inner side of a spoke and the heart of the generator is 30 mm. This give a rotor radius R of $500 + 200 + 30 - 40 = 690$ mm = 0.69 m. So the rotor diameter is $2 * 0.69 = 1.38$ m and the windmill is therefore called the VIRYA-1.38.

A blade is 7.14 % cambered and the chord is therefore is little smaller than the sheet width. This results in a chord of 329 mm = 0.329 m. A blade is connected to the front side of a spoke. A spoke is twisted 27° right hand in between the blade and the generator to give the blade the correct blade angle at the blade root.

A high starting torque coefficient is only gained if the design tip speed ratio λ_d is chosen rather low. So it is chosen that $\lambda_d = 2.5$ for the VIRYA-1.38. The overlap in between a blade and a spoke is 40 mm and so the free blade length is $500 - 40 = 460$ mm which is expected to be short enough for a rotor with a design tip speed ratio of 2.5 and a blade thickness of 1 mm to prevent flutter in the blades at high wind speeds.

A blade is 7.14 % cambered. It might be possible to camber the blade with a small standard roll bender and to get the correct blade twist by giving the sheet sides a certain angle with the axis of the rolls. Another option might be to fold the blade around a cylinder with the correct diameter. Characteristics of a 7.14 % cambered plate are given in report KD 398 (ref. 4).

The spoke width is only 200 mm at the inner and the outer side. The width at the middle part is reduced to 150 mm otherwise twisting of the spokes is difficult. The inside corners are rounded with $r = 5$ mm. A spoke is flat at the inner side and cambered for the 40 mm wide outer side with the same radius as used for the blade. The middle part is twisted over a length of 90 mm to give the blade the correct blade angle of 27° at the blade root. Six 10 mm long bushes with an inside diameter of 5 mm and an outside diameter of 8 mm are placed in between the three overlapping spokes and the generator flange.

A spoke is connected to the generator flange by three stainless steel bolts M5 * 25 and three stainless steel self locking nuts M5. A blade is connected to a spoke by three stainless steel bolts M5 * 10 and three stainless steel self locking nuts M5. The mass of the whole rotor is about 5.5 kg which is 1 kg lighter than that of the VIRYA-1.4 rotor. The mass of the generator is about 5.5 kg and so the total mass is about 11 kg which seems acceptable.

Three blades of the VIRYA-1.38 rotor have the same mass as four blades of the VIRYA-1.4 rotor. The three spokes of the VIRYA-1.38 rotor are about 1 kg lighter than the hub plate of the VIRYA-1.4 rotor. However, for the hub plate of the VIRYA-1.4 rotor, a sheet size 500 * 500 * 2 mm is needed. This sheet has a mass of 3.9 kg. Three sheets size 200 * 200 * 2 mm are needed for the spokes of the VIRYA-1.38. These sheets have a total mass of 1.87 kg. So the difference in needed stainless steel is $3.9 - 1.87 = 2.03$ kg.

A sketch of the rotor is given in figure 10. All parts are drawn flat in this sketch.

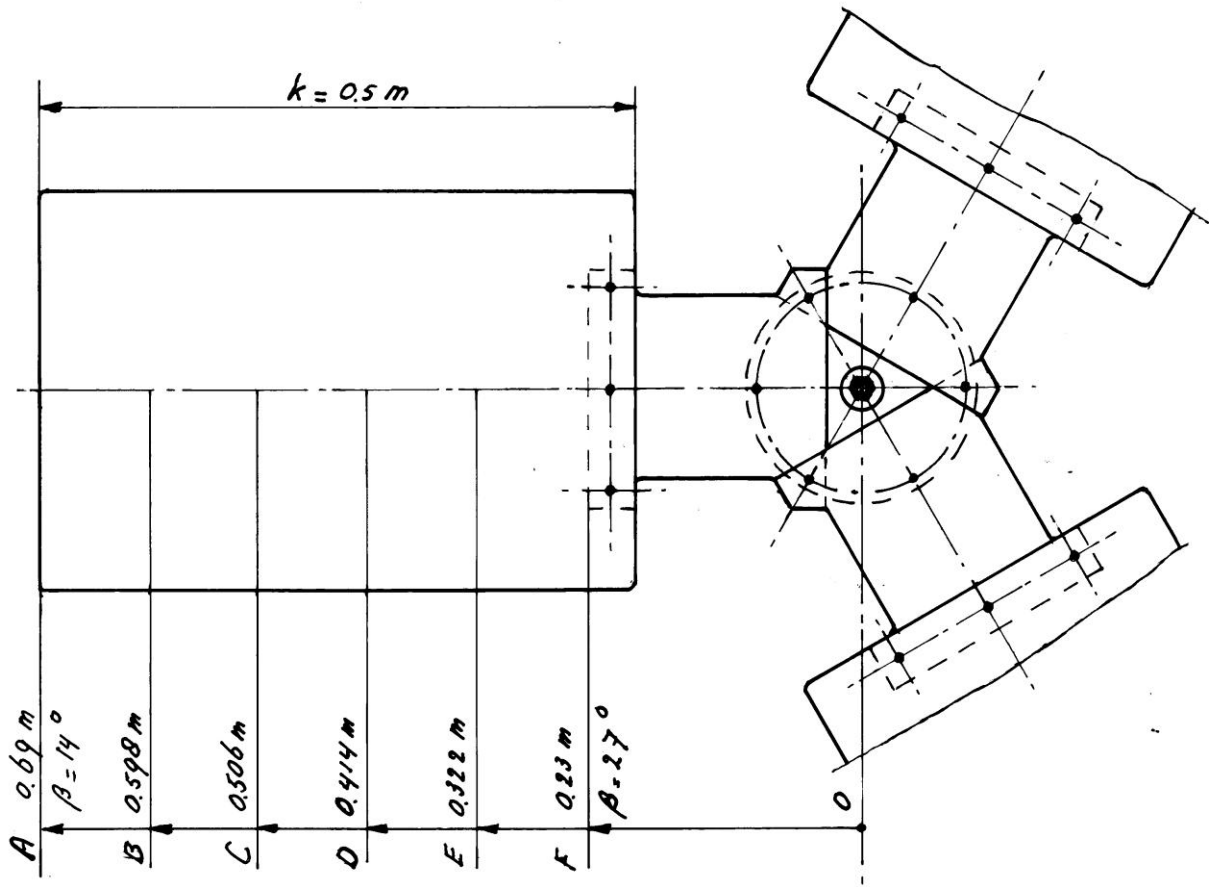


fig. 10 Sketch of the VIRYA-1.38 rotor

The required formulas for the rotor calculations are given in chapter 5 of report KD 35 (ref. 1). Substitution of $\lambda_d = 2.5$ and $R = 0.69$ m in formula (5.1) of KD 35 gives:

$$\lambda_{rd} = 3.6232 * r \quad (-) \quad (10)$$

Formula's (5.2) and (5.3) of KD 35 stay the same so:

$$\beta = \phi - \alpha \quad (^\circ) \quad (11)$$

$$\phi = 2/3 \arctan 1 / \lambda_{rd} \quad (^\circ) \quad (12)$$

Substitution of $B = 3$ and $c = 0.329$ m in formula (5.4) of KD 35 gives:

$$C_1 = 25.464 r (1 - \cos\phi) \quad (-) \quad (13)$$

Substitution of $V = 5$ m/s and $c = 0.329$ m in formula (5.5) of KD 35 gives:

$$R_{er} = 1.097 * 10^5 * \sqrt{(\lambda_{rd}^2 + 4/9)} \quad (-) \quad (14)$$

The blade is calculated for six stations A till F which have a distance of 0.092 m of one to another. Station F corresponds to the end of a spoke. The blade has a constant chord and the calculations therefore correspond with the example as given in chapter 5.4.2 of KD 35. This means that the blade is designed with a low lift coefficient at the tip and with a high lift coefficient at the root. First the theoretical values are determined for C_l , α and β and next β is linearised such that the twist is constant and that the linearised values for the outer part of the blade correspond as good as possible with the theoretical values. The result of the calculations is given in table 3.

The aerodynamic characteristics of 7.14 % cambered airfoil are given in figure 1 and 2 of report KD 398 (ref. 4). The Reynolds values for the stations are calculated for a wind speed of 5 m/s because this is a reasonable wind speed for a good wind regime. Those airfoil Reynolds numbers are used which are lying closest to the calculated values.

station	r (m)	λ_{rd} (-)	ϕ (°)	c (m)	C_{lth} (-)	C_{lin} (-)	$Re_r * 10^{-5}$ V = 5 m/s	$Re^* 10^{-5}$ 7.14 %	α_{th} (°)	α_{lin} (°)	β_{th} (°)	β_{lin} (°)	C_d/C_{lin} (-)
A	0.69	2.5	14.5	0.329	0.56	0.79	2.84	2.5	-0.9	0.5	15.4	14	0.036
B	0.598	2.167	16.5	0.329	0.63	0.71	2.49	2.5	-0.5	-0.1	17.0	16.6	0.041
C	0.506	1.833	19.1	0.329	0.71	0.71	2.14	2.5	-0.1	-0.1	19.2	19.2	0.041
D	0.414	1.5	22.5	0.329	0.80	0.72	1.80	1.7	1.5	0.7	21.0	21.8	0.046
E	0.322	1.167	27.1	0.329	0.90	0.92	1.47	1.7	2.5	2.7	24.6	24.4	0.041
F	0.23	0.833	33.5	0.329	0.97	1.28	1.17	1.2	2.9	6.5	30.6	27	0.050

table 3 Calculation of the blade geometry of the VIRYA-1.38 rotor

The theoretical blade angle β_{th} for stations A up to F varies in between 15.4° and 30.6° . If the blade angle is chosen 14° at the blade tip and 27° at the blade root, the linearised blade angles are lying close to the theoretical values for the most important outer part of the blade. The spokes are twisted 27° right hand to get the correct blade angle at the blade root. The blade itself is twisted 13° left hand to get the correct blade angle at the blade tip. A sketch of the rotor and the PM-generator is given in figure 10.

The determination of the C_p - λ and C_q - λ curves is given in chapter 6 of KD 35. The average C_d/C_l ratio for the most important outer part of the blade is about 0.04. Figure 4.7 of KD 35 (for $B = 3$) and $\lambda_{opt} = 2.5$ and $C_d/C_l = 0.04$ gives $C_{pth} = 0.42$ (interpolation in between the lines for $C_d/C_l = 0.03$ and $C_d/C_l = 0.05$).

Substitution of $C_{pth} = 0.42$, $R = 0.69$ m and blade length $k = 0.5$ m in formula 6.3 of KD 35 gives $C_{pmax} = 0.39$. This is a little higher than for the VIRYA-1.4 rotor for which it was found that $C_{pmax} = 0.38$. The reasons are that a real 7.14 % cambered airfoil is used for the VIRYA-1.38 and that the Reynolds values are higher because of the larger chord.

$C_{qopt} = C_{pmax} / \lambda_{opt} = 0.39 / 2.5 = 0.156$. Substitution of $\lambda_{opt} = \lambda_d = 2.5$ in formula 6.4 of KD 35 gives $\lambda_{unl} = 4$. The starting torque coefficient is calculated with formula 6.12 of KD 35 which is given by:

$$C_{qstart} = 0.75 * B * (R - \frac{1}{2}k) * C_l * c * k / \pi R^3 \quad (-) \quad (15)$$

The average blade angle is 20.5° . For a non rotating rotor, the average angle of attack α is therefore $90^\circ - 20.5^\circ = 69.5^\circ$. The C_l - α curve for large values of α is given as figure 5 of KD 398 for 10 % camber. It is assumed that this figure can also be used for 7.14 % camber if the angle of attack is very large. For $\alpha = 69.5^\circ$ it can be read that $C_l = 0.69$.

Substitution of $B = 3$, $R = 0.69$ m, $k = 0.5$ m, $C_l = 0.69$ and $c = 0.329$ m in formula 15 gives that $C_{qstart} = 0.109$. For the ratio in between the starting torque and the optimum torque we find that it is $0.109 / 0.156 = 0.7$. This is rather high for a rotor with a design tip speed ratio of 2.5.

The starting wind speed V_{start} of the rotor is calculated with formula 8.6 of KD 35 which is given by:

$$V_{\text{start}} = \sqrt{\left(\frac{Q_s}{C_{q \text{ start}} * \frac{1}{2}\rho * \pi R^3} \right)} \quad (\text{m/s}) \quad (16)$$

For the peak on the sticking or cogging torque Q_s it has been measured that $Q_s = 0.692 \text{ Nm}$. Substitution of $Q_s = 0.692 \text{ Nm}$, $C_{q \text{ start}} = 0.109$, $\rho = 1.2 \text{ kg/m}^3$ and $R = 0.69 \text{ m}$ in formula 16 gives that $V_{\text{start}} = 3.2 \text{ m/s}$. This is the same as for the VIRYA-1.4 rotor for which it was also found that $V_{\text{start}} = 3.2 \text{ m/s}$. This seems acceptable for regions with a moderate wind regime.

Once the rotor is started, it will stop at a much lower wind speed than the starting wind speed of 3.2 m/s . This has two reasons. The first reason is that now the rotor is working as a flywheel and therefore only the average cogging torque has to be supplied. It has been measured that this average torque is 0.315 Nm for low rotational speeds. However, during these measurements only the generator housing was working as a flywheel. The moment of inertia is much larger if the windmill rotor is connected to the generator and the real average cogging torque will therefore be lower than 0.315 Nm . Assume $Q_s = 0.25 \text{ Nm}$. The second reason is that the rotor stops only if the peak torque of the rotor is lower than the average generator torque. The peak torque is gained for the peak value of the torque coefficient $C_{q \text{ peak}}$. In figure 12 it can be seen that $C_{q \text{ peak}} = 0.1725$ for $\lambda = 2$. Formula 16 can also be used to calculate the stopping wind speed V_{stop} but only V_{start} has to be changed by V_{stop} and $C_{q \text{ start}}$ has to be changed by $C_{q \text{ peak}}$. Substitution of $Q_s = 0.25 \text{ Nm}$, $C_{q \text{ peak}} = 0.1725$, $\rho = 1.2 \text{ kg/m}^3$ and $R = 0.69 \text{ m}$ in formula 16 gives that $V_{\text{stop}} = 1.5 \text{ m/s}$. So the starting behaviour seems acceptable for this 3-bladed rotor with $D = 1.38 \text{ m}$ and $\lambda_d = 2.5$ in combination with the tested Sparta Ion wheel motor. But to be sure, it must be tested and I won't do that.

In chapter 6.4 of KD 35 it is explained how rather accurate C_p - λ and C_q - λ curves can be determined if only two points of the C_p - λ curve and one point of the C_q - λ curve are known. The first part of the C_q - λ curve is determined according to KD 35 by drawing an S-shaped line which is horizontal for $\lambda = 0$. Kragten Design developed a method with which the value of C_q for low values of λ can be determined (see report KD 97 ref. 5). With this method, it can be determined that the C_q - λ curve is rising slowly for low values of λ if a 7.14 % cambered airfoil is used. This effect has been taken into account and the estimated C_p - λ and C_q - λ curves for the VIRYA-1.38 rotor are given in figure 11 and 12.

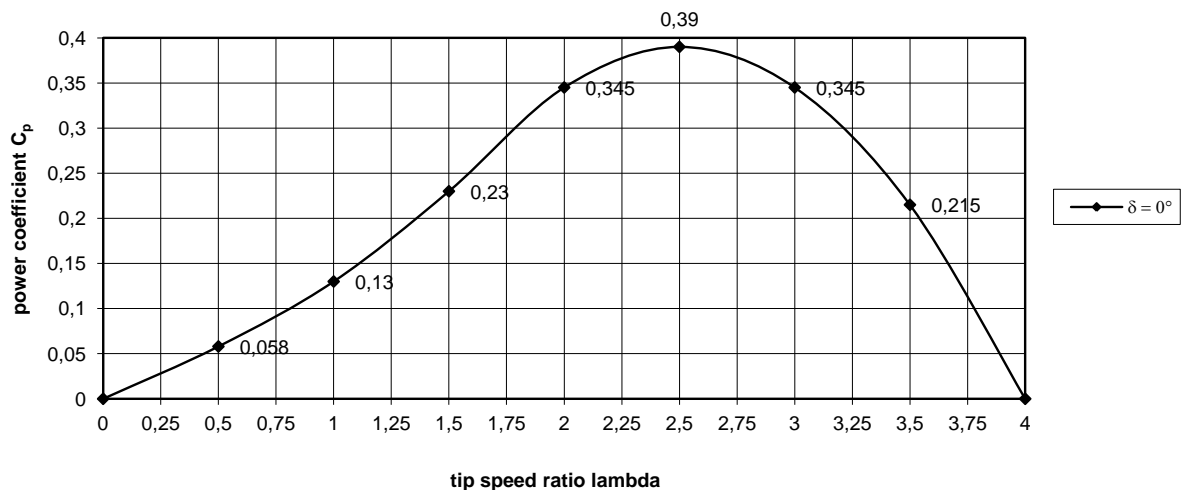


fig. 11 Estimated C_p - λ curve for the VIRYA-1.38 rotor for the wind direction perpendicular to the rotor ($\delta = 0^\circ$)

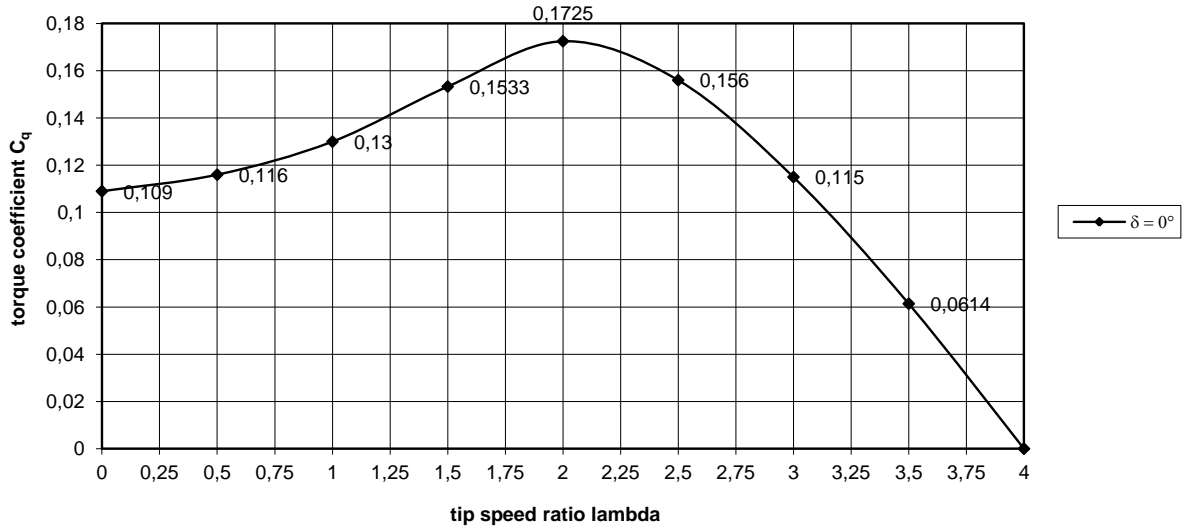


fig. 12 Estimated C_q - λ curve for the VIRYA-1.38 rotor for the wind direction perpendicular to the rotor ($\delta = 0^\circ$)

The C_p - λ curve can be used to derive the P-n curves for different wind speeds but for higher wind speeds one needs the δ -V curve of the chosen safety system. Assume that the same head is used as the head which is used for the VIRYA-1.4. This head is derived from the head of the VIRYA-1.8 by increasing the eccentricity e . Assume that the δ -V curve of the VIRYA-1.38 in combination with this head is the same as the curve given in figure 8.

The rotor starts turning out of the wind at a wind speed of about 5 m/s. The ideal curve is followed for wind speeds higher than 9 m/s which means that the P-n curve of the rotor for $V = 9$ m/s is also valid for higher wind speeds than 9 m/s.

The P-n curves are used to check the matching with the P_{mech} -n curve of the generator for a certain gear ratio i (the VIRYA-1.38 has no gearing so $i = 1$). The P-n curves are determined for wind the speeds 3, 4, 5, 6, 7, 8 and 9 m/s. At high wind speeds the rotor is turned out of the wind by a yaw angle δ and therefore the formulas for P and n are used which are given in chapter 7 of KD 35.

Substitution of $R = 0.69$ m in formula 7.1 of KD 35 gives:

$$n_\delta = 13.840 * \lambda * \cos\delta * V \quad (\text{rpm}) \quad (17)$$

Substitution of $\rho = 1.2$ kg / m³ and $R = 0.69$ m in formula 7.10 of KD 35 gives:

$$P_\delta = 0.8974 * C_p * \cos^3\delta * V^3 \quad (\text{W}) \quad (18)$$

The P-n curves are determined for C_p values belonging to $\lambda = 1, 1.5, 2, 2.5, 3, 3.5$ and 4 (see figure 11). For a certain wind speed, for instance $V = 3$ m/s, related values of C_p and λ are substituted in formula 17 and 18 and this gives the P-n curve for that wind speed. For the higher wind speeds the yaw angle as given by figure 13, is taken into account. The result of the calculations is given in table 4.

λ	C_p	V = 3 m/s $\delta = 0^\circ$		V = 4 m/s $\delta = 0^\circ$		V = 5 m/s $\delta = 0^\circ$		V = 6 m/s $\delta = 4^\circ$		V = 7 m/s $\delta = 12^\circ$		V = 8 m/s $\delta = 21^\circ$		V = 9 m/s $\delta = 30^\circ$	
		n (rpm)	P (W)	n (rpm)	P (W)	n (rpm)	P (W)	n_s (rpm)	P_s (W)	n_s (rpm)	P_s (W)	n_s (rpm)	P_s (W)	n_s (rpm)	P_s (W)
1	0.13	41.5	3.15	55.4	7.47	69.2	14.58	82.8	25.02	94.8	37.45	103.4	48.60	107.9	55.24
1.5	0.23	62.3	5.57	83.0	13.21	103.8	25.80	124.3	44.26	142.1	66.26	155.0	85.99	161.8	97.73
2	0.345	83.0	8.36	110.7	19.81	138.4	38.70	165.7	66.39	189.5	99.38	206.7	128.98	215.7	146.60
2.5	0.39	103.8	9.45	138.4	22.40	173.0	43.75	207.1	75.05	236.9	112.35	258.4	145.81	269.7	165.72
3	0.345	124.6	8.36	166.1	19.81	207.6	38.70	248.5	66.39	284.3	99.38	310.1	128.98	323.6	146.60
3.5	0.215	145.3	5.21	193.8	12.35	242.2	24.12	289.9	41.37	331.7	61.93	361.8	80.38	377.6	91.36
4	0	166.1	0	221.4	0	276.8	0	331.4	0	379.1	0	413.5	0	431.5	0

table 4 Calculated values of n and P as a function of λ and V for the VIRYA-1.38 rotor

The calculated values for n and P are plotted in figure 13. The optimum cubic line which can be drawn through the maximum of all P-n curves is also given in figure 13.

Unfortunately no measurements have been performed for a 12 V battery load. So the $P_{\text{mech}}-n$ and the $P_{\text{el}}-n$ curves have not been measured and therefore it isn't possible to check the matching in between rotor and generator and to determine the $P_{\text{el}}-V$ curve. However, the unloaded voltage has been measured as a function of the rotational speed and is given in figure 4. Assume that charging of a 12 V lead acid battery starts if the open voltage is 12.5 V. In figure 4 it can be seen that this voltage is gained at a rotational speed of about 135 rpm. In figure 13 it can be seen that this rotational speed is gained for an unloaded rotor, so for $\lambda = 4$, at a wind speed below 3 m/s. It can be calculated that this unloaded rotational speed is already gained at a cut-in wind speed of 2.5 m/s (if the rotor has started). So the $P_{\text{mech}}-n$ curve for a 12 V battery load will increase suddenly above $n = 135$ rpm and therefore it can be expected that the matching with the optimum cubic line of the rotor will be rather good.

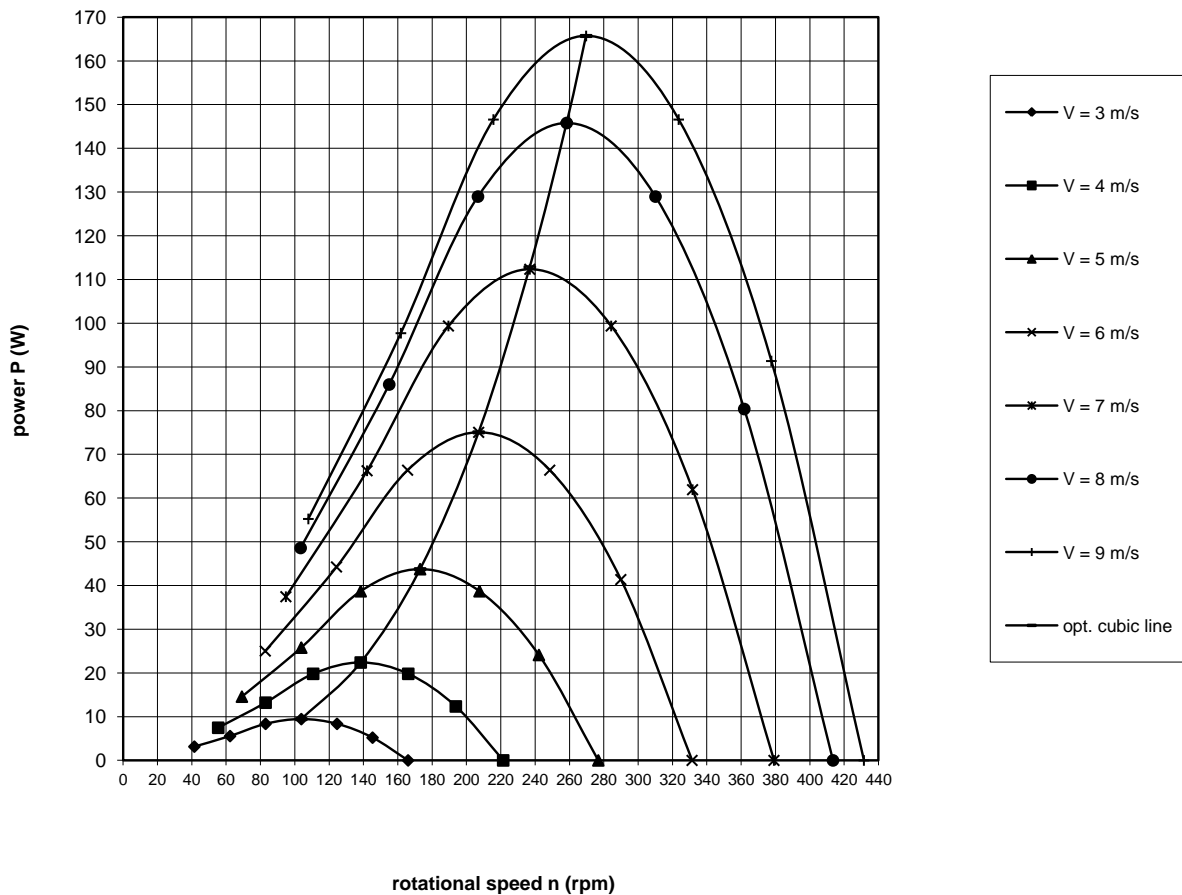


fig. 13 P-n curves of the VIRYA-1.38 rotor and optimum cubic line

If figure 13 is compared to figure 9 it can be seen that the maximum mechanical power of the VIRYA-1.38 is about the same as that of the VIRYA-1.4. The slightly lower rotor diameter of the VIRYA-1.38 is compensated by the slightly higher maximum C_p . As less material is needed for the VIRYA-1.38, the VIRYA-1.38 seems a better choice than the VIRYA-1.4.

If the matching is good at a wind speed of 9 m/s and if the generator efficiency is about 50 % at maximum power, the maximum electrical power will be about 75 W. This is certainly acceptable and relatively much higher than the maximum power of 6 W of the VIRYA-1.04 or the VIRYA-0.98 which make use of a Nexus hub dynamo from a bicycle wheel. So it seems worthwhile to make a detailed drawing of the rotor and build a prototype and test this rotor with the generator on a modified head of the VIRYA-1.8. However, I won't build a prototype of the rotor.

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